Theoretical calculation of the 'natural flock restoration' time of a cotton comber

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Based on the overhanging beam theory, the stress on the flock of a cotton comber has been analyzed, and the natural frequency of the 'natural flock restoration' is calculated in combination with energy method. Finally, a theoretical model of the 'natural flock restoration' time has been developed. The model illustrates that the 'natural flock restoration' time of a cotton comber is determined by cotton properties, combing process and mechanical structure. The established theoretical model corrects the previous calculation method for the 'natural flock restoration' time of a cotton comber. The finding provides information for the development of a new type of cotton comber with high velocity.

Keywords: Bending rigidity, Cotton comber, Cotton fibre, Elastic modulus, Natural restoration of flock

1 Introduction

The maximum velocity of a cotton comber is an important measure of its modernization level. The theoretical maximum velocity is closely related to the 'natural flock restoration' time. Recent studies on flock restoration mainly concentrate on the properties of cotton and examine 'natural flock restoration' using the mechanical theory of the cantilever. However, this method has several defects. First, only the stress on the flock is analyzed without considering the structure of the cotton comber. Thus, the flock is generally equivalent to an overhanging beam with respect to its stress pattern, which is inconsistent with the stress on the flock of a cotton comber. Second, the influence of the combing process on the 'natural flock restoration' time is not considered. Therefore, the maximum velocity of a cotton comber is often incorrect, which seriously hinders the cotton comber development¹⁻¹⁰.

In view of the above, present study has been undertaken to analyze the stress on the flock in a cotton comber and proposes a mathematical model for overhanging beam displacement through mechanical displacement superposition. A calculation model has also been developed for the natural frequency of 'natural flock restoration' using the energy method. The parameters for the cotton properties, mechanical structure and combing process can be represented in the mathematical model. The development of a

^aCorresponding author. E-mail: lixinrong7507@hotmail.com theoretical model for the 'natural flock restoration' time of a cotton comber has several benefits. The model corrects the previous calculation method for 'natural flock restoration' time of a cotton comber and provides information for studying the basic combing process. The study also provides a theoretical calculation model for investigating the maximum velocity of a cotton comber. This study may also help in the development of a new type of cotton comber with high velocity.

2 Methodology

Flock restoration refers to the process in which the flock bounces back and straightens out after comber cylinder combing. Flock restoration consists of "active flock restoration" and 'natural flock restoration.' The former indicates that the flock bounces back and straightens out under an external force such as the relative motion of air flow or bottom nipper plate, whereas the latter indicates that the flock bounces back and straightens out through its own elasticity after comber cylinder combing. The combing process can only be smoothly completed when the straightened out flock links to the flock returned by the detaching roller. Thus, the calculation of the maximum velocity of a cotton comber should be based on the analysis of "flock restoration." The study of "active restoration flock" must also be based on a thorough understanding of 'natural flock restoration'. Therefore, calculating 'natural flock restoration' time has practical value and scientific significance.

To analyze the stress on the flock, the flock model should be first simplified into a cotton fibre pole with elastic modulus (E), density (ρ), cross-sectional area (S) and moment of inertia of cross-sectional area (I) (Fig.1). The cotton fibre pole was used for mechanics analysis. Then, the combing location parameter of the flock in a cotton comber was determined with the flock length (L_1) inside the nipper jaw (Fig.2). The combing process parameters were determined with the flock length (L_2) outside the nipper jaw (Fig.2). Based on Fig.2, the flock was found equivalent to an overhanging beam with respect to its stress pattern. Based on the mechanical theory of an overhanging beam, the above-mentioned parameters were used to establish the mathematical model of for analyzing 'natural flock restoration' time of a cotton comber. The established mathematical model consists

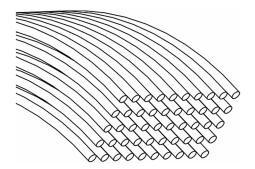


Fig.1—Flock in a cotton comber

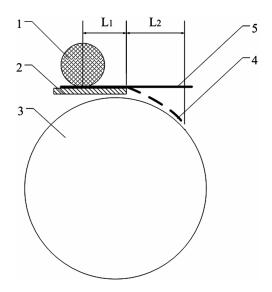


Fig.2—'Natural flock restoration' in a cotton comber [1-feeding roller, 2-bottom nipper plate, 3-cylinder, 4-location before 'flock restoration', and 5- location after 'flock restoration', L_1 -flock length of flock inside the nipper jaw, and L_2 -flock length outside the nipper jaw]

of parameters for flock properties, flock location in a cotton comber and combing processing.

2.1 Parameters for Flock Properties and Flock Location

It is assumed that the top nipper plate remains open upon the completion of comber cylinder combing (Fig.2). Then, the flock begins to bounce back and straighten out through its elasticity. By analyzing the stress acting on the flock in a cotton comber, it is found that 'natural flock restoration' has a close relationship with the following parameters.

2.1.1 Flock Properties

Cotton fibre elastic modulus (E) is an important mechanical constant in the stress-strain relationship of the material and indicates the deformation resistance of the material to external force. The elastic modulus of the cotton fibre is usually determined by the slope of the first part of the stress-strain curve in the tensile test and is referred to as the "initial elastic modulus"¹¹. The elastic modulus of the flock determines the velocity at which the flock bounces back, i.e. the length of "flock restoration" time. Therefore, the elastic modulus of the flock should be first analyzed when studying the 'natural flock restoration' time of a cotton comber. In textile engineering practice, direct measuring of elastic modulus (E) and moment of inertia of a crosssectional area (I) in the cotton fibre is rarely possible. Therefore, we usually define other technical parameters such as specific bending rigidity $(R_{\rm f})$, as shown below:

$$R_f = \frac{R}{N_t^2} \qquad \dots (1)$$

where R is the bending rigidity; and N_t , the cotton fibre fineness. Based on the definition of bending rigidity, R = EI, and hence

$$EI = R_f N_t^2 \qquad \dots (2)$$

2.1.2 Flock Location

The flock state in a cotton comber was analyzed as shown in Fig.2. Based on the stress pattern, the flock can be equivalent to an overhanging beam. L_1 is the distance between the flock holding point of the feeding roller and front end of the bottom nipper. L_1 is also the mid-span length of the overhanging beam and closely related to the restoration of the free end of the overhanging beam. In addition, L_1 affects the 'natural flock restoration' time of a cotton comber and is the location parameter of the flock in a cotton comber.

2.1.3 Process Parameters

Figure 2 shows that the flock length (L_2) outside the nipper jaw is the length of the free end of the overhanging beam. The flock length (L_2) outside the nipper jaw has a close relationship with the restoration of the free end of the flock, influencing the 'natural flock restoration' time of a cotton comber. L_2 is determined by the detaching distance B (the distance between the flock holding point of the detaching roller and the front end of the bottom nipper), feeding length S (feeding flock length per nipper) and feeding method of the feeding roller¹².

2.2 Mathematical Calculation Model

To analyze the flock stress and displacement, the constraint and load on the flock were first simplified. As shown in Fig.2, the flock holding point of the feeding roller was simplified into a fixed support hinge, and the front end of the bottom nipper was simplified into a roller support hinge. The front end of the flock was stretched out of the bottom nipper plate. That is, the flock in a cotton comber was simplified into an overhanging beam as illustrated in Fig.3. In this figure, Y=flock displacement, L_1 =distance between the flock holding point A and the front end B of the bottom nipper plate, and L_2 (length of flock stretching out of nipper)= distance between the front end B of the bottom nipper plate and the front end C of flock. The dashed line represents the location before flock restoration, and the solid line represents the location after flock restoration.

The stress acting on the flock at section AC was analyzed. To calculate the displacement, section AC was divided into section AB, where the stress acted on

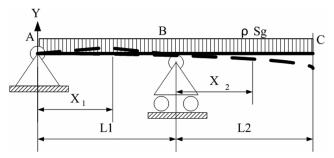


Fig.3—'Diagram of the simplified flock stress pattern [*Y*-flock displacement, L_1 -distance between the point A and B, L_2 -distance between the point B and C, X_1 -distance between the point A and the point which displacement is maximum at section AB, X_2 -distance between the point B and the point where the stress acted on the flock]

the flock inside the nipper plate, and section *AC*, where the stress acted on the flock outside the nipper plate.

First, the stress acting on section AB was analyzed (Fig.4). Two simple loads acted on the flock in section AB, its own weight and torque at point B. The flock displacement (Y_1) was obtained using the following method.

In the premise of small deformation and line elastic deformation, it proves a linear relationship between the displacement and the load of the beams. When beam was acted by multiple load, the displacement of beam could be calculated by superimposing the displacement acted by single load, called mechanical superposition method¹³. As shown in Fig 4, the flock *AB* was forced by gravity and torque load on point B, and the force was decomposed (Fig. 5). Here Y_{11} was the displacement of beam acted by gravity and Y_{12} was the displacement of beam acted by torque load on point B. Y_{11} and Y_{12} could be obtained by adopting differential method¹⁴, as shown below:

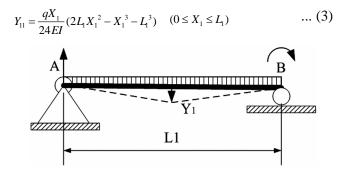


Fig.4—'Force diagram of section AB [L₁ -distance between the point A and B; and Y_1 - flock displacement]

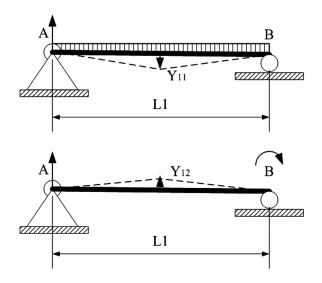


Fig 5-Force decomposition diagram of section AB

$$Y_{12} = \frac{qL_1^2 X_1}{2 \times 6L_1 EI} (L_1^2 - X_1^2) \quad (0 \le X_1 \le L_1) \qquad \dots (4)$$

 Y_1 was obtained by adopting the mechanical superposition method, as shown in the following equation :

$$Y_{1} = Y_{11} + Y_{12}$$

$$= \frac{qX_{1}}{24EI} (2L_{1}X_{1}^{2} - X_{1}^{3} - L_{1}^{3}) + \frac{qL_{1}^{2}X_{1}}{2 \times 6L_{1}EI} (L_{1}^{2} - X_{1}^{2}) \dots (5)$$

$$= \frac{q}{24EI} (L_{1}^{3}X_{1} - X_{1}^{4}) \qquad (0 \le X_{1} \le L_{1})$$

where $q=\rho Sg$.

To calculate the displacement of point *C*, the overhanging beam *AC* is divided into cantilever *BC* and simply supported beam *AB* (Fig. 6). Flock displacement Y_2 was obtained by the superposition method using the following equation.

$$Y_{2} = Y_{1}'(L_{1})X_{2} + \frac{qX_{2}^{2}}{24EI}(4L_{2}X_{2} - 6L_{2}^{2} - X_{2}^{2})(0 \le X_{2} \le L_{2})$$
...(6)

$$Y_{1}'(L_{1}) = \frac{q}{24EI}(L_{1}^{3} - 4X_{1}^{3}) = -\frac{qL_{1}^{3}}{8EI} \qquad \dots (7)$$

By substituting Eq.(7) into Eq.(4), following equation is obtained:

$$Y_{2} = -\frac{qL_{1}^{3}}{8EI}X_{2} + \frac{qX_{2}^{2}}{24EI}(4L_{2}X_{2} - 6L_{2}^{2} - X_{2}^{2}) \quad (0 \le X_{2} \le L_{2})$$
$$= \frac{q}{24EI}(4L_{2}X_{2}^{3} - 6L_{2}^{2}X_{2}^{2} - X_{2}^{4} - 3L_{1}^{3}X_{2}) \quad \dots (8)$$

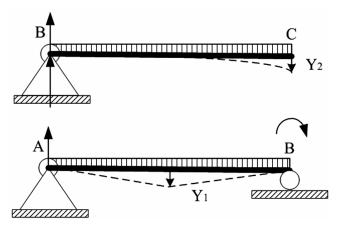


Fig.6—Force diagram of section *BC* [Y_1 - flock displacement on half of the beam; and Y_2 - flock displacement of point C]

Because the movement of flock restoration is under the condition of simple harmonic vibration, the system observes the law of conservation of energy. It means that energy transforms between the kinetic energy and potential energy. All harmonic vibration system, including vibration system with multiple freedom degree, are following the above rules. The angular frequency of simple harmonic vibration system could be obtained directly by using energy equation. In this study, only the elastic potential energy was considered in analyzing the stress acting on the flock. The natural frequency (ω) was calculated via energy method using the following equation^{13,14}.

$$\omega^{2} = \frac{EI \int_{0}^{L_{1}} (\frac{\partial^{2} Y_{1}}{\partial X_{1}^{2}})^{2} dX_{1} + EI \int_{0}^{L_{2}} (\frac{\partial^{2} Y_{2}}{\partial X_{2}^{2}})^{2} dX_{2}}{\rho S \int_{0}^{L_{1}} Y_{1}^{2} d(X_{1}) + \rho S \int_{0}^{L_{2}} Y_{2}^{2} d(X_{2})}$$
$$= \frac{EI}{\rho S} \frac{1296(L_{1}^{5} + L_{2}^{5})}{5L_{1}^{9} + L_{2}^{3}(135L_{1}^{4} + 234L_{1}^{3}L_{2}^{3} + 104L_{2}^{6})} \qquad \dots (9)$$

Hence

$$\omega = \sqrt{\frac{EI}{\rho S} \frac{1296(L_1^5 + L_2^5)}{5L_1^9 + L_2^3(135L_1^4 + 234L_1^3L_2^3 + 104L_2^6)}}$$

The flock restoration time (t) is found to be 1/4 of the vibration period T. Thus, following equation of t was obtained:

$$t = \frac{T}{4} = \frac{\pi}{2\omega} = \frac{\pi}{2\sqrt{\frac{EI}{\rho S} \frac{1296(L_1^5 + L_2^5)}{5L_1^9 + L_2^3(135L_1^4 + 234L_1^3L_2^3 + 104L_2^6)}}} = \frac{\pi}{2\sqrt{R_f N_t} \frac{1296(L_1^5 + L_2^5)}{5L_1^9 + L_2^3(135L_1^4 + 234L_1^3L_2^3 + 104L_2^6)}} \dots (10)$$

3 Results and Discussion

We used a Rieter Combers E60 for this experiment to verify the correctness of the calculation model. The flock analyzed in the paper was manufactured from pure cotton with a specific bending rigidity (R_f) of 0.53×10^{-3} Nmm²/tex² (ref. 15). The cotton fibre fineness (N_t) ranged from 1.45×10^{-1} tex to 1.77×10^{-1} tex (ref. 16). For this study, we set $N_t = 1.61 \times 10^{-1}$ tex.

To ensure the spinnability and reliability of the cotton comber, the study used the minimum forward feeding $(L_{2\min})$ and maximum backward feeding

 $(L_{2\text{max}})$ for the analysis. For the forward feeding, $L_2 = B$, and for the backward feeding, $L_2 = B + S^{12}$. The minimum detaching distance for the Rieter Combers E60 was $B_{\text{min}} = 6.24 + 25 / 2 = 18.74$. The distance of the feeding roller ranges from 4.3 mm to 6.7 mm. Then, $L_{2\text{min}} = B = 18.74$ mm, and $L_{2\text{max}} = B + S = 18.74 + 6.7 = 25.44$ mm. Therefore, we set $L_2 = 25.44$ mm.

By measuring the nipper plate structure of the cotton comber, the distance from the flock holding point of the feeding roller to the front end of the bottom nipper plate was $L_1 = 27.3$ mm. Substituting the above parameters into Eq.(10), yielded t = 0.0485.

The cotton comber considered one rotation of the cylinder shaft as the motion cycle, being divided into 40 indexes. The indexing number of the "flock restoration" of the Rieter Combers E60 is γ , and $L_2 = 25.44$ mm corresponds to $\gamma_2 = 19.25$. By substituting the value of *t* and γ into following equation [Eq.(11)], the following equation [Eq.(12)] was obtained:

$$t = \frac{60}{n_{\max}} \cdot \frac{\gamma}{40} = \frac{3\gamma}{2n_{\max}} \qquad \dots (11)$$

$$n_{\max} = \frac{3\gamma}{2t} = \frac{3 \times 19.25}{2 \times 0.0485} \approx 600 \qquad \dots (12)$$

Thus, the theoretical maximum velocity of the cotton comber was 600 nips/min. It proves that the calculation model for 'natural flock restoration' is correct.

4 Conclusion

The maximum velocity of cotton comber is an important measure of modernization level, and theoretical maximum velocity is closely related to the time of flock natural restoration. This study analyzes the actual force on the flock in the cotton comber by leading the process parameters and proposes the mathematical model of displacement of overhanging beam. In addition, we build a calculation model of the natural frequency of flock natural restoration by energy method, i.e. the model of time of flock natural restoration. The following conclusions are obtained from the preceding analysis and calculation:

- **4.1** The time of flock natural restoration is not a fixed value but depends on the flock properties, the length of the flock outside nipper plate and the length of the flock inside nipper plate.
- **4.2** The performance indicators of cotton fibre such as bending rigidity and fineness decide the time of flock natural restoration.

- **4.3** The length of the flock outside nipper plate depends on detaching distance, feeding length and feeding method, i.e. combing process.
- **4.4** The length of the flock inside nipper plate depends on the location of feeding roller above the bottom nipper plate, i.e. nipper mechanism of comber.
- **4.5** The time of flock natural restoration on a cotton comber is a variable value, which depends on flock properties, combing process and combing mechanism.

This study combines flock properties, combing process and comber structure for the first time. A calculation model of flock natural restoration time based on cotton comber is deduced. Researchers can design cotton comber with higher velocity and better adaptability according to the calculation model proposed in this study, by taking into account of flock properties, mechanical structure and combing process, to increase the output of cotton comber and to reduce cost.

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