Indian Journal of Chemical Technology
Vol. 23, January 2016, pp. 47-52

Analytical solution of unsteady GO-water nanofluid flow and heat transfer between two parallel moving plates

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Received 2 September 2013; accepted 28 October 2014

Unsteady squeezing nanofluid flow between parallel plates has been analyzed analytically. The based fluid is water containing graphene oxide. The Reconstruction of Variational Iteration Method is used to solve this problem. Similarity transformations are used to transform the governing nonlinear equations of momentum and thermal energy to a system of nonlinear ordinary coupled differential equations with fitting boundary conditions. The transmuted model is shown to be controlled by a number of thermo-physical parameters, viz. moving parameter, graphene oxide nanoparticles solid volume fraction, Eckert and Prandtl number. Nusselt number and skin friction parameter are achieved for various values of GO solid volume fraction and Eckert number. The comparison assures us about validity and accuracy of solution.

Keywords: Squeezing flow, Nanofluid flow, Heat transfer enhancement, Analytical solution, Graphene oxide.

Most common fluids such as water, ethylene, glycol, toluene or oil generally have poor heat transfer characteristics owing to their low thermal conductivity. A recent technique to improve the thermal conductivity of these fluids is to suspend nano-sized metallic particles such as aluminum, titanium, gold, copper, iron or their oxides in the fluid to enhance its thermal properties.

The term nanofluid was envisioned to describe a fluid in which nanometer-sized particles were suspended in conventional heat transfer basic fluids. Nanotechnology aims to manipulate the structure of the matter at the molecular level with the goal for innovation in virtually every industry and public endeavor including biological sciences, physical sciences, electronics cooling, transportation, the environment and national security[1-3].

Graphene was found to display high quality electron transport at room temperature. Theoretical study was performed on determination of thermal conductivity of graphene and suggests that it has unusual thermal conductivity[4].

There are four possible mechanisms in nanofluids contribute to thermal conduction: (a) ballistic nature of heat transport in nanoparticles, (b) Brownian motion of nanoparticles, (c) liquid layering at the liquid/particle interface, and (d) nanoparticle clustering in nanofluids. The Brownian motion of nanoparticles is too slow to directly transfer heat through nanofluid; however, it could have an indirect role to produce a convection like micro environment around the nanoparticles and particle clustering to increase the heat transfer[5] (Table 1).

Squeezing flows have many applications in food industry, especially in chemical engineering. The determination of squeeze flow characteristics has attracted the attention of several investigators due to its importance in the practical problems of improving the performance of hydraulic machine elements, food industry, chemical engineering, polymer processing, compression, and injection molding[6-10].

There are a number of approaches for solving non-linear equations, which range from completely analytical to completely numerical ones. Besides all advantages of using numerical methods, closed form solutions appear more appealing because they reveal physical insights through the physics of the problem. Also, parametric studies become more convenient with

Table 1 — Thermo physical properties of water and GO nanopartic

	$\rho(kg / m^3)$	$C_p (j / kgk)$	$k(W / m.k$
Pure water	997.1	4179	0.613
Graphene Oxide	1800	717	5000

applying analytical methods. Some of these methods are Homotopy Perturbation Method (HPM)[11,12], Differential Transformation Method (DTM)[13], Galerkin Optimal Homotopy Asymptotic Method[5], Variational Iteration Method (VIM)[14], Adomian Decomposition Method (ADM)[15] and other[16].

In this study, the Reconstruction of Variational Iteration Method[17], is applied to find the semi-analytical solutions of nonlinear differential equations governing the problem of unsteady nanofluid flow between two moving plates and heat transfer. The effects of the moving parameter, type of nanoparticle, the nanofluid volume fraction and Eckert number on Nusselt number are investigated.

Mathematical Formulation

The unsteady flow and heat transfer in a two-dimensional nanofluid between two infinite parallel plates is considered in this study. Figure 1 shows the problem schematic. The two plates are placed at $z = \pm l(1-\alpha t)^{1/2} = \pm h(t)$ for $\alpha > 0$, and moved until they touch $t = 1/\alpha$. For $\alpha < 0$ the two plates are separated. The viscous dissipation effect and heat generation due to friction caused by shear in the flow, is retained. This behavior occurs at high Eckert number (<<1). The Eckert number expresses the relationship between a flow's kinetic energy and enthalpy. The fluid is a water based nanofluid containing Graphene Oxide. The nanofluid is a two component mixture with the following assumptions: Incompressible; No-chemical reaction; Negligible viscous dissipation; Negligible radiative heat transfer; Nano-solid-particles and the base fluid are in thermal equilibrium and no slip occurs between them.The governing equations are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \ldots (1)$$

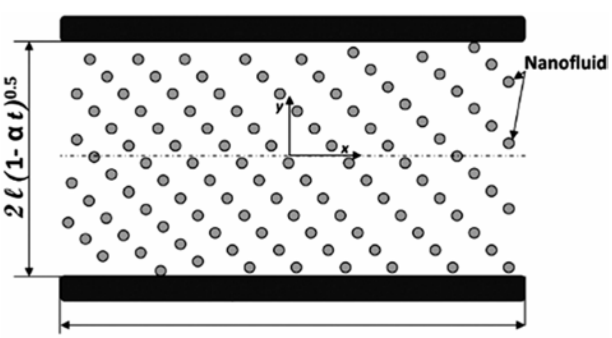

Fig. 1 — Geometry of problem

$$\rho_{nf}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu_{nf}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad \ldots (2)$$

$$\rho_{nf}\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu_{nf}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad \ldots (3)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu_{nf}}{(\rho C_p)_{nf}}\left[4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] \quad \ldots (4)$$

where u and v are the velocities in the x and y directions, respectively. Effective density (ρ_{nf}), the effective dynamic viscosity (μ_{nf}), effective heat capacity $(\rho_{nf})_{nf}$ and the effective thermal conductivity k_{nf} of the nanofluid are defined as [13]:

$$\begin{aligned}
&\rho_{nf} = \rho_f(1-\phi) + \rho_s\phi_s \\
&(\rho C_p)_{nf} = (\rho C_p)_f(1-\phi) + (\rho C_p)_s \\
&\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \\
&\frac{k_{ns}}{k_f} = \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + 2\varphi(k_f - k_s)} \\
&v_{nf} = \frac{\mu_f}{\rho_{nf}}
\end{aligned} \quad \ldots (5)$$

The relevant boundary conditions for the problem are:

$$\begin{aligned}
&y = h(t) \rightarrow v = v_w = \frac{dh}{dt}, \quad T = T_H \\
&y = 0 \rightarrow v = \frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = 0
\end{aligned} \quad \ldots (6)$$

We can simplify above equations by introducing following parameters:

$$\begin{aligned}
&\eta = \frac{y}{[l(1-\alpha t)^{1/2}]}, \quad u = \frac{\alpha x}{[2(1-\alpha t)]} f'(\eta), \quad v = -\frac{\alpha l}{[2(1-\alpha t)^{1/2}]} f(\eta), \\
&\theta = \frac{T}{T_H}, \quad A = (1-\varphi) + \varphi\frac{\rho_s}{\rho_f}, \quad B = (1-\varphi) + \varphi\frac{(\rho C_p)_s}{(\rho C_p)_f}, \quad C = \frac{k_{nf}}{k_f}
\end{aligned} \quad \ldots (7)$$

The above parameters are substituted into equations. (2) and (3). Then the pressure gradient is eliminated from the resulting equations:

$$f^{(IV)} - SA(1-\varphi)^{2.5}(\eta f''' + 3f'' + f'f'' - ff''') = 0 \quad \ldots (8)$$

Using equation (7), Equations (3) and (4) simplify to following equations:

$$\theta'' + \Pr S\left(\frac{B}{C}\right)(f\theta' - \eta\theta') + \frac{\Pr Ec}{C(1-\varphi)^{2.5}}\left(f''^2 + 4\delta^2 f'^2\right) = 0 \quad \text{... (9)}$$

With the following boundary conditions:

$$f(0)=0,\quad f''(0)=0,\quad f(1)=1,\quad f'(1)=0,\quad \theta'(0)=0,\quad \theta(1)=1 \quad \text{... (10)}$$

where S is moving parameter, $\Pr$ is the Prandtl number and Ec is the Eckert number, Which are defined as:

$$S = \frac{\alpha l^2}{2v_f},\quad \Pr = \frac{\mu_f(\rho C_p)_f}{\rho_f k_f},\quad Ec = \frac{\rho_f}{(\rho C_p)_f}\left(\frac{\alpha x}{2(1-\alpha t)}\right)^2,\quad \delta = \frac{l}{x} \quad \text{... (11)}$$

moving parameter S describes the movement of the plates. $S>0$ corresponds to the plates moving apart, while $S<0$ corresponds to the plates moving together (also called squeezing flow).

Solution Procedure

In the following section, an alternative method for finding the optimal value of the Lagrange multiplier by the use of the Laplace transform will be investigated. Suppose x, t are two independent variables, consider t as the principal variable and x as the secondary variable. if $u(x,t)$ is function of two variables x and t, when the Laplace transform is applied with t as a variable, definition of Laplace transform is:

$$L(u(x,t);s) = \int_0^\infty e^{-st}u(x,t)dt \quad \text{... (12)}$$

We have some preliminary notations as:

$$L\left(\frac{\partial u}{\partial t};s\right) = \int_0^\infty e^{-st}\frac{\partial u}{\partial t}dt = sU(x,s) - u(x,0) \quad \text{... (13)}$$

$$L\left(\frac{\partial^2 u}{\partial t^2};s\right) = s^2U(x,s) - sU(x,s) - u_t(x,0) \quad \text{... (14)}$$

$$U(x,s) = L(u(x,t);s) \quad \text{... (15)}$$

We often come across functions which are not the transform of some known function but then they can possibly be as a product of two function. Thus we may be able to write the given function as $U(x,s)$, $V(x,s)$ where $U(s)$ and $V(s)$ are known to the transform of the function $u(x,t)$, $v(x,t)$, when the Laplace transform is applied to t as a variable, respectively; then $U(x,s)$, $V(x,s)$ is the Laplace Transform of $\int_0^t u(x,t-\varepsilon)v(x,\varepsilon)d\varepsilon$:

$$L^{-1}(U(x,s),V(x,s)) = \int_0^t u(x,t-\varepsilon)v(x,\varepsilon)d\varepsilon \quad \text{... (16)}$$

To facilitate our discussion of Reconstruction of Variational Iteration Method (RVIM), introducing the new linear or nonlinear function $h(u(x,t)) = f(x,t) - N(u(x,t))$ and considering the new equation, rewrite $h(u(x,t)) = f(x,t) - N(u(x,t))$ as:

$$L(u(t,x)) = h(t,x,u) \quad \text{... (17)}$$

Now, for implementation the correctional function of VIM based on new idea of Laplace transform, applying Laplace Transform to both sides of the above equation so that we introduce artificial initial conditions to zero for main problem, then left hand side of equation after transformation is featured as:

$$L[L\{u(x,t)\}] = U(x,s)P(s) \quad \text{... (18)}$$

where $P(s)$ is polynomial with the degree of the highest order derivative of linear operator :

$$L[L\{u(x,t)\}] = U(x,s)P(s) = L[h\{(x,t,u)\}] \quad \text{... (19)}$$

$$U(x,s) = \frac{L[h\{(x,t,u)\}]}{P(s)} \quad \text{... (20)}$$

Suppose that $D(s) = 1/P(s)$, Using the convolution theorem, Taking the inverse Laplace transform on both side of Equation.(22),

$$u(x,t) = \int_0^t d(t-\varepsilon)h(x,\varepsilon,u)d\varepsilon \quad \text{... (21)}$$

$$u_0(x,t) + \int_0^t d(t-\varepsilon)h(x,\varepsilon,u)d\varepsilon \quad \text{... (22)}$$

And $u_0(x,t)$ is initial solution with or without unknown parameters. In absence of unknown parameters, $u_0(x,t)$ should satisfy initial boundary conditions.

Results and Discussion

The results of squeezing Go-Water nanofluid flow between infinite parallel plates have been obtained. The effects of active parameter on heat and mass characteristics were also examined.

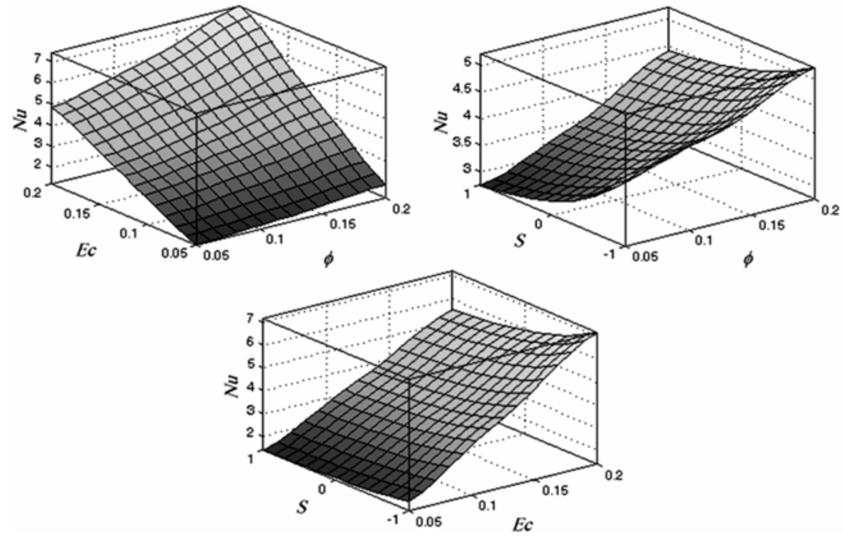

Fig. 2 — Effects of (a) solid volume fraction and Eckert number, (b) moving parameter and solid volume fraction and (c) moving parameter and Eckert number on Nusselt number.

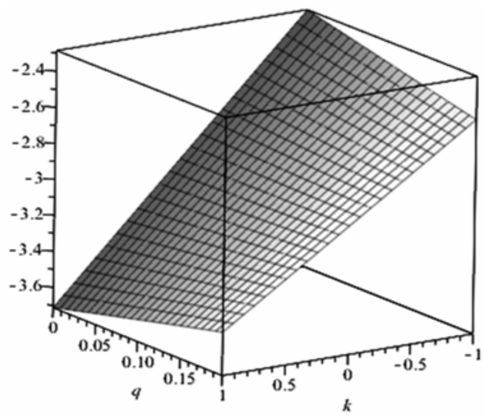

Fig. 3 — Effects of solid volume fraction and moving parameter on Skin friction coefficient

Physical quantities of interest are the skin friction coefficient and Nusselt number which are defined as follows:

$$Cf = \frac{\mu_f \left(\frac{\partial u}{\partial y} \right)_{y=h(t)}}{\rho_{nf} v_w^2}, \quad Nu = \frac{-lk_{nf} \left(\frac{\partial T}{\partial y} \right)_{y=h(t)}}{kT_H} \qquad \dots (23)$$

Effects of Graphene Oxide (GO) nanoparticles solid volume fraction, *Eckert* number and *moving parameter* on *Nusselt number* are shown in Fig. 2 a-c. As it is evident from the

Fig. 2a, for constant S value, when the Ec is lower than or equal to 0.05, the increase in nano particle volume fraction does not result in a significant change in the thermal boundary layer thickness and Nu value. While, for high Ec values, the increase in

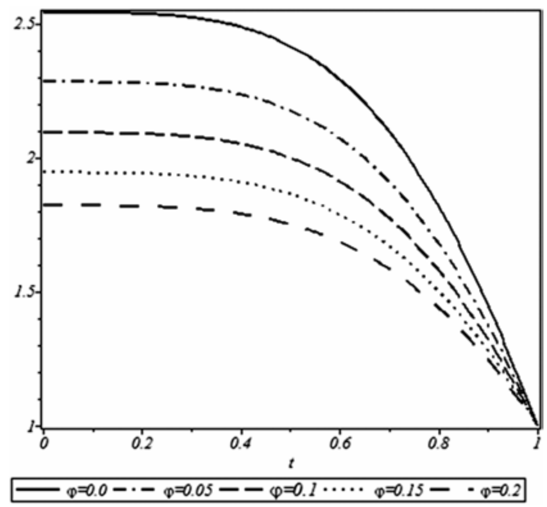

Fig. 4 — Effect of φ on Temperature profile when $\delta = 0.1$, $\Pr = 6.2$, $Ec = 0.2$, $S = -1$

nanoparticles volume fraction causes more increment of thermal boundary layer thickness, and therefore a higher Nu is obtained. Another consequence that can be achieved from Fig. 2 is by increasing the solid volume fraction from 0.1 to 0.2 we can increase the Nusselt number about 20 or 30% for each case. As a result, it can be illustrated that Eckert number has strong effect on Nusselt number. Also Fig. 3 shows that the absolute values of skin friction coefficient have reverse relationship with the squeeze number and GO nanoparticles solid volume fraction .

Effects of the nanofluid volume fraction and the squeeze number on the temperature profile are shown in Figs 4 and 5, respectively. According to these

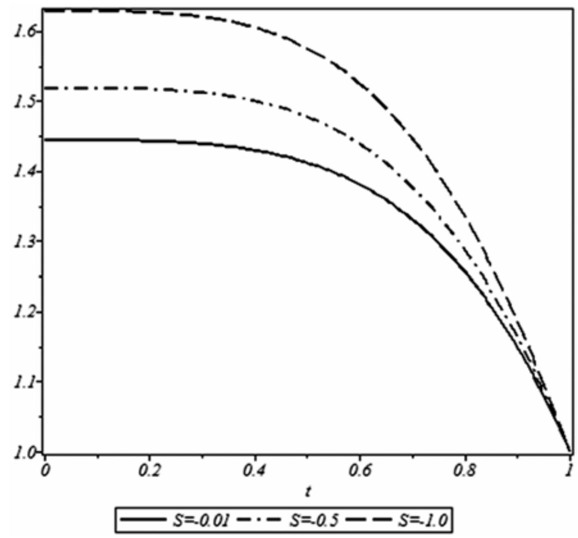

Fig. 5 — Effect of S on Temperature profile when $\delta = 0.1,\ \Pr = 6.2,\ Ec = 0.1,\ \varphi = 0.15$

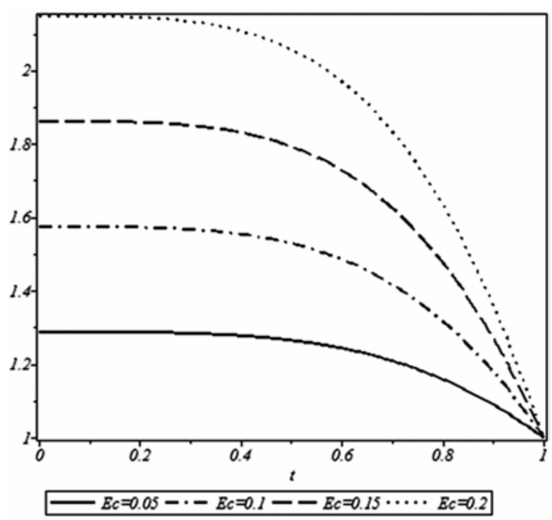

Fig. 6 — Effect of Ec on Temperature profile when $\delta = 0.1,\ \Pr = 6.2,\ \varphi = 0.1,\ S = -0.5$

figures, by increasing GO nanoparticle solid volume fraction, heat transfer enhancement can be achieved. In other words, Increasing the volume fraction of nanofluid leads to decrease in the thermal boundary layer thickness.

Figure 6 shows the effect of Eckert number on temperature profile. As it can be seen in Figure 6 by increasing Eckert number, temperature gets higher value. This is due to the fact that the viscous dissipation effect significantly increases the temperature of the fluid between two plates.

In Fig. 7 the comparison of approximate results with numerical solutions have been presented when

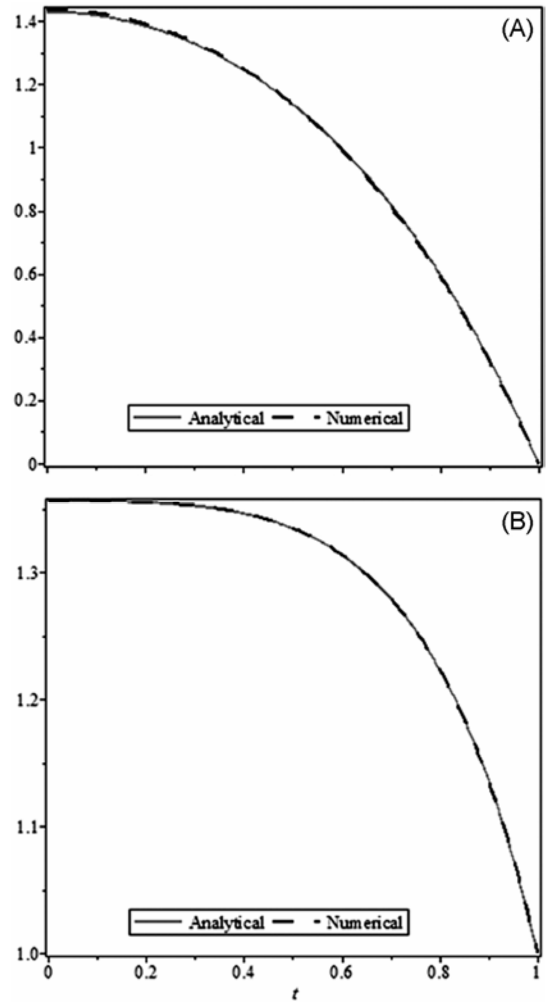

Fig. 7 — Comparison of analytical results and numerical ones when $S = 1,\ \varphi = 0.05,\ \delta = 0.1,\ \Pr = 6.2,\ Ec = 0.1$ for A-*Velocity* B-Temperature

$S = 1,\ \varphi = 0.05,\ \delta = 0.1,\ \Pr = 6.2,\ Ec = 0.1$. In first plot, the velocity profile has been shown and in second plot we can see the results comparison for temperature profile. As it can be seen numerical results have good agreement with obtained results for both temperature and velocity.

Conclusion

In this paper, the heat transfer in the unsteady nanofluid flow between two moving parallel plates has been investigated using RVIM. The effect of solid volume fraction on heat transfer enhancement for case plates moving together (squeezing flow) have been studied. The effective thermal conductivity and viscosity of the nanofluid are calculated using the Maxwell–Garnetts (MG) and Brinkman models, respectively. The analytical investigation is carried out for various governing parameters such as the squeeze number, nanoparticle volume fraction and Eckert number. The results show that for the case in which two plates moving together, the Nusselt number increases with increase of nanoparticle volume fraction and Eckert number while it decreases with growth of the squeeze number. The comparison of obtained results with numerical solutions assures us about the validity and accuracy of the current study.

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