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MHD boundary layer flow and heat transfer along an infinite porous hot horizontal continuous moving plate

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Analysis is to study the two-dimensional magnetohydrodynamic (MHD) boundary layer flow and heat transfer along an infinite porous hot horizontal continuous moving plate. The governing partial differential equations are transformed into self-similar ordinary differential equations using similarity transformations before being solved analytically. Numerical results for the dimensionless velocity profiles, the temperature profiles, the skin friction coefficient and the Nusselt number are present graphically and discuss briefly for various physical parameters, such as magnetic parameter M, plate velocity α, Prandtl number Pr, Eckert number Ec and heat source/sink parameter S. It has been found that these parameters have significantly effects on the flow and heat transfer.

Keywords: MHD, Boundary layer flow, Moving plate, Laminar flow, Heat transfer.

The problem concerning the flow of an electrically conducting fluid past a continuous moving surface in a quiescent fluid has copious applications in many engineering processes, such as materials manufactured by polymer extrusion, artificial fibers, hot rolling, wire drawing, glass fiber, metal extrusion and metal spinning, cooling of metallic sheets. In addition, they also find very useful applications in the design of insulation systems employing porous media. In all these cases, a study of the flow field and the heat transfer can be of significant importance since the quality of the final product depends to a large extend on the skin friction coefficient and the surface heat transfer rate. In view of these applications the classical problem was introduced by Blasius[1] where he considered the boundary layer flow on a fixed flat plate. Different from Blasius[1], Sakiadis[2] initiated the study of boundary layer flow over a continuous solid surface moving at a constant speed through an otherwise quiescent fluid environment. Later, Crane[3] extended this idea for the two-dimensional problem where the velocity is proportional to the distance from the plate. Subsequently, several investigators Tsou *et al.*[4], Vleggaar[5], Banks[6], Jeng*et al.*[7], Vajravelu[8], Char *et al.*[9], Buhler *et al.*[10], Takhar *et al.*[11], Pop *et al.*[12], Andersson[13], Lin *et al.*[14], Hassanien[15], Seddeek[16], Muondwal *et al.*[17], Agarwal *et al.*[18] have considered various aspects of this problem, such as the effect of mass transfer, wall temperature, variable fluid properties and magnetic field.

Motivated by works mentioned above and practical applications, the main concern of the present paper is to study the two-dimensional magnetohydrodynamic (MHD) boundary layer flow and heat transfer along an infinite porous hot horizontal continuous moving plate. Using similarity transformation, the governing partial differential equations are transformed into a set of self-similar ordinary differential equations, which are then solved analytically. The numerical results are plotted in some figures and the variations in physical characteristics of the flow dynamics and heat transfer for several parameters involved in the equations are discussed.

Problem formulation

Let us consider two dimensional laminar steady boundary layer flows and heat transfer of a viscous incompressible electrically conducting fluid along an infinite hot continuous moving flat plate in the presence of constant section at the surface, constant free stream, U_∞ and heat generation (or adsorption). It is assumed that external fluid owing polarization of charges and Hall-effect are neglected. The plate is moving in flow direction with constant velocity U_w and maintain at constant temperature, where x-axis is along the flow and y-axis is perpendicular to it, the applied magnetic field B_0 is transversely to x-axis. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. Under the usual boundary layer approximations, the governing equation of continuity, momentum and energy (Pai[19], Schlichting[20], Bansal[21]) under the influence of externally imposed transverse magnetic field (Jeffery[22], Bansal[23]) are:

$$\frac{\partial v^*}{\partial y^*} = 0 \Rightarrow v^* = -v_0 \text{ (constant)}, v_0 > 0 \quad \dots (1)$$

$$\rho\left(-v_0 \frac{\partial u^*}{\partial y^*}\right) = \mu \frac{\partial^2 u^*}{\partial y^{*2}} - \sigma_e B_0^2 u^* \quad \dots (2)$$

$$\rho C_p\left(-v_0\frac{\partial T^*}{\partial y^*}\right)=\kappa\frac{\partial^2 T^*}{\partial y^{*2}}+\mu\left(\frac{\partial u^*}{\partial y^*}\right)^2+Q(T^*-T_\infty) \quad \dots (3)$$

Along with the appropriate boundary conditions for the problem are given by:

$$y=0: u^*=U_w, v^*=-v_0, T^*=T_w$$
$$y\to\infty: u^*\to U_\infty, T^*\to T_\infty \quad \dots (4)$$

Analysis

The momentum and energy equations can be transformed into the corresponding ordinary differential equation by using the following non-dimensional parameters:

$$y=y^*\frac{v_0}{\nu},\ u=\frac{u^*}{U_\infty},\ \theta=\frac{T^*-T_\infty}{T_w-T_\infty},\ \alpha=\frac{U_w}{U_\infty},\ Pr=\frac{\mu C_p}{\kappa},$$
$$S=\frac{Qv^2}{\kappa v_0^2},\ Ec=\frac{U_\infty^2}{C_p(T-T_\infty)} \quad \dots (5)$$

The transformed ordinary differential equations are:

$$u''+u'=Mu \quad \dots (6)$$

$$\theta''+Pr\,\theta'+S\theta=-EcPr(u')^2 \quad \dots (7)$$

The transformed boundary conditions are:

$$u(0)=\alpha, \theta(0)=1 \text{ and } u(\infty)\to 1, \theta(\infty)\to 0. \quad \dots (8)$$

where prime denotes differentiation with respect to y, $M=\frac{\sigma_e B_0^2 \upsilon}{\rho v_0^2}$ is the dimensionless magnetic parameter, $Pr=\frac{\mu c_p}{\kappa}$ is the Prandtl number, α is the velocity of the plate, $S=\frac{Qv^2}{\kappa v_0^2}$ is the heat source (S<0) or sink (S>0) parameter and $Ec=\frac{U_\infty^2}{C_p(T-T_\infty)}$ is the Eckert number.

Solving equations (6) and (7) subject to boundary condition (8), we have:

$$u(y)=A_1e^{n_1y}+A_2e^{n_2y} \quad \dots (9)$$

$$\theta(y)=-A_3e^{2n_1y}-A_4e^{2n_2y}-A_5e^{(n_1+n_2)y}+A_6e^{n_3y}+A_7e^{n_4y} \quad \dots (10)$$

where the constants A_i (i = 1-7) and n_j(j =1-4) are given in "Appendix".

Skin friction and Nusselt number

Having known the velocity and temperature fields we can now obtain the expression for the dimensionless Skin friction C_f and Nusselt number Nu, which are given by

$$C_f=\left(\frac{\partial u}{\partial y}\right)_{y=0}=A_1n_1+A_2n_2 \quad \dots (11)$$

And

$$Nu=-\left(\frac{\partial\theta}{\partial y}\right)_{y=0}=-2n_1A_3-2n_2A_4-A_5(n_1+n_2)+A_6n_3+A_7n_4 \quad \dots (12)$$

The numerical results of C_f and Nu are shown in Figs 7 and 8, respectively.

Results and Discussion

In order to discuss the effect of various parameters on the velocity field, thermal boundary layer, shearing stress and coefficient of rate of heat on the wall, the numerical computation of the solution, obtained in preceding section, have been carried out and they are represented in Figs 1-8.

Figure 1 illustrates the effect of plate velocity α on velocity profile for M = 0. We infer from this figure that velocity increases considerably as plate velocity α increases. Moreover, when $0<\alpha<1$ (that is when plate velocity is less than free stream velocity), the profile for u is concave down, and when $\alpha>1$ (that is when free stream velocity less than plat velocity), the profile for u is concave up.

The effect of magnetic parameter M on the velocity near the plate is presented for α = 0 in Fig. 2. Application of a transverse magnetic field to an electrically conducting flow gives rise to a resistive type of force called Lorenz force. This force has the tendency to slow down the motion of the fluid in the boundary layer. As expected, as M increases the

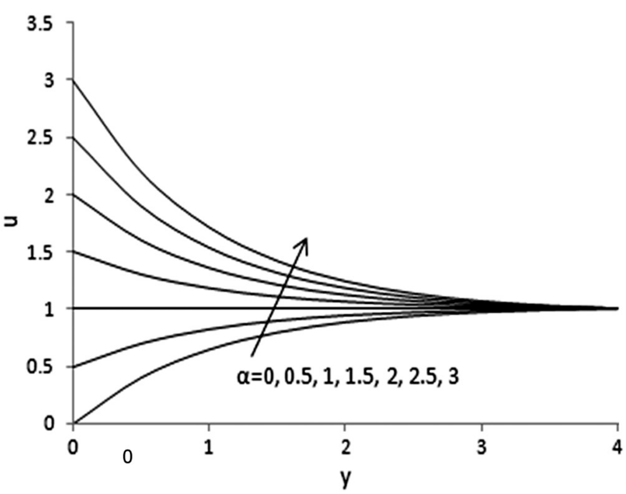

Fig. 1 — Effect of α on the velocity profile for M=0.

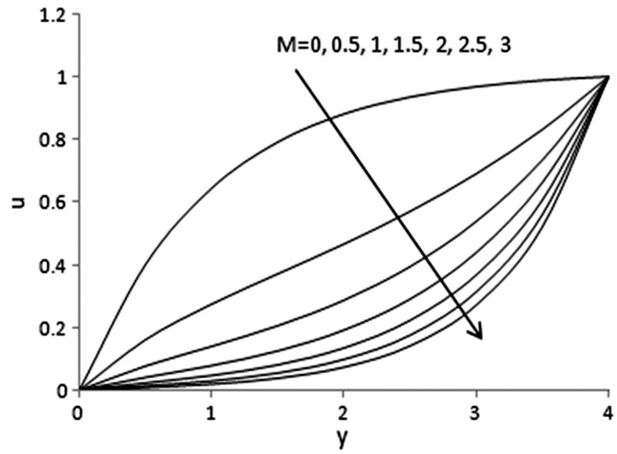

Fig. 2 — Effect of M on the velocity profile for α=0.

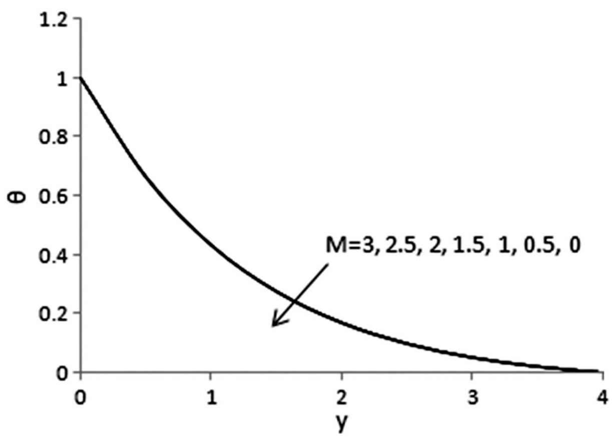

Fig. 3 — Effect of M on the temperature profile for α= 0.5, S=0.2, Ec=0.01 and Pr=1.

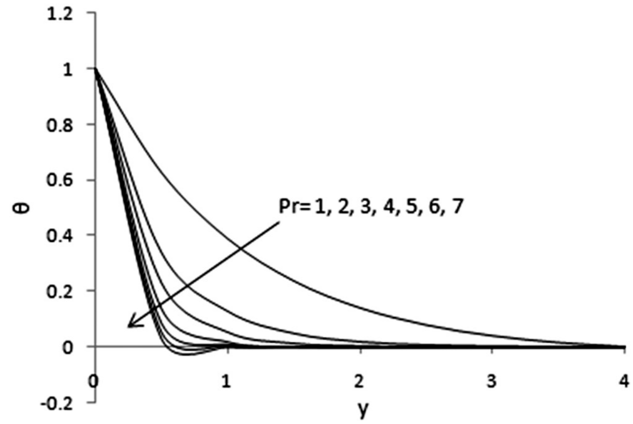

Fig. 4 — Effect of Pr on the temperature profile for α= 0.5, S=0.1, Ec=0.01 and M=1.

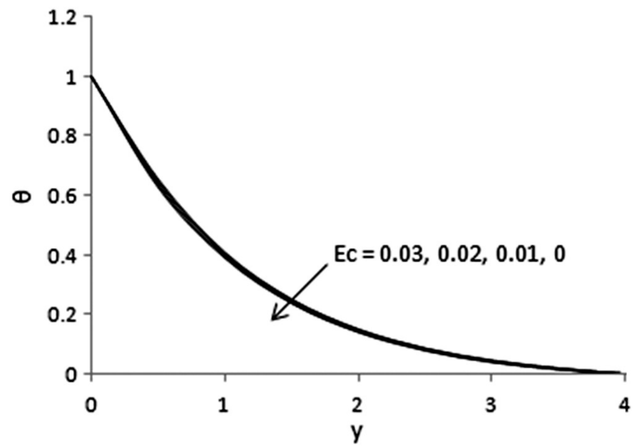

Fig. 5 — Effect of Ec on the temperature profile for α= 0.5, S=0.1 Pr=1, Ec=0.01 and M=1.

velocity decreases. On the contrary, Fig. 3 shows that the temperature is not much influenced by the magnetic parameter (M).

Figure 4 illustrates the effect of Prandtl number (Pr) on the temperature profiles. We infer from this figure that temperature decreases with an increase in the Prandtl number, which implies viscous boundary layer is thicker than the thermal boundary layer. From this plot, it is evident that large values of Prandtl number result in thinning of the thermal boundary layer. In this case, temperature asymptotically approaches to zero in free stream region. Since the momentum equation is independent ofθ, so no effect of Pr on the velocity field is observed.

Figure 5 is plotted for temperature profile for different values of Eckert number (Ec). We observe that the effect of increasing values of Eckert number is to enhance the temperature at a point. Physically, it means that the heat energy is stored in the fluid due to the frictional heating.

Figure 6 shows the effect of heat source/sink parameter (S) on the temperature profile for given values of Prandtl number (Pr), Eckert number (Ec) plate velocity α and magnetic parameter M. From this plot, it is observed that the effect of increasing values of heat source/sink parameter is to increases the temperature.

The local skin friction coefficient against plate velocity (α) for various values of magnetic parameter (M), is illustrated in Fig. 7. The local skin friction coefficient decreases with an increase in magnetic parameter.

Figure 8, which is a representation of the local dimensionless coefficient of heat transfer $-\theta'(0)$, knows as the Nusselt numberfor various values of heat source/sink parameter (S) versus magnetic parameter (M) for the given values of Prandtl number (Pr), Eckert number (Ec) and plate velocity α. We observe from this

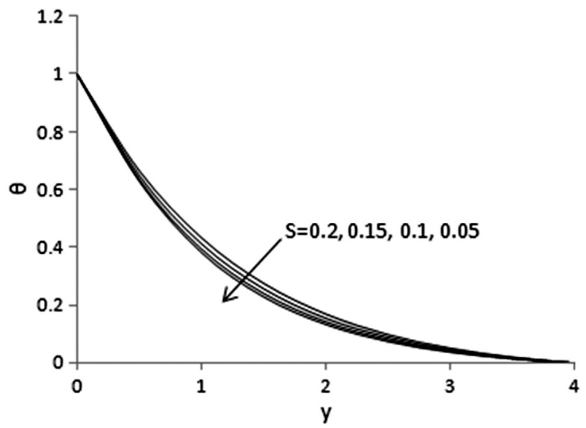

Fig. 6 — Effect of S on the temperature profile for α= 0.5, Pr=1, Ec=0.01 and M=1.

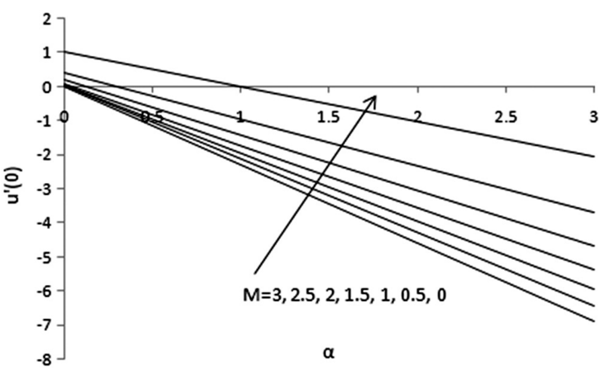

Fig. 7 — Skin friction against α for different values of M.

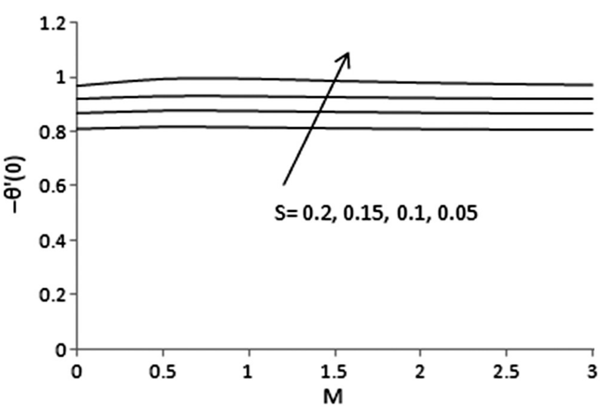

Fig. 8 — Nusselt number against M for various values of S when Pr = 1.0, α = 0.5 and Ec = 0.01.

figure that the rate of heat transfer decreases with an increase in heat source/sink parameter.

Conclusion

A mathematical model has been presented for the MHD boundary layer flow and heat transfer along an infinite porous hot horizontal continuous moving plate. The governing partial differential equations are converted into ordinary differential equations by using similarity transformations. The effect of several parameters controlling the velocity and temperature profiles are shown graphically and discussed briefly. The influence of the parameters α, M, Pr, Ec and S on dimensionless velocity and temperature profiles were examined. The main physical results of the study may be summarized as follows.

(i) The mean velocity profile is concave down for $0 < \alpha < 1$ and is concave up for $\alpha > 1$.
(ii) As the magnetic parameter increases, we can find the velocity profile decreases in the flow region. Thus, we conclude that we can control the velocity field and temperature by introducing magnetic field.
(iii) The boundary layer is highly influenced by the Prandtl number. The effect of Prandtl number is to decrease the thermal boundary layer thickness.
(iv) Eckert number has significant effect on the boundary layer growth.
(v) The effect of heat source/sink parameter is to increase the thermal boundary layer thickness.
(vi) The Skin friction coefficient decrease as magnetic parameter increases.
(vii) The Nusselt number decrease as heat source/sink parameter increases.

Appendix

$n_1 = \frac{-1+\sqrt{1+4M}}{2}$,

$n_2 = \frac{-1-\sqrt{1+4M}}{2}$,

$n_3 = \frac{-Pr+\sqrt{Pr^2-4S}}{2}$,

$n_4 = \frac{-Pr-\sqrt{Pr^2-4S}}{2}$,

$A_1 = \frac{1-\alpha e^{4n_2}}{e^{4n_1}-e^{4n_2}}$,

$A_3 = \frac{\alpha e^{4n_1}-1}{e^{4n_1}-e^{4n_2}}$,

$A_3 = \frac{EcPrA_1^2 n_1^2}{4n_1^2+2Prn_1+S}$,

$A_4 = \frac{EcPrA_2^2 n_2^2}{4n_2^2+2Prn_2+S}$,

$A_5 = \frac{2PrEcA_1A_2n_1n_2}{(n_1+n_2)^2+Pr(n_1+n_2)+S}$,

$A_6 = \frac{e^{4n_4}+(A_3+A_4+A_5)e^{4n_4}-A_3e^{8n_1}-A_4e^{8n_2}-A_5e^{4(n_1+n_2)}}{e^{4n_4}-e^{4n_3}}$,

$A_7 = 1 + A_3 + A_4 + A_5 - A_6$.

Nomenclature

B_0 Constant applied magnetic field, [Wb m^{-2}]
C_f Skin friction coefficient, [−]
Ec Eckert number ($= U_\infty^2/C_p(T - T_\infty)$), [−]

M Magnetic parameter $(= \sigma_e B_0^2/\rho c)$, $[-]$
Nu Nusselt number, $[-]$
Pr Prandtl number $(= \mu C_p/\kappa)$, $[-]$
Q Volumetric rate of heat generation, [K]
S Heat source/sink parameter $(= Qv^2/\kappa v_0^2)$, $[-]$
T Temperature of the fluid, [K]
u, v Velocity component of the fluid along the x and y directions, respectively, $[\mathrm{m\ s^{-1}}]$
x, y Cartesian coordinates along the surface and normal to it, respectively, [m]

Greek symbols

α Dimensionless velocity of the plate, $[= U_w/U_\infty]$
ρ Density of the fluid, $[\mathrm{Kg\ m^{-3}}]$
μ Viscosity of the fluid, $[\mathrm{Kg\ m\ s^{-1}}]$
$\boldsymbol{\sigma_e}$ Electrical conductivity, $[\mathrm{m^2 s^{-1}}]$
κ Thermal conductivity, $[\mathrm{W\ m^{-2} K^{-4}}]$
υ Kinematic viscosity, $[\mathrm{m^2 s^{-1}}]$
θ Dimensionless temperature, $[= (T - T_\infty)/(T_w - T_\infty)]$

Superscript

′Derivative with respect to y

Subscripts

w Properties at the plate
∞ Free stream condition

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