

Mass transfer characteristics of activated sludge process at high MLSS and power consumption

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Mass transfer coefficients (k_{La}) of activated sludge process are measured over a wide range of mixed liquor suspended solids (MLSS). Total three sets of experiments have been used (with steady state dissolved oxygen (DO) of 1.0, 2.5 mg L⁻¹ with varying agitator speeds and constant agitator speeds with varying-unsteady state DO beginning from 1.0 mg L⁻¹). The results show rise in k_{La} with MLSS. The rise in DO is observed when the agitator speeds are kept constant throughout because of reduction in chemical oxygen demand (COD). Attempts have also been made to treat the entrapped/adsorbed COD on the biomass.

Keywords: Aerobic activated sludge treatment, Mass transfer coefficients, Mixed liquor suspended solids, High speed agitation, Kinetics, Power requirement

The oxygen transfer is necessary for maintaining dissolved oxygen (DO) level because it governs the removal of pollutants in the effluent by biodegradation with the help of micro-organisms. The biological treatments have become more acceptable because of effective removal of organic compounds from wastewaters, easy handling of huge amount of effluents, less cost, abundant organisms' availability, effective coagulation for removing the non-settleable colloidal solids, simplicity of designing and treatment to a high degree of reduction to meet the pollution limits¹⁻¹⁰.

The waste sludge generated at the end of treatment is utilized for the treatment again and practically no external input is required. The feed sludge is enriched with broad-spectrum of microbes well acclimatized with the effluent. As the substrate in effluent being their food for survival, they digest the substrate responsible for pollution. Additionally, their high population density enhances the treatment efficiently. It is to be noted that the presence of variety of microbes in the sludge readily treats the generation of toxic intermediates. Further, the open system adds the atmospheric microbes responsible for substrate degradation automatically. In short, the capability of mixed culture present in waste sludge is more advantageous than the utilization of pure cultures¹¹⁻¹⁴.

The stirred/aeration tank bio-reactors are more popular due to their simplicity, capacity of handling a

huge quantity of effluent and reducing pollution level as per standards. At present, for any capacity of aeration tank (bioreactor), the residence time is of the order of 24-100 h. As a result the capital costs are high due to the high capacity tanks. Hence efforts are required to enhance the rates of biological oxidation. One such possibility is to operate the biological oxidation with high mixed liquor suspended solids (MLSS).

Oxygen mass transfer studies related to high MLSS aerobic membrane bioreactors have been reported in the literature¹⁵⁻¹⁹. The kinetics of oxidation and oxygen transfer strongly depend upon the nature of wastewater, the micro-organisms, MLSS concentration, agitation speeds, temperature, etc^{8,20-25}.

It has been reported⁶ that the intrinsic rate of bio-reaction increases with an increase in the MLSS. However, as the MLSS increases the viscosity of suspension increases. As a result, under otherwise identical conditions, the values of k_{La} decrease with an increase in MLSS. Therefore, it is desirable to understand the effect of MLSS as well as the mass transfer. Therefore, in the present work an attempt has been made to make these measurements. In the second part of this study, the effect of power consumption on the rates of biological oxidation has been investigated. The first motivation is to increase the mass transfer rate at high MLSS which is needed

for enhanced rates of bio-oxidation at high MLSS. The second motivation addresses one very important issue in activated sludge process. It is known that, when sludge is added to the effluent, there is sudden reduction in the values of chemical oxygen demand (COD). This is because, some COD gets occluded 'in' the biomass and some gets adsorbed 'on' the biomass. Such an occluded and adsorbed COD may not get oxidised in the power consumption range employed in the industry. Therefore, it was thought desirable to investigate the effects of power consumption on the overall extent of COD reduction.

Theory of mass transfer, kinetics and power consumption in biological oxidation: Mass transfer (oxygen transfer)^{7,11,26}

At unsteady state

From material balance (law of conservation of mass),

$$\text{Accumulation} = \text{Input} + \text{Output} \quad \dots (1a)$$

$$\text{O}_2 \text{ accumulation rate } (dC/dt) = \text{O}_2 \text{ transfer rate (OTR)} + \text{O}_2 \text{ utilisation rate (OUR)} \dots (1b)$$

$$dC/dt = \text{OTR} - \text{OUR} \quad \dots (1c)$$

$$dC/dt = k_L a (C^* - C) - k_0 CX \quad \dots (2)$$

where C is DO concentration (mg L^{-1}), t is time (s), $k_L a$ is O_2 transfer coefficient (s^{-1}), k_0 is O_2 utilization rate constant ($\text{L}(\text{mg s}^{-1}) X$ is MLSS concentration (mg L^{-1}).

As $dC/dt \ll \text{OUR}$, then $\text{OTR} \approx \text{OUR}$

At steady state

At steady state, accumulation, $dC/dt=0$, hence

$$\text{OTR} = \text{OUR} \quad \dots (3)$$

$$k_L a = k_0 CX / (C^* - C) \quad \dots (4)$$

Kinetics of COD reduction

A modified Monod type model reported in literature^{2,8} describes the dependence of the rate of substrate consumption on DO level. The substrate consumption rate can be given as:^{2,8}

$$-dS/dt = a(S/S_0)^b (C/(c+C))^d X^e \quad \dots (5)$$

where S is substrate/COD concentration (mg L^{-1}), a is reaction rate constant (s^{-1}), b is order of MLSS concentration (dimensionless), c is DO level constant (mg L^{-1}), d is order of DO level (dimensionless) and e is order of MLSS concentration (dimensionless). Equation (5) particularly incorporates the effect of the initial COD (S_0) of the system, the effect of DO and MLSS of the system, on substrate consumption rate. As the term $[C/(c+C)]^d \approx \text{constant}$, Eq. (5) results as:

$$-dS/dt = a'(S/S_0)^b X^e \quad \dots (6)$$

where $a' = a[C/(c+C)]^d$, modified reaction rate constant (s^{-1}).

Experimental results can be fitted to Eq. (6) to find the parameters a , b , c , d and e . The rates of oxidation of the constituents decide the overall rate of oxidation for aerobic biological treatment. The treatment rates can be expressed in many ways as a function of instantaneous substrate concentration based upon the nature of substrates having different degradation profiles and concentration of micro-organisms. In the literature a number of such models are available²⁷⁻³⁰. The total rate of substrate consumption can be obtained for growth of bacteria metabolizing one compound using other low concentration compounds. Monod type model is wide accepted by many researchers because of its simplicity^{2,8}.

Power consumption

Mechanical agitator power consumption for stirred tank batch reactors are calculated by knowing power number (N_p) and the Reynolds number (N_{Re}). The definite correlation between shape factors etc. is available in literature at N_{Re} values. Relation between the power requirement and N_{Re} can be written as:

$$N_{Re} = D^2 N_p / \mu_l \quad \dots (7)$$

where D is impellor diameter (m), N is agitation speed in revolutions per second (s^{-1}), ρ_l is liquid density (kg m^{-3}) and μ_l is liquid viscosity (mN s m^{-2}). The relation for theoretical power requirement (P_{th}) in Watts can be written as:

$$P_{th} = N_p (N^3 D^5 \rho_l) \quad \dots (8)$$

The designed power is normally 30% extra to overcome losses in power transmission (gear box, electrical motor etc). For pitched blade turbine agitators, with $N_{Re} > 10000$, $N_p=1.5$, and thus, power requirement can be calculated³¹⁻³³.

Correlation between oxygen mass transfer and kinetics

Steady state oxygen mass transfer

Oxygen consumption is same and $C=C_s$ with reference to Eqs. (3) and (4) for steady state. As OUR is directly proportional to substrate utilization (COD reduction) rate (dS/dt), OUR (and O_2 utilization rate coefficient (k_0)) values were calculated using dS/dt values by Monod kinetics and relation between (S_0-S) with time. As $\text{OUR} = \text{OTR}$, $k_L a$ values were also predicted. Mathematically it can be written as:

$$k_0 CX = -KdS / dt \quad \dots (9)$$

where, K is constant of proportionality and is assumed to be 1.25 to take care of additional consumption of O₂ by the simultaneous side reactions (like oxidation of ammonia, etc) difficult to define.^{1,22,26} Eq. (9) is further modified and correlated as:

$$k_0 = K a'(S/S_0)^b X^e \quad \dots (10)$$

$$k_L \underline{a} = [K a'(S/S_0)^b X^e / (C^* - C_s)] \quad \dots (11)$$

Thus, $k_L \underline{a}$ and k_0 can be predicted for different MLSS and DO.

Unsteady state oxygen mass transfer

During unsteady state DO, the agitation speed is constant, i.e., O₂ transfer rate 'OTR' is constant but O₂ utilization rate 'OUR' is varying due to variation in COD. As a result DO level increases constantly and reaches to a steady state in due course. In other words, 'OUR' is proportional to rate of COD reduction. Using and rearranging Eqs. (2) and (6) we can get:

$$k_0 = K a'(S/S_0)^b X^e / CX \quad \dots (12)$$

$$k_L \underline{a} = [dC/dt + K a'(S/S_0)^b X^e / (C^* - C)] \quad \dots (13)$$

As $dC/dt \ll$ OTR, then

$$k_L \underline{a} \approx K a'(S/S_0)^b X^e / (C^* - C) \quad \dots (13a)$$

Experimental Section

The primary treated wastewater from an agrochemicals plant was used for the experiments. The wastewater contained many complex organic/inorganic compounds; having COD of 1200 mgL⁻¹ ($\pm 10\%$), high TDS, chlorides and salinity of 13%. The pH of primarily treated wastewater was in the range of 6-8 and biochemical oxygen demand (BOD) varied from 400 to 500 mg L⁻¹. The COD was considered as an overall measure of substrate in the wastewater. Activated sludge used for the treatability study was collected from the effluent treatment plant of the industry. Analysis of primary treated wastewater is given in Table 1.

The system being aerobic suspended growth, the required COD: N: P is 100:10:1^{13,27}. The F/M ratio was within the range of 0.03-0.2 (i.e. starving condition) and mixed liquor volatile suspended solids (MLVSS)/MLSS ratio of the activated sludge at the beginning of all the experiments was 0.37.

The treatability experiments were performed in batch mode. Specially fabricated 2L cylindrical

baffled glass reactor having 1.7L effluent loading was used. The reactor had four standard baffles of 10 mm width. Oxygen mass transfer, mixing and cell suspension were provided by means of impeller agitation at bottom with a least possible clearance to avoid dead zones. A Remi make 50 Volt DC motor coupled with a speed regulator equipped with digital speed display was used for variable speed agitation to achieve required DO levels. The system had pitched blade turbine impeller as shown in Fig. 1.

Rotation of the impeller gave an up-flow to the content at the time of experiments and baffles avoided vortex formation giving proper mixing. The details of the system geometry are given in Table 2 (Refs 2,8).

The experiments were carried out at temperature of 30-32°C in 3 batch reactors simultaneously at different MLSS levels with same DO levels. The experiments of first two sets were performed for 60 h where as the third set operated for 41 h. Samples were withdrawn from the reactor at a fixed time interval of 8 h. DO was measured using Digital DO Meter (HACH, USA). Other parameters, such as colour, suspended solids (SS), chemical oxygen demand

Table 1 — Primary treated wastewater analysis.

Parameters	Value
pH	6.5-8.0
Salinity (%)	13.0
SS (mg L ⁻¹)	400
TDS (mg L ⁻¹)	15000
BOD (mg L ⁻¹)	500
COD (mg L ⁻¹)	1600
Ammonical N ₂ (mg L ⁻¹)	30
Cl ⁻ (mg L ⁻¹)	15000
Phenol (mg L ⁻¹)	8.0
Cyanide (mg L ⁻¹)	0.05
KCl (mg L ⁻¹)	500
Hexane, Toluene, etc. (mg L ⁻¹)	Traces

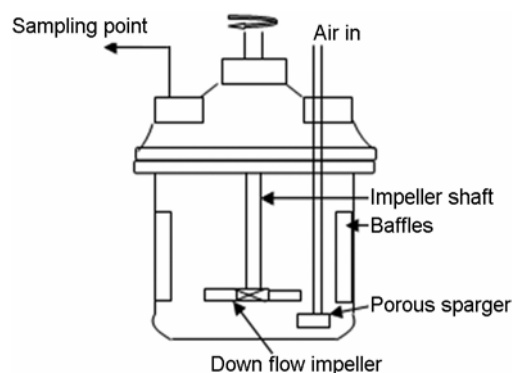


Fig. 1 — Schematic diagram of reactor.

Table 2 — Details of system geometry.

Design details	Parameters
Reactor specifications	2 L capacity glass reactor, 130 mm Ø Baffles, 04 nos. = 10 mm
Working volume	1.7 L
Impeller system	Pitched blade turbine type– Up flow
Design details of impeller	Ø = 60 mm, Width = 15 mm Number of blades = 06

Table 3 — Nutrient analysis of effluent and make of nutrients

Parameters (mg L ⁻¹)	COD	TN	TP
	1174		
Required		117.4	11.74
Present		58.00	14.30
Added 127.15 mg L ⁻¹ of 46.66% urea		59.33	Adequate

(COD), chlorides (Cl⁻), were measured using DR-2800 spectrophotometer (HACH, USA) and total nitrogen (TN) was measured using Schimadzu-10000 online TOC/N meter. The parameters of wastewater are analyzed by using standard methods³⁴.

Results and Discussion

Nutrient analysis and makeup

Table 3 shows the effluent analysis indicating nutrient deficiency particularly; total nitrogen (TN) and required make-up amount of TN for aerobic suspended growth process, whereas the amount of total phosphorous (TP) was sufficient. As per literature, an aerobic suspended growth activated sludge process requires minimum COD: TN: TP = 100:10:1, the nutrient make up was done accordingly by adding deficit nitrogen (TN) as urea. However, TP was adequate in the primary treated effluent which was to be treated further²¹.

Oxygen accumulation-unsteady state DO

During unsteady state aeration, the DO level is changing with respect to time as under:

- (i) As the aeration is stopped (by stopping air sparging and/or agitation), the DO level constantly decreases due to consumption of dissolved oxygen by the microbes in the effluent and becomes zero. As no oxygen is available in the bulk due to reduced solubility, the aerobic microbes present at the surface survive may remain active.
- (ii) If the aeration is continuous (by virtue of air sparging and/or appropriate agitation), the DO level continuously rises as a result of increased difference between oxygen transfer, and reduction in OUR is due to reduction in substrate

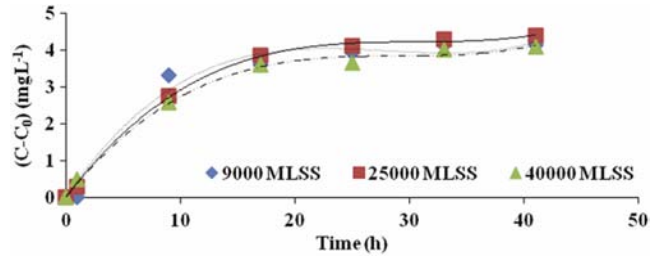


Fig. 2 — DO (C-C₀) vs. time (t-t₀), (for unsteady state condition).

Table 4 — Summary of Monod type kinetic parameters

Parameters	a' (h ⁻¹)	b	e	R ²
Set 1	-0.0929	1.9084	0.6394	0.9434
Set 2	-0.1044	2.02	0.633	0.977
Set 3	-0.09	1.9	0.64	0.981

concentration (COD) because of efficient microbial activity. Moreover, reduction in substrate concentration raises oxygen transfer. The rate of change of DO (rate of change of oxygen accumulation) with respect to time, dC/dt, was predicted by plotting (C-C₀) with time as shown in Fig. 2.

The curves in Fig. 2 clearly indicate two trends; i.e., as the rate of increase in DO concentration (rate of accumulation of oxygen with respect to time) is faster and more or less linear till about 17 h and thereafter, and then it becomes steady for all MLSS concentrations. However, at 25000 mg L⁻¹ MLSS, the steady state DO value is highest.

Estimation of kinetic parameters

The values of the parameters a', b and e of Eq. (9) have a very marginal difference and can easily be treated as shown in Table 4. The average predicted values are: a' = -0.1, b = 2.0 and e = 0.64. A comparison between predicted and experimental values of rate of change in substrate concentration was made. It clearly validated the agreement between predicted and experimental values. The relationship predicted is shown in Eq. (14) utilizing Eq. (6):

$$-dS/dt = -0.1(S/S_0)^2 X^{0.64} \dots (14)$$

Prediction of rate of change of COD (substrate) with time

The rate of change of COD (substrate) with time dS/dt was predicted by two ways as: (i) by using Monod type Eq. (6) gave direct values of dS/dt, and (ii) the equations to curves from Fig. 3, (S₀-S) vs. time and differentiating the same; dS/dt at particular time can be predicted. The curves in Fig. 3 clearly indicate two trends as: (i) the rate of reduction of substrate

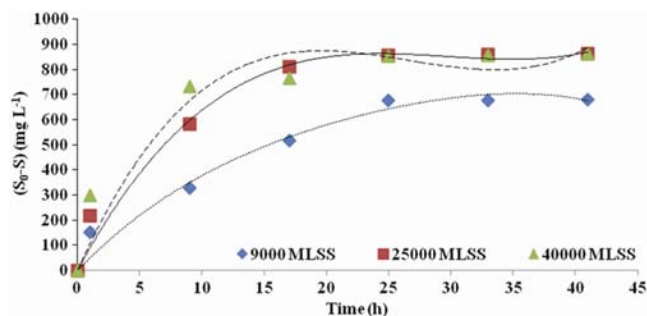


Fig. 3 — COD (S_0-S) reduction vs. time, (for unsteady state condition).

Table 5 — Steady state rising DO: Comparison of k_0 and k_{La} at different MLSS.

DO (mg L^{-1})	2.5		1.0	
MLSS (mg L^{-1})	k_0 ($\times 10^{-3}$) ($\text{L}(\text{mg h})^{-1}$)	k_{La} (h^{-1})	MLSS (mg L^{-1})	k_0 ($\times 10^{-3}$) ($\text{L}(\text{mg h})^{-1}$)
9000	0.35-0.55	5.09-0.81	9000	0.35-0.55
25000	2.07-0.15	8.44-0.63	25000	2.07-0.15
40000	1.58-0.13	10.32-1.00	40000	1.58-0.13

(COD) concentration is faster and linear till about 17 h, and thereafter (ii) it becomes steady (i.e., the rate of COD reduction with time is negligible or $dS/dt \approx 0$). Normally, the beginning linear trend is considered.³⁵

Prediction of k_0 and k_{La} for different MLSS

With reference to section 2 elaborated above, on the basis of dS/dt and dC/dt values, prediction of k_0 at different MLSS was done for unsteady state. Whereas at steady state, $dC/dt=0$, k_{La} and k_0 values were dependent on dS/dt only. OUR can be correlated with Monod kinetic equation for predicting k_0 and k_{La} , referring Eqs. (9-11). Table 5 shows a comparison of k_0 and k_{La} at different MLSS. The observations have indicated that as MLSS increases k_{La} also increases invariably.

Effect of high agitation speeds

At the beginning, the high agitation speed broke the flocs of microbes and maintained DO level. The breakage of flocs has increased the interfacial oxidation area. Further, the high agitation speeds increased microbial mobility and compelled them to release the occluded polluting substrate inside the cellular level and the adsorbed COD on the body yielding complete treatment.⁶ The reduction of agitation speeds is required to reduce OTR for maintaining constant DO. As COD reduces, OUR also reduces as a result of reduction in driving force required for mass transfer. Thus, the agitation speeds were reduced to about 33% at the end of first two sets of experiments. Whereas, for last set of experiments,

Table 6 — Agitation speed range at different MLSS.

DO (mg L^{-1}) ↓	Agitation speed (rpm)	MLSS (mg L^{-1}) ↓		
		9000	25000	40000
2.5 (steady state)		636-392	614-209	875-314
1.0 (Steady state)		582-184	563-190	631-209
1.0 (at $t=0$) (unsteady state, rising DO)		600	559	631
		throughout constant speed.		

the initial high speeds were set for 1.0 mg L^{-1} and were maintained till the end of experiment. As a result, the oxygen transfer OTR remained constant and oxygen consumption, OUR was reduced because of reduced COD. This process allowed more accumulation of oxygen. Experiments have been performed at high initial agitation speeds of 559-631 rpm and at MLSS concentrations of 9000, 25000 and 40000 mg L^{-1} . As the systems were self aspirating, the oxygen transfer was only due to surface aeration. Table 6 shows a comparison of initial agitator speeds for steady state and unsteady state DO. It shows that the agitation speeds were least for 25000 MLSS. This can be justified as the appropriate synchronisation of microbial population, tolerable torque developed for their survival. As the speed was least at 25000 MLSS; the power requirement has been naturally low.

At 9000 MLSS the COD reduction was affected due to lesser microbial population and high agitation torque killing the microbes and at 40000 MLSS, COD reduction was affected due to increased viscosity levels and high agitation speeds for oxygen transfer have generated high torque killing the microbes.

The relationship between agitation speeds and k_{La}/k_0 values was studied. A rise in both the values at higher agitation speeds and vice versa was seen. At high speeds the rise in interfacial area due to breakage of flocs and discouraged coalescence of air bubbles enhances the mass transfer. Particularly, k_{La} has shown a linear relation with the cube of agitation speed (N^3). As N^3 is directly proportional to the power consumption, the theoretical power requirement is calculated using Eq. (8). A linear relationship was seen between k_{La} and power requirement. The relations predicted are represented as Eqs. (15) and (16) with reference to Eqs. (7) and (8) as:

$$k_{La} = 2 \times 10^{-6} N^3 \quad \dots (15)$$

$$k_{La} = 0.087P \quad \dots (16)$$

$$P = 11.4943k_{La} \quad \dots (16a)$$

This has indicated that as COD reduces, the agitation speed has to reduce to reduce OTR for maintaining DO; as a result the k_0 reduces.

Effect of COD reduction

Figures 4 and 5 show a relationship between k_0 vs. COD reduction, and $k_{L\bar{a}}$ vs. COD reduction, respectively. The reduction in $k_{L\bar{a}}/k_0$ is due to reduction in agitation speeds, as discussed earlier.

Behaviour of $k_{L\bar{a}}$ and k_0

The k_0 decreases with decrease in $k_{L\bar{a}}$ and vice-versa. Figure 6 represents a relationship between $k_{L\bar{a}}$ and k_0 , i.e., at higher OTR, OUR is also higher because of availability of adequate oxygen.

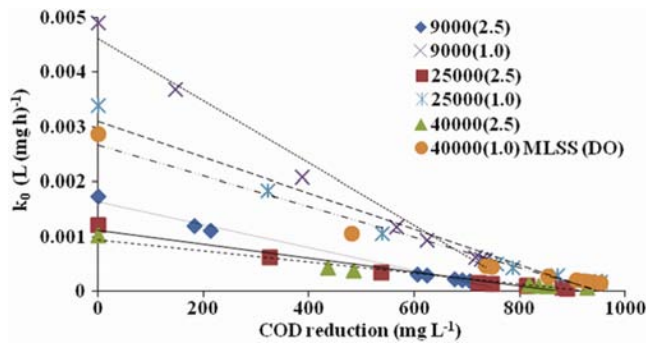


Fig. 4 — Relation between k_0 vs. COD reduction.

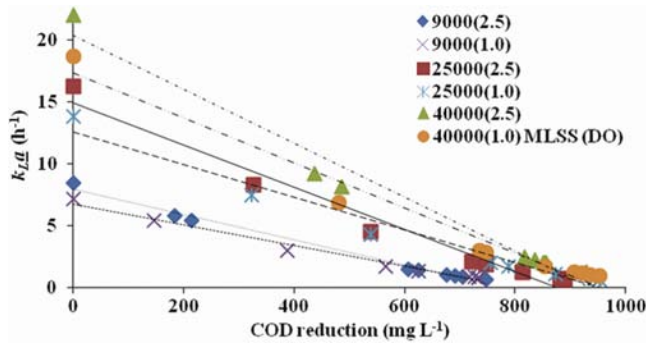


Fig. 5 — Relation between $k_{L\bar{a}}$ vs. COD reduction.

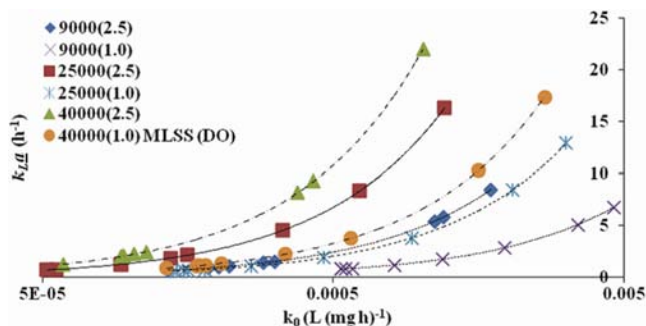


Fig. 6 — Relation between $k_{L\bar{a}}$ vs. k_0

Effect of DO

At 2.5 DO, the COD reduction is higher because of availability of sufficient O_2 for the oxidation of substrate.

Physical observations about the exit sludge

The physical observations about the sludge produced at the end of study were: (i) the colour of the sludge after treatment was reduced as compared to the sludge fed for maintaining MLSS concentration; (ii) the exit sludge was smelling less obnoxious as compared to the sludge used for maintaining MLSS at the beginning of the treatment; (iii) the exit sludge had neutral pH (≈ 7.0), and (iv) the COD of exit sludge was same as exit effluent. This indirectly indicates the removal and treatment to the maximum possible extent.

Conclusion

The sludge utilised from the same treatment plant for desirable high MLSS concentration naturally contain a high microbial population right from the beginning saves the time required for the growth and multiplication of microbes. Since the nutrient balance of effluent is done initially, the microbes get sufficient nutrition and food both that required for their healthy growth for the effective reduction in COD. The high agitation speeds rise oxygen transfer rate, discouraging coalescence of air bubbles resulting rise in their interfacial area and residence time, brake the flocs increasing interfacial area, rising mobility of microbes avoiding the tendency of settling and the compulsive release of occluded (inside cells) and adsorbed (on the cell body) polluting substrate making available for treatment arresting the transfer of pollution from effluent to sludge generated. A successful effort of correlating kinetics with mass transfer (by predicting substrate consumption rate (dS/dt), OUR coefficient (k_0) and OTR coefficient ($k_{L\bar{a}}$) and optimisation of MLSS (25000 mg L^{-1}), DO (2.5 mg L^{-1}), agitation speed (least of all other MLSS levels) and power requirement, thereby, is hoped to be helpful in designing the bioreactor due to development of mathematical models. The predicted correlations for kinetics, mass transfer coefficient with agitation speed and mass transfer coefficient with power requirement are $-dS/dt = -0.1(S/S_0)^2 X^{0.64}$, $k_{L\bar{a}} = 2 \times 10^{-6} N^3$ and $P_{th} = 11.4943 k_{L\bar{a}}$, respectively.

Nomenclature

- a -- Reaction rate constant, h^{-1}
- a' -- Modified reaction rate constant, h^{-1}

a	--	Interfacial area, m^{-1}
b	--	Order to initial substrate concentration
C	--	DO at any time t , $mg L^{-1}$
C^*	--	DO at saturation, $mg L^{-1}$, ($= 7.12 mg L^{-1}$ at temp. $32^{\circ}C$)
C_0	--	DO at $t=0$, $mg L^{-1}$
C_s	--	DO at steady state, $mg L^{-1}$
c	--	Coefficient of DO, $mg L^{-1}$
COD	--	Chemical oxygen demand, $mg L^{-1}$
D	--	Diameter of agitator, m
DO	--	Dissolved oxygen, $mg L^{-1}$
d	--	Order of DO term, dimensionless
e	--	Order of MLSS concentration
dC/dt	--	Dissolved oxygen rate, $mg (L h)^{-1}$
dS/dt	--	Microbial substrate utilization rate, $mg (L h)^{-1}$
K	--	Constant of other oxidation reactions
k_L	--	Real mass transfer coefficient, $(m^2 h)^{-1}$
k_{La}	--	Gas-liquid surface Oxygen transfer coefficient, h^{-1}
k_0	--	Apparent O_2 uptake rate coefficient, $L (h mg)^{-1}$
N	--	Agitation speed revolutions per second, s^{-1}
N_p	--	Power no., dimensionless
N_{Re}	--	Reynolds no., dimensionless
OTR	--	Oxygen transfer rate, $mg (L h)^{-1}$
OUR	--	Oxygen uptake rate, $mg (L h)^{-1}$
P_{th}	--	Theoretical power requirement, w
S_0	--	Initial substrate concentration, $mg L^{-1}$
S	--	Substrate concentration at any time t , $mg L^{-1}$
TN	--	Total Nitrogen, $mg L^{-1}$
TP	--	Total Phosphorous, $mg L^{-1}$
t	--	Time, h
X	--	MLSS concentration, $mg L^{-1}$
<i>Greek symbols</i>		
α	--	Sign of proportionality
\emptyset	--	Diameter, mm or m
ρ_t	--	Density of effluent, $kg m^{-3}$
μ_t	--	Viscosity of effluent, $mN s m^{-2}$

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