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Analytical investigation of unsteady CuO nanofluid flow, heat and mass transfer between two parallel disks

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The heat transfer in the unsteady CuO nanofluid flow between two moving parallel disks has been investigated using analytical method called Galerkin Optimal Homotopy Asymptotic Method (GOHAM). The effect of Brownian motion on heat transfer enhancement has been shown. The analytical investigation is carried out for various governing parameters such as the squeeze parameter, Hartman number, Brownian motion and thermophoretic parameters. The results show that concentration is an increasing function of Brownian motion parameter while it is a decreasing function of the thermophoretic parameter.The comparison of obtained results with numerical solutions assures us about the validity and accuracy of the current study.

Keywords: Squeezing Flow, Nanofluid Flow, Heat Transfer Enhancement, GOHAM, CuO

Nanofluid, a name conceived by Choi, in Argonne National Laboratory to describe a fluid in which nanometer-sized particles are suspended. Nanoparticles have unique properties, such as large surface area to volume ratio, and lower kinematic energy which can be exploited in various applications. Nanoparticles are better stable when dispersed in base fluids, due to their large surface area and they are more stable when compared to micro fluids which lead to many practical problems. In recent years, nanofluids have attracted more and more attention[1,2].

In paper, by Azimi and Azimi[3], DTM have successfully applied to a non-linear MHD Jeffery Hamel problem with Graphene Oxide (GO) nanoparticle. The effects of graphene oxide solid volume fraction, Reynolds number, Hartman number and the angle between parallel plates on velocity components were investigated. Their results showed that the velocity profile is strongly influenced by solid volume fraction of GO nanoparticles.

The unsteady mixed convection squeezing flow of an incompressible graphene oxide water nanofluid between two vertical parallel planes is discussed in paper by Azimi and Riazi[4]. The buoyancy force due to thermal and molecular diffusion is taken as the source of the convective flow. They concluded that when Graphene oxide solid volume fraction increases, the rate of heat transfer increases. Eckert number has significant effect on temperature profile and it can increase the rate of heat transfer by increasing and their results showed that the temperature field T decreases by increasing the mixed convection parameter.

The squeezing flow between two parallel boundaries is an interesting topic of research due to its abundant applications. Examples of such flows are quite prevalent in polymer processing, compression and injection modeling. The lubrication system can be discussed through the squeezing flow. The initial work on the squeezing flow was investigated by Stefan[5].

Azimi and Riazi[6] used analytical method called Reconstruction of Variational Iteration Method (RVIM) in order to fine approximate solution for nanofluid squeezing flow and heat transfer between two moving parallel plates. They concluded that the

Nusselt number increases with increase of Eckert number and solid volume fraction of graphene oxide nanoparticles in water.

The effect of different types of nanoparticles (graphene oxide, aliminium oxide, titanium oxide, silver) on the Nusselt numberin unsteady squeezing flow between two moving parallel plates (which is filled with nanofluid) problem was investigated by Azimi and Mirzaei[7]. The results showed the nanoparticle type is an important factor in the cooling and heating processes and silver can cause most heat transfer enhancement rate. Velocity profiles for various moving number have been also obtained in their study.

In the heart of all the different engineering sciences, everything showed itself in the mathematical relation that most of these problems and phenomena are modeled by ordinary or partial differential equations. In most cases, scientific problems are inherently of nonlinearity that does not admit exact solution, so these equations should be solved using special techniques. Some of these methods are Homotopy Perturbation Method (HPM)[8], Reconstruction of Variational Iteration Method (RVIM)[9], Glerkin Optimal Homotopy Asymptotic Method (GOHAM)[10] and others[11,12].

In this study, the Galerkin Optimal Homotopy Asymptotic Method (GOHAM), is applied to find the semi-analytical solutions of nonlinear differential equations governing the problem of unsteady CuO nanofluid flow, heat and mass transfer between two moving disks. The effect of Brownian motion on nanoparticle concentration was also studied.

Mathematical Formulation

Figure 1 shows the geometry of the squeezing flow of an incompressible viscous MHD nanofluid between two circular plates separated by a distance $z=\pm l\left(1-\alpha t\right)^{1/2}=\pm h\left(t\right)$. A uniform magnetic field of strength $B\left(t\right)=B_0\left(1-\alpha t\right)^{-0.5}$ is applied perpendicular to the disks. The upper disk at $z=h\left(t\right)$ approaching the stationary lower disk with the velocity dh/dt. The flow is axisymmetric about $r=0$. The velocity components along the radial and axial directions are $u\left(r,z,t\right)$, $w\left(r,z,t\right)$, respectively. Now specify the basic equations for an unsteady axisymmetric flow and assume $v=\left[\left(u\left(r,z,t\right),0,w\left(r,z,t\right)\right)\right]$ thus, the unsteady mass and conservation Equations become:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \quad \text{... (1)}$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu_{nf}\left(\frac{\partial^2 u}{\partial r^2}+\frac{\partial^2 u}{\partial z^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right) \quad \text{...(2)}$$

$$\rho_{nf}\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu_{nf}\left(\frac{\partial^2 w}{\partial r^2}+\frac{\partial^2 w}{\partial z^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right) \quad \text{...(3)}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\frac{k_{nf}}{\left(\rho C_p\right)_{nf}}\left(\frac{\partial^2 T}{\partial r^2}+\frac{\partial^2 T}{\partial z^2}+\frac{1}{r}\frac{\partial^2 T}{\partial z^2}\right)+\tau\left[D_B\left(\frac{\partial C}{\partial r}\frac{\partial T}{\partial r}+\frac{\partial C}{\partial z}\frac{\partial T}{\partial z}\right)+\frac{D_T}{T_m}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right]\right] \quad \text{...(4)}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial r}+w\frac{\partial C}{\partial z}=D_B\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}+\frac{\partial^2 C}{\partial z^2}\right)+\frac{D_B}{T_m}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right) \quad \text{...(5)}$$

where u and w are the velocities in the r and z directions, respectively, $p$is pressure, $T$is temperature, C is the nanoparticle concentration, D_B is the Brownian motion coefficient, D_T is the termophoretic diffusion coefficient, T_m is the mean fluid temperature and k is the thermal conductivity. The last term in the energy equation is the total diffusion mass flux for nanoparticles, given as sum of two diffusion terms. τ is the dimensionless parameter that gives the ratio of

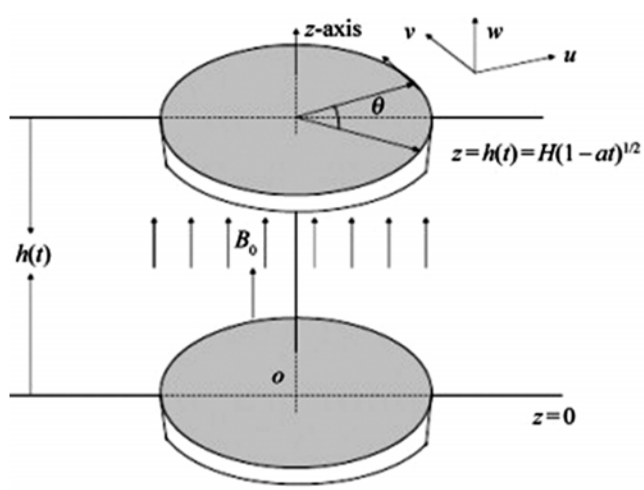

Fig.1 — Physical model.

effective heat capacity of the nanoparticle material to heat capacity of the fluid. Effective density (ρ_{nf}), the effective dynamic viscosity (μ_{nf}), effective heat capacity (C_{nf}) and the effective thermal conductivity k_{nf} of the nanofluid are defined as[8]:

$$\rho_{nf}=\rho_f(1-\varphi)+\rho_s\varphi_s$$
$$(\rho C_p)_{nf}=(\rho C_p)_f(1-\varphi)+(\rho C_p)_s$$
$$\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}}$$
$$\frac{k_{ns}}{k_f}=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+2\phi(k_f-k_s)}$$
$$\nu_{nf}=\frac{\mu_f}{\rho_{nf}} \quad ...(6)$$

The relevant boundary conditions for the problem are:

$$z=h(t)\rightarrow u=0,\ w=w_w=\frac{dh}{dt},\ T=T_H,\ C=C_h$$
$$z=0\rightarrow u=0,\ w=-\frac{w_0}{\sqrt{1-\alpha t}},\ T=T_w,\ ,C=C_w \quad ...(7)$$

By introducing following parameters, the above Equation can be easily simplified:

$$u=\frac{\alpha r}{\left[2(1-\alpha t)^{1/2}\right]}f'(\eta),\ w=-\frac{\alpha H}{\left[(1-\alpha t)^{1/2}\right]}f(\eta),\ \eta=\frac{z}{H\left[(1-\alpha t)^{1/2}\right]}$$
$$\theta=\frac{T-T_H}{T_H-T_H},\ B=\frac{B_0}{\left[(1-\alpha t)^{1/2}\right]},\ \varphi=\frac{C-C_h}{C_w-C_h} \quad ...(8)$$

The above parameters are substituted into Equations. (2) and (3). Then the pressure gradient is eliminated from the resulting Equations. We finally yield:

$$f^{(IV)}-S(\eta f'''+3f''-2ff''')-M^2f''=0 \quad ...(9)$$

Using equation (8), Equations (3) and (4) simplify to following equations:

$$\theta''+\Pr S(2f\theta'-\eta\theta')+\Pr Nb\theta'\varphi'+\Pr Nt\theta'^2=0 \quad ...(10)$$
$$\varphi''+LeS(2f\varphi'-\eta\varphi')+\frac{Nt}{Nb}\theta''=0 \quad ...(11)$$

With the following boundary conditions:

$$f(0)=0,\quad f'(0)=0,\quad f(1)=1, \quad ...(12)$$
$$f'(1)=0,\ \theta(0)=\varphi(0)=1,\ \theta(1)=\varphi(1)=0$$

where S is squeeze parameter, $\Pr$ is the Prandtl number, M is Hartman number, Nb is the Brownian motion parameter Nt is thermophoretic parameter and Le is the Lewis number which are defined as:

$$S=\frac{\alpha H^2}{2\nu_f},\ \Pr=\frac{\nu_f}{\alpha},\ M=\sqrt{\frac{\sigma B_0^2H^2}{\nu}},\ Le=\frac{\nu}{D_e}$$
$$Nb=\frac{(\rho c)_s D_B(C_w-C_h)}{(\rho c)_f\nu},\ Nt=\frac{(\rho c)_s D_T(T_w-T_h)}{(\rho c)_f T_m\nu} \quad ...(13)$$

It is important to note that $A>0$ indicates the suction of fluid from the lower disk while $A<0$ represents injection flow.

Solution Procedure

Following differential Equation is considered:

$$L(u(t))+N(u(t))+g(t)=0,\ B(u)=0 \quad ...(14)$$

where L is a linear operator, τ is an independent variable, $u(t)$ is an unknown function, $g(t)$ is a known function, $N(u(t))$ is a nonlinear operator and B is a boundary operator. By means of OHAM, one first constructs a set of Equations:

$$(1-p)\left[L(\varpi(\tau,p)+g(\tau))\right]-H(p)$$
$$\left[L(\varpi(\tau,p))+g(\tau)+N(\varpi(\tau,p))\right]B(\varpi(\tau,p))=0 \quad ...(15)$$

where $p\in[0,1]$ is an embedding parameter, $H(p)$ denotes a nonzero auxiliary function for $p\neq 0$ and $H(0)=0$, ϖ is an unknown function. Obviously, when $p=0$ and $p=1$, it holds that:

$$\varpi(\tau,0)=u_0(\tau),\ \varpi(\tau,1)=u(\tau) \quad ...(16)$$

Thus, as p increases from *0* to *1*, the solution $\varpi(\tau,p)$ varies from $u_0(\tau)$ to the solution $u(\tau)$, where $u_0(\tau)$ is obtained from Eq. (16) for $p=0$:

$$L(u_0(\tau))+g(\tau)=0,\ B(u_0)=0 \quad ...(17)$$

We choose the auxiliary function $H(p)$ in the form:

$$H(p)=p_1C_1+p_2C_2+ \quad ...(18)$$

where C_1, C_2, … are constants which can be determined later. Expanding $\phi(\tau, p)$ in a series with respect to p, one has:

$$\varpi(\tau, p, C_i) = u_0(\tau) + \sum_{k>1} u_k(\tau, C_i)\, p_k, \; i = 1, 2, \qquad \text{...(19)}$$

Substituting Equation.20 into Equation.16, collecting the same powers of p, and equating each coefficient of p to zero, we obtain set of differential equation with boundary conditions. Solving differential Equations by boundary conditions $u_0(\tau), u_1(\tau, C_1), u_2(\tau, C_2), \ldots$ are obtained. Generally speaking, the solution of Equation.15 can be determined approximately in the form:

$$\varpi(\tau, p, C_i) = u_0(\tau) + \sum_{k>1} u_k(\tau, C_i)\, p_k, \; i = 1, 2, \qquad \text{... (20)}$$

$$\tilde{u}(m) = u_0(\tau) + \sum_{k=1}^{m} u_k(\tau, C_i) \qquad \text{...(21)}$$

Note, that the last coefficient C_m can be function of τ. Substituting Equation.20 into Equation.14, there results the following residual:

$$R(\tau, C_i) = L\left(\tilde{u}^{(m)}(\tau, C_i)\right) + g(\tau) + N\left(\tilde{u}^{(m)}(\tau, C_i)\right) \qquad \text{... (22)}$$

If $R(\tau, C_i) = 0$ then $\tilde{u}^{(m)}(\tau, C_i)$ happens to be the exact solution. Generally, such a case will not arise for nonlinear problems, but we can minimize the functional by Galerkin method:

$$w_i = \frac{\partial R(\tau, C_1, C_2, \cdots, C_m)}{\partial C_i}, \; i = 1, 2, \ldots, m \qquad \text{... (23)}$$

The unknown constants $C_i\,(i = 1, 2, \ldots, m)$ can be identified from the conditions:

$$J(C_1, C_2) = \int_a^b w_i R(\tau, C_1, C_2, \ldots, C_m)\, d\tau = 0 \qquad \text{... (24)}$$

where a and b are two values, depending on the given problem. With these constants, the approximate solution (of order m) (Eq. (24)) is well determined. It can be observed that the method proposed in this work generalizes these two methods using the special (more general) auxiliary function $H(p)$.

Results and Discussion

In this section, we will discuss about the obtained results of squeezing CuO-Water nanofluid flow between parallel disks problem for various solid volume fraction and moving parameter. The physical properties of Copper Oxide- Water nanofluid can be found in Table.1.

Figure 2 shows the effect of the squeeze number on the temperature profile in the case of $H = 2, \Pr = 7, Ec = 0.05, N_t = 0.15, N_b = 0.5, Sc = 3$ .As it can be seen in Fig.2 the non-dimensional temperature is direct function of squeezing parameter. In the other words, an increase in the squeeze number can be related with the decrease in the kinematic viscosity, an increase in the distance between the plates and an increase in the speed at which the plates move. Thermal boundary layer thickness increases as the squeeze number increases.

It is important to note that parameters N_b and N_t characterize the strengths of Brownian motion and thermophoresis effects.

Table1 — Thermo physical properties of water and CuO nanoparticle

	$\rho(kg/m^3)$	$C_p(j/kgk)$	$k(W/m.k)$
Pure water	997.1	4179	0.613
Copper Oxide	8933	385	401

... (22)

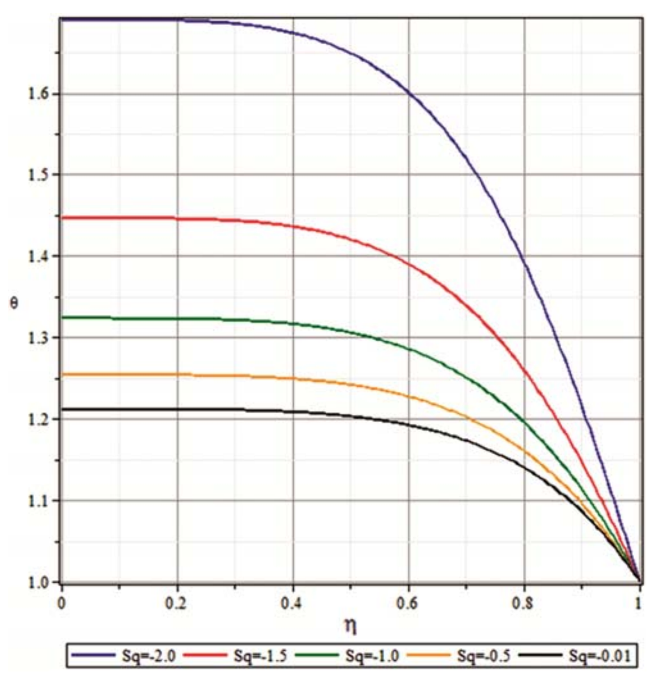

Fig.2 — Effect of squeeze parameter on temperature.

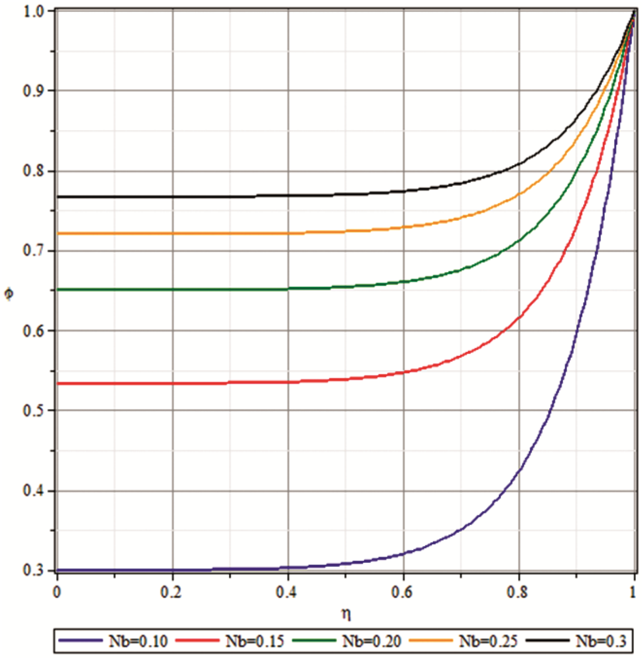

Fig.3 — Effect of Brownian motion parameter on temperature.

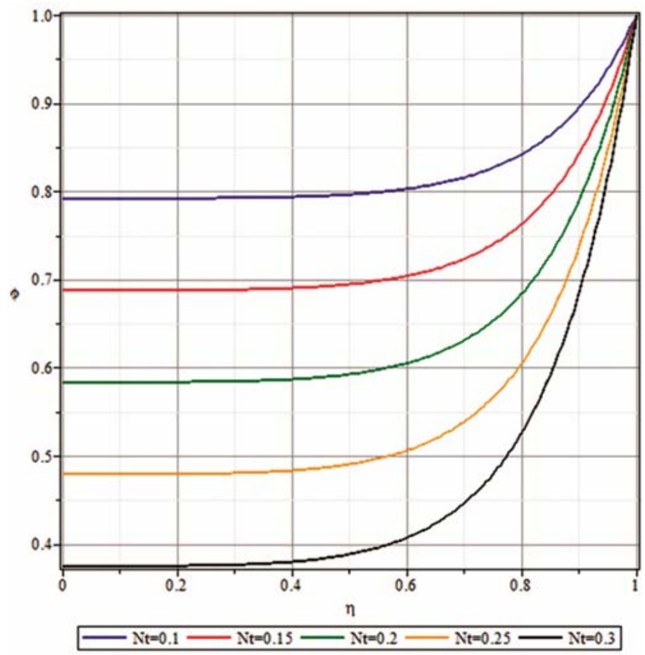

Fig.4 — Effect of thermophoretic parameter on concentration.

Figure 3 shows the effect of squeezing parameter on non-dimensional concentration profile in the case of $H=4,\Pr=7,Ec=0.1,N_t=0.25,Sq=0,5,Sc=3$.

As it can be seen in Fig.3, an increase in N_b effectively increases the nanoparticles concentration. This increase is due to the effective movement of nanoparticles from the upper disk to the fluid.

Figure 4 shows the influence of thermophoretic parameter on concentration function in the case of $H=3,\Pr=7,Ec=0.1,N_b=0.15,Sq=0.5,Sc=3$ .The non-dimensional concentration function decreases by increasing the thermophoretic parameter. From the physical point of view, an increase in the thermophoretic effect generates the larger mass flux due to temperature gradient which decreases the concentration.

Figure 5 shows the effect of Hartman number on velocity profile when $Sq=1$.

It is important to note that the influence of external magnetic field is to decrease the value of the velocity magnitude throughout the enclosure because the presence of magnetic field introduces a force called the Lorentz force, which acts against the flow, if the magnetic field is applied in the normal direction. The figure also gives information about the accuracy of our solution by presenting a comparison between analytical solutions obtained by GOHAM and numerical ones achieved by forth order Runge Kutta method. As it can be illustrated in Fig.5, analytical solutions have good agreement with numerical ones. This figure assures us about the accuracy and validity of our approximate analytical solution.

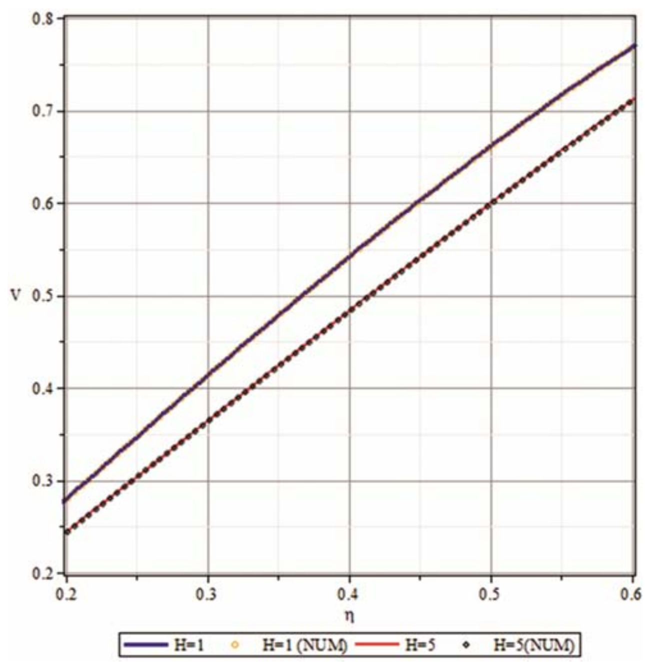

Fig.5 — Influence of H on velocity.

Conclusion

In this study, unsteady MHD nanofluid flow and heat transfer between parallel disks are investigated.

GOHAM is used to solve the governing equations. The effect of the squeeze number on heat and mass transfer are investigated. The results show that the higher values of heat transfer enhancement are obtained when Brownian motion increases. Also, it can be found that concentration is an increasing function of Brownian motion parameter while it is a decreasing function of the thermophoretic parameter. Velocity is decreasing function of magnetic effect. The comparison between GOHAM and Runge Kutta method assures us about the validity and accuracy of our solution.

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