Indian Journal of Fibre & Textile Research
Vol. 40, September 2015, pp. 236-242

Estimating engineering constants of a selected model of textile composite

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Received 24 March 2014; revised received and accepted 25 July 2014

This study deals with the development of a simple geometric model of fibre-reinforced plastic (FRP) composite. The composite uses an epoxy resin as the matrix, and one layer of a plane wave E-glass fabric as the reinforcement. Using theoretical considerations (based on this model), it is feasible to calculate engineering constants useful for stress analysis. The work consists of developing constitutive model of specific textile composite material (plain weave). There is a numerical constitutive model which approximates the mechanical behavior at the local scale. The method of calculation presented in this study can be applied to other models of FRP composites with other woven fabric geometries, such as Pierce's model of plane wave, Pierce's elliptic model, Kemp model with race track cross-section, and Hearle's model.

Keywords: Effective moduli, Fibreglass fabric, Fibre-reinforced composites, Textile mechanics, Unit cell model, Woven fabric

1 Introduction

Fibre-reinforced plastic (FRP) composites represent a class of advanced materials, which are reinforced with textile preforms for structural or load bearing applications. In general, composites can be defined as a select combination of dissimilar materials with a specific internal structure and external shape. The unique combination of two material components leads to singular mechanical properties, and performance characteristics do not depend in any of the components alone[1-4]. Additionally, composite materials are often overwhelmingly superior to other materials such as metals on a strength-to-weight or stiffness-to-weight basis. With this reassurance, the range of applications for composite materials appears to be limitless. FRP composites can be defined as the combination of a resin system with a textile fibre, yarn, or fabric system. They may be either flexible or quite rigid. This study reports how to build a simple geometrical model of FRP composite and how to get from this model certain geometrical parameters, which are very useful for further strength analysis.

Some of the most important and earliest studies carried out on textile composite science were gathered by Poe and Harris[5]. One of the first attempts to model thermo-elastic behavior of two-dimensional woven fabric composites was presented by Chou and Ishikawa[6-8]. They developed three analytical models for 2D woven composites based on classical lamination theory (CLT). Naik[9,10] presented a general purpose micromechanics analysis that discretely modeled the yarn architecture within representative unit cell (RUC) of the textile. This model was developed to predict overall, 3D, thermal and mechanical properties based on an iso-strain assumption and using a stress averaging technique[11]. The model was also implemented in a computer program called Textile Composite Analysis for Design (TEXCAD)[9] to analyze plain, 5-harness satin and 8-harness satin woven composites along with 2D and 2×2 2D triaxial braided composites.

Naik and Ganesh[12] presented an analytical model for woven composites based on CLT that accounted for undulations in both the fill and warp directions of the yarns. They computed overall stiffness properties by assuming either iso-strain or iso-stress, or a combination of these, and analytically integrated through the volume of the RUC. They developed two models to describe the geometry of the undulating region by assuming either a circular or sinusoidal yarn path. Scida *et al.*[13] used the model of Naik[9] to make comparison with test data and to investigate the relative accuracy of the results. Masters *et al.*[14] investigated the tensile properties of 2D triaxial braided composites from both an

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experimental and analytical viewpoints. Various computer programs that can predict the properties of textile composites were also presented by Cox and Flanagan[4], most of these programs were developed within the Advanced Composites Technology (ACT) Program sponsored by NASA Langley Research Center. Byun[15] also developed his own analytical model to determine three-dimensional elastic stiffness properties for 2D triaxial braided fabrics using either iso-strain or iso-stress assumption.

Although numerous analytical and numerical techniques have been used to predict the stiffness properties of both woven and braided composites, there are only a few models that have been developed for the strength prediction of textile composites. Ko and Pastore[16] used the yarn orientations to first estimate the strength of the fabric preform and then computed composite strength using a simple rule of mixture. Dow and Ramnath[17] modeled woven fabric composites using a simple geometry model that assumed a linear undulation path for the fill and warp yarns. They computed constituent fibre and matrix stresses from local stresses which were calculated using an iso-strain assumption and predicted failure based on the average stresses in the fibre and the matrix along with a maximum stress criterion.

An important characteristic of textile composites is that they exhibit non-linear shear behavior. The earlier analysis techniques to model damage propagation and strength of textile composites often made simplifying assumptions regarding the fabric architecture and did not account for both geometric and material nonlinearities. Naik[18] developed a general-purpose analysis technique for the prediction of failure initiation, damage progression and strength of 2D woven and braided composite materials, including the effects of nonlinear shear response and nonlinear material response. Naik also predicted failure within the RUC by discretizing the yarns into slices, averaging the stresses over the volume of the RUC to get the overall stiffness matrix. Theoretical calculation of predicted properties of textile composites needs a proper constructed and used geometric model.

Optimal design problems of textile composite structures, taking into account shape and orientation of fibres, is the main topic of many studies and works. In separate studies[19,20] various methods of solving that problem have been presented, using traditional optimization methods. Optimal design of multilayer composite reinforced with fibres was a topic of another work[21], where the main method in optimization process is the method based on genetic algorithms. Two different approaches to design reinforced composite flywheels are also presented[22]. The first approach is based on a discrete model of reinforcement. In the second approach, the material of reinforced flywheel is subjected to homogenization procedure using the Halpin-Tsai assumption and then the continuity of both static and kinematic fields within flywheel domain is preserved.

This study deals with the development of a simple geometric model of fibre-reinforced plastic (FRP) composite, which is not limited to small crimp angles only.

2 Materials and Methods

2.1 Materials

In this study, for numerical calculations, epoxy resin was used as the matrix, and fibreglass yarn for the plane wave fabric was used as the reinforcement to prepare composite.

2.2 Methods

2.2.1 Calculation of the Effective Moduli of FRP Composite

Sinusoidal model of FRP composite, as developed earlier[23], was used to predict its geometric properties, for the next step in mechanical analysis. Present study is the continuation of earlier work[23], and hence the information and details are already reported therein.

Figure 1 presents the real cross-sectional area of FRP composite. The composite is made of a single layer plain woven fabric as reinforcement and thermoset resin. Figure 2 represents a cross-section along the warp of an idealized unit cell of a single layer plain woven fabric (the representative unit cell - RUC). More details about this geometric model of FRP composite are reported in earlier study[23].

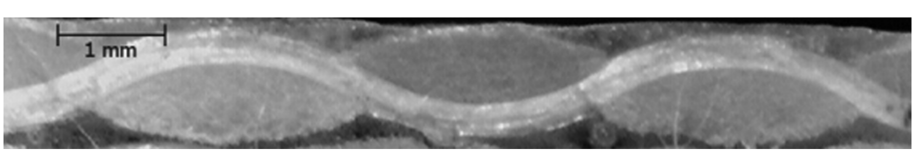

Fig. 1—Real cross-sectional area of FRP composite

Let the stress vector and strain vector in the global coordinate system (*x*, *y*, *z*) be denoted by σ and ε, as shown below:

$$\sigma^{\mathrm{T}} = [\sigma_x, \sigma_y, \sigma_z, \sigma_{yz}, \sigma_{zx}, \sigma_{xy}], \quad \ldots (1)$$

$$\varepsilon^{\mathrm{T}} = [\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{yz}, \gamma_{zx}, \gamma_{xy}] \quad \ldots (2)$$

As it is known[3], the stress-strain form of Hooke's law in the on-axis coordinates is $\sigma = C\varepsilon$, where *C* is the symmetric 6×6 elasticity matrix. In general, the strain energy of a linear elastic solid is

$$U = \frac{1}{2}\int_V \sigma^{\mathrm{T}} \varepsilon \, dV = \frac{1}{2}\int_V \varepsilon^{\mathrm{T}} C \varepsilon \, dV, \quad \ldots (3)$$

where $V = 4a^2 H$ is the volume of the RUC.

The RUC is assumed to be composed of three linear elastic phases, namely two warp yarns, two fill yarns, and resin matrix. Therefore, the strain energy can be expressed as

$$U = \frac{1}{2}\int_{V_w} \varepsilon_w^{\mathrm{T}} C_w \varepsilon_w \, dV + \frac{1}{2}\int_{V_f} \varepsilon_f^{\mathrm{T}} C_f \varepsilon_f \, dV + \frac{1}{2}\int_{V_r} \varepsilon_r^{\mathrm{T}} C_r \varepsilon_r \, dV \quad \ldots (4)$$

Indexes *w*, *f*, *r* denote warp, fill and resin matrix respectively. Homogenization of the RUC to determine its effective moduli is based on the iso-strain assumption (the strains are spatially uniform). By this iso-strain assumption the strain vector can be taken out of the integrand *U* and the strain energy can be written in the following form:

$$U = \frac{1}{2}\varepsilon^{\mathrm{T}}\left(\int_{V_w} C_w dV + \int_{V_f} C_f dV + \int_{V_r} C_r dV\right)\varepsilon = \frac{V}{2}\varepsilon^{\mathrm{T}} C_{eq} \varepsilon, \quad \ldots (5)$$

where C_{eq} is the equivalent elasticity matrix of the RUC, defined by

$$C_{eq} = \frac{1}{V}\int_V C \, dV \ . \quad \ldots (6)$$

Therefore, the equivalent elasticity matrix is the sum of the volume integral of the elasticity matrices over each phase, as shown below:

$$C_{eq} = \frac{1}{V}\left(\int_{V_w} C_w dV + \int_{V_f} C_f dV + \int_{V_r} C_r dV\right) . \quad \ldots (7)$$

The volume of the RUC is subdivided into the volume of two warp yarns $V_w = 2Al$, the two fill yarns $V_f = 2Al$ and the volume of the resin matrix $V_r = V - 4Al$, where *A* is the cross-sectional area of the yarns (same for warp and fill), and *l* denotes the arc length of one of these identical yarns. Let C_{eqw}, C_{eqf}, C_{eqr} denote

$$C_{eqw} = \frac{1}{2Al}\int_{V_w} C_w \, dV, \quad C_{eqf} = \frac{1}{2Al}\int_{V_f} C_f \, dV,$$

$$C_{eqr} = \frac{1}{V_r}\int_{V_r} C_r \, dV \quad \ldots (8)$$

Thus,

$$C_{eq} = \nu_w C_{eqw} + \nu_f C_{eqf} + \nu_r C_{eqr} \ . \quad \ldots (9)$$

The volume fractions of the warp yarns, fill yarns and resin are given by the following equations:

$$\nu_w = \nu_f = 2Al/V, \ \nu_r = V_r/V = 1 - \nu_w - \nu_f \quad \ldots (10)$$

According to sinusoidal model assumptions[23], the centerline of the warp and fill yarns path in RUC is specified by:

$$z_w(x) = \begin{cases} (t/2)\sin(\pi x/L_u) & \text{for} \quad x \in (0, L_u/2), \\ t/2 & \text{for} \quad x \in (L_u/2, a - L_u/2), \\ -(t/2)\sin(\pi(x-a)/L_u) & \text{for} \quad x \in (a - L_u/2, a), \end{cases} \quad \ldots (11)$$

$$z_f(y) = \begin{cases} -(t/2)\sin(\pi y/L_u) & \text{for} \quad y \in (0, L_u/2), \\ t/2 & \text{for} \quad y \in (L_u/2, a - L_u/2), \\ (t/2)\sin(\pi(y-a)/L_u) & \text{for} \quad y \in (a - L_u/2, a), \end{cases} \quad \ldots (12)$$

The derivatives of a functions $z_w(x)$ and $z_f(y)$ for $x \in (0, L_u/2)$, $y \in (0, L_u/2)$ are as given below:

$$\frac{dz_w(x)}{dx} = z_w' = \tan\beta_w = [\pi t/(2L_u)]\cos(\pi x/L_u), \ldots (13)$$

$$\frac{dz_f(y)}{dy} = z_f' = \tan\beta_f = [-\pi t/(2L_u)]\cos(\pi y/L_u) \quad \ldots (14)$$

The crimp angle β_c (Fig. 2) can be expressed as:

$$\tan\beta_c = \tan\beta_w(x=0) = |\tan\beta_f(y=0)| = \pi t/(2L_u) \quad \ldots (15)$$

The functions z'_w and z'_f over one-half of the length of the warp and fill yarn are

$$z'_w = \begin{cases} \tan\beta_c \cos(\pi x/L_u) & \text{for} \quad x \in (0,\, L_u/2)\,, \\ 0 & \text{for} \quad x \in (L_u/2,\, a - L_u/2)\,, \\ -\tan\beta_c \cos[\pi(x-a)/L_u] & \text{for} \quad x \in (a - L_u/2,\, a)\,, \end{cases} \quad \ldots (16)$$

$$z'_f = \begin{cases} -\tan\beta_c \cos(\pi y/L_u) & \text{for} \quad y \in (0,\, L_u/2)\,, \\ 0 & \text{for} \quad y \in (L_u/2,\, a - L_u/2)\,, \\ \tan\beta_c \cos[\pi(y-a)/L_u] & \text{for} \quad y \in (a - L_u/2,\, a)\,. \end{cases} \quad \ldots (17)$$

The volume elements of a warp (w) and fill (f) yarn are represented by

$$dV_w = A\sqrt{1+(z'_w)^2}\,dx\,,\; dV_f = A\sqrt{1+(z'_f)^2}\,dy \quad \ldots (18)$$

The filaments composing the yarn are assumed to be parallel and not twisted. Denote the orthogonal, on-axis directions at an arbitrary point in a yarn as (1,2,3), where the 1-axis is parallel to the filaments, and the 2-axis is normal to the z-axis of the RUC. It is a local coordinate system connected with yarn. Let θ denotes the angle of rotation about the z-axis transforming the Cartesian systems $(x, y, z) \to (x_1, y_1, z_1)$, where z_1-direction is the same as the z-direction. Consider a second rotation through angle β about the y_1-axis transforming the Cartesian systems $(x_1, y_1, z_1) \to (1, 2, 3)$, where 2-direction is the same as the y_1-direction. Figure 3 shows rotations from RUC directions (x,y,z) off-axis configuration to yarn directions (1,2,3) on-axis configuration. (x,y,z) is the global coordinate system.

The direction cosine matrix of the rotations from RUC direction to the yarn (1,2,3) directions[3] are given by

$$\begin{array}{cc} & \begin{array}{ccc} x & y & z \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \end{array} = \begin{bmatrix} \cos\beta\cos\theta & \cos\beta\sin\theta & \sin\beta \\ -\sin\theta & \cos\theta & 0 \\ -\sin\beta\cos\theta & -\sin\beta\sin\theta & \cos\beta \end{bmatrix} \quad \ldots (19)$$

In the on-axis system, let σ_0 denotes the vector of stress components; ε_0 the vector of engineering strains; C_0 the symmetric 6×6 elasticity matrix; and S_0, the symmetric 6×6 compliance matrix. The components of the stress and strain vectors are

$$\sigma_0^{\mathrm{T}} = [\sigma_1, \sigma_2, \sigma_3, \sigma_{23}, \sigma_{31}, \sigma_{12}]\,, \quad \ldots (20)$$

$$\varepsilon_0^{\mathrm{T}} = [\varepsilon_1, \varepsilon_2, \varepsilon_3, \gamma_{23}, \gamma_{31}, \gamma_{12}]\,. \quad \ldots (21)$$

The stress-strain form of Hooke's law in the on-axis coordinates is $\sigma_0 = C_0\varepsilon_0$ and the strain-stress form is $\varepsilon_0 = S_0\sigma_0$. The elasticity matrix is the inverse of the compliance matrix $C_0 = S_0^{-1}$ or $S_0 = C_0^{-1}$. The yarn is assumed to be a transversely isotropic material in the (1,2,3) directions. The five independent material properties are:

- the modulus of elasticity along the 1-axis E_1,
- the modulus of elasticity along the 2- and 3-axes E_2,
- Poisson's ratio in the 1-2 and 1-3 planes ν_{12},
- Poisson's ratio in the 2-3 plane ν_{23},
- shear modulus in the 1-2 and 1-3 planes G_{12}.

The on-axis compliance matrix is

$$S_0 = \begin{bmatrix} E_1^{-1} & -\nu_{12}E_1^{-1} & -\nu_{12}E_1^{-1} & 0 & 0 & 0 \\ -\nu_{12}E_1^{-1} & E_2^{-1} & -\nu_{23}E_2^{-1} & 0 & 0 & 0 \\ -\nu_{12}E_1^{-1} & -\nu_{23}E_2^{-1} & E_2^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu_{23})E_2^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{12}^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{12}^{-1} \end{bmatrix} \quad \ldots (22)$$

Let the stress vector and strain vector in the off-axis system (x,y,z) be denoted by σ and ε, as given below:

$$\sigma^{\mathrm{T}} = [\sigma_x, \sigma_y, \sigma_z, \sigma_{yz}, \sigma_{zx}, \sigma_{xy}]\,, \quad \ldots (23)$$

$$\varepsilon^{\mathrm{T}} = [\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{yz}, \gamma_{zx}, \gamma_{xy}]\,. \quad \ldots (24)$$

The transformation of the stress and strain vectors to the on-axis directions from the off-axis system is accomplished by the orthogonal 6×6 matrices T_σ and T_ε as follows $\sigma_0 = T_\sigma\sigma_1, \varepsilon_0 = T_\varepsilon\varepsilon_1$. The form of matrices C_0 and $\mathrm{T}_\varepsilon T_\varepsilon$ can be found in literature, especially from the area of the composite materials[2,24]. Using transformation relations, the direction cosine matrix and the definition of the engineering strain vector are shown in Eq. (25).

$$T_\varepsilon = \begin{bmatrix} a_{11}^2 & a_{12}^2 & a_{13}^2 & a_{12}a_{13} & a_{11}a_{13} & a_{11}a_{12} \\ a_{21}^2 & a_{22}^2 & a_{23}^2 & a_{22}a_{23} & a_{21}a_{23} & a_{21}a_{22} \\ a_{31}^2 & a_{32}^2 & a_{33}^2 & a_{32}a_{33} & a_{31}a_{33} & a_{31}a_{32} \\ 2a_{21}a_{31} & 2a_{22}a_{32} & 2a_{23}a_{33} & (a_{23}a_{32}+a_{22}a_{33}) & (a_{23}a_{31}+a_{21}a_{33}) & (a_{22}a_{31}+a_{21}a_{32}) \\ 2a_{11}a_{31} & 2a_{12}a_{32} & 2a_{13}a_{33} & (a_{13}a_{32}+a_{12}a_{33}) & (a_{13}a_{31}+a_{11}a_{33}) & (a_{12}a_{31}+a_{11}a_{32}) \\ 2a_{11}a_{21} & 2a_{12}a_{22} & 2a_{13}a_{23} & (a_{13}a_{22}+a_{12}a_{23}) & (a_{13}a_{21}+a_{11}a_{23}) & (a_{12}a_{21}+a_{11}a_{22}) \end{bmatrix}$$

Equation (25)

Substitute the stress and strain transformations into the on-axis material law to get $T_\sigma \sigma_1 = C_0 T_\varepsilon \varepsilon_1$. Next, $\sigma_1 = T_\sigma^{-1} C_0 T_\varepsilon \varepsilon_1 = C_1 \varepsilon_1$, where the off-axis elasticity matrix for the yarn is denoted by C_1, as shown below:

$$C_1 = T_\sigma^{-1} C_0 T_\varepsilon \ . \quad \ldots (26)$$

From the tensor transformation equations it can be shown that the inverse of the stress transformation matrix T_σ is equal to the transpose of the strain transformation matrix $T_\varepsilon^{\mathrm{T}}$, so $T_\sigma^{-1} = T_\varepsilon^{\mathrm{T}}$. Hence, we can write the off-axis elasticity matrix C_1 as

$$C_1(\theta, \beta) = T_\varepsilon^{\mathrm{T}} C_0 T_\varepsilon \ . \quad \ldots (27)$$

For the warp yarn angle $\theta = 0$ and the angle $\beta = \beta_w(x)$, and for the fill yarn angle $\theta = \pi/2$ and the angle $\beta = \beta_f(x)$, $\beta_w = \arctan(z'_w)$ and $\beta_f = \arctan(z'_f)$. The resin matrix is assumed to be homogenous and isotropic. Hence, there are two independent material properties, namely the modulus of elasticity *Er* and Poisson's ratio *Vr*. The compliance matrix for the resin is

$$S_r = \begin{bmatrix} E_r^{-1} & -\nu_r E_r^{-1} & -\nu_r E_r^{-1} & 0 & 0 & 0 \\ -\nu_r E_r^{-1} & E_r^{-1} & -\nu_r E_r^{-1} & 0 & 0 & 0 \\ -\nu_r E_r^{-1} & -\nu_r E_r^{-1} & E_r^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu_r)E_r^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu_r)E_r^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu_r)E_r^{-1} \end{bmatrix} \quad \ldots (28)$$

The elasticity matrix for the resin is $C_r = S_r^{-1}$. From Eqs (8), (16), (17), (18), (27) the phase equivalent elasticity matrices for the warp and fill yarns are expressed as

$$C_{eqw} = \frac{1}{2Al} \int_{V_w} C_w \, dV_w = \frac{1}{l} \int_0^{2a} C_1(0, \beta_w) \sqrt{1 + (z'_w)^2} \, dx \, , \quad \ldots (29)$$

$$C_{eqf} = \frac{1}{2Al} \int_{V_f} C_f \, dV_f = \frac{1}{l} \int_0^{2a} C_1(\pi/2, \beta_f) \sqrt{1 + (z'_f)^2} \, dx \quad \ldots . (30)$$

The resin is assumed to be homogeneous and isotropic, so its equivalent elasticity matrix is equal to its elasticity matrix, i.e.

$$C_{eqr} = \frac{1}{V_r} \int_{V_r} C_r \, dV = C_r \ . \quad \ldots (31)$$

It turns out that the equivalent elasticity matrices for the warp and fill yarns are the same if the integrations in Eqs (29) and (30) are performed over the interval from 0 to *a* as for the interval of *a* to 2*a*. As a result, we can integrate over one quarter of the RUC to determine the equivalent moduli for the entire RUC, as given below:

$$C_{eqw} = \frac{2}{l} \int_0^{a} C_1(0, \beta_w) \sqrt{1 + (z'_w)^2} \, dx \, , \quad \ldots (32)$$

$$C_{eqf} = \frac{2}{l} \int_0^{a} C_1(\pi/2, \beta_f) \sqrt{1 + (z'_f)^2} \, dy \ . \quad \ldots (33)$$

The integrations indicated in Eqs (32) and (33) have been calculated numerically using a computer program in Mathematica environment by Wolfram Research[25].

Note that $C_1(0, \beta_w)$ and $C_1(\pi/2, \beta_f)$ are the elasticity matrices for the warp and fill yarns in the global coordinate directions (*x*,*y*,*z*) so-called off-axis system (Fig. 3). The equivalent elasticity matrix C_{eq} of the RUC is calculated using Eqs (9) and (10).

3 Results and Discussion

The numerical results for the effective moduli of FRP composite have been presented below. The calculations were made for the following geometric parameters: volume fraction V_f = 0.64, diameter of filament d_f = 0.007 mm, yarn spacing a = 1.411 mm, yarn packing density p_d = 0.75, and number of filaments from n = 2000 to n = 6000. The geometric parameters (Table 1) were obtained using procedures described earlier[23]. The composite used was an epoxy resin as the matrix, and fibre glass yarn for the plane wave fabric as the reinforcement. Yarn and resin properties are presented in Table 2.

Next, computer program (created in Mathematica environment) has generated results in the form of engineering constants (Table 3). It should be pointed out that only a fragment of results has been included in the tables. For comparison, Table 3 presents engineering constants obtained by Naik's model[9] (in parenthesis).

The results obtained are the same as obtained from Naik for very small crimp angles, but differ as the crimp angle increases. These differences for increasing crimp angles are expected, since the preform geometry[9] is limited to small crimp angles. Besides, if the fibre volume fraction specified for the unit cell is too small, it is necessary to add an additional resin layer (not considered by Naik) of thickness t_r to the unit cell (Fig. 2). Compared with the results obtained by Naik[9] it can be pointed out that the effective Young moduli and shear moduli are essentially the same as obtained by Naik[9] for n = 2000 (where n is the number of filaments). As the number of filaments in the yarns increases, the effective in-plane moduli E_{xx}, E_{yy}, G_{xy} decrease and the transverse moduli E_{zz}, G_{yz}, G_{zx} increase. The Young moduli E_{xx}, E_{yy}, E_{zz} from the analysis are, in general, less than those from the earlier analysis[9], but transverse shear moduli G_{yz}, G_{zx} are greater than those obtained eralier[9]. The in-plane Poisson ratio ν_{xy} decreases with increasing n, and the in-plane

Table 1—Geometric parameters of the model

Number of filaments	a mm	t mm	L_u mm	β_c deg
2000	1.411	0.0857	0.5869	12.90
4000	1.411	0.1739	0.6297	23.50
6000	1.411	0.2666	0.6826	31.53

Table 2—Yarn and resin properties

Material	E_1 GPa	E_2 GPa	G_{12} GPa	ν_{12} -	ν_{23} -
Yarn	144.80	11.73	5.52	0.23	0.30
Resin	3.45	3.45	1.28	0.35	0.35

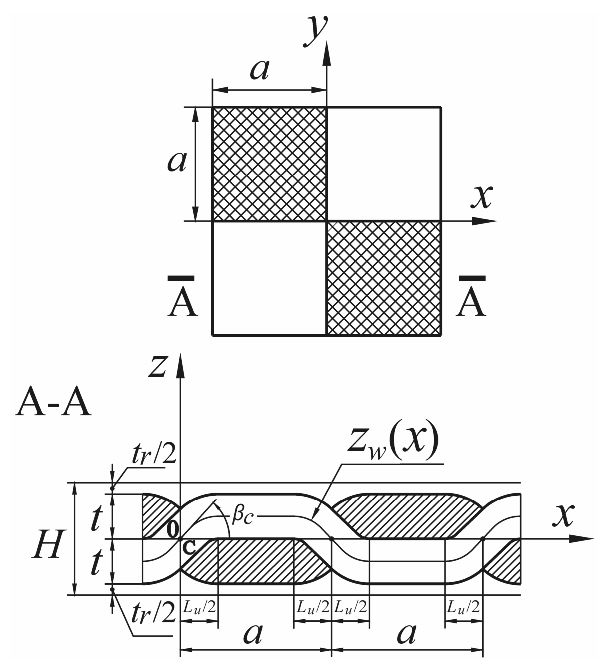

Fig. 2—Cross-section of representative unit cell (RUC) along the warp yarn [$2a \times 2a$–the dimensions of the RUC; H–the thickness of the RUC; t–thickness of the yarn; t_r–thickness of resin layer; L_u –undulation length; and β_C –crimp angle]

Table 3—Engineering constants for present model

Number of filaments	$E_{xx} = E_{yy}$ GPa	E_{zz} GPa	$G_{yz} = G_{zx}$ GPa	G_{xy} GPa	ν_{xy} -	$\nu_{xz} = \nu_{yz}$ -
2000	66.0648 (66.0613)	11.4475 (11.4495)	5.0405 (5.0415)	4.8886 (4.8885)	0.03292 (0.03292)	0.35231 (0.35232)
4000	62.2436 (62.3531)	11.6091 (11.6922)	6.3593 (6.3138)	4.8646 (4.8648)	0.01937 (0.02021)	0.44319 (0.43692)
6000	57.4660 (58.3916)	12.4008 (12.8242)	7.8662 (7.5309)	4.8313 (4.8343)	0.01828 (0.00858)	0.51117 (0.47495)

Values in parentheses are obtained by Naik's model.

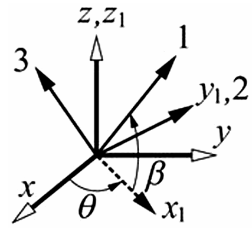

Fig. 3—Off-axis and on-axis coordinates system

Poisson ratios from the present analysis are less than those from the Naik's analysis. Finally, the transverse Poisson ratios v_{xz}, v_{yz} from the present analysis are greater than those predicted from the Naik's analysis, and both analyses show that v_{xz}, v_{yz} initially increase with n and then decrease with further increasing n.

4 Conclusion

Proposed method of analysis can be used for theoretical modeling of FRP composites. The work consists of developing constitutive model of specific textile composite material, presently plain weave. There is a numerical constitutive model which approximate the mechanical behavior at the local scale. It can be applied to other models of FRP composites with other woven fabric geometries, such as Pierce's model of plane wave, Pierce's elliptic model, Kemp model with race track cross-section, and Hearle's model. Using geometrical considerations it is possible to obtain from this model basic mechanical parameters such as Young's modulus, useful for stress analysis. It is typical theoretical work, and all results have been obtained by numerical calculations. For this purpose, a computer program has been developed. The program code has been written in Mathematica language.

Present analysis compares very well with the analysis performed by Naik for small crimp angles, which correspond to small values of n. Discrepancies occur for larger crimp angles, because Naik's model is based on the small crimp angles.

References

1 Hull D & Clyne T W, *An Introduction to Composite Materials* (Cambridge University Press, Cambridge, UK), 1997, 158.
2 Jones R M, *Mechanics of Composite Materials*, 2nd edn (Taylor & Francis, UK), 1998, 26.
3 Tsai S W & Hahn H T, *Introduction to Composite Materials* (Technomic Publishing Co., Westport CT), 1980, 277.
4 *Cox B N & Flanagan G,* Handbook of Analytical Methods for Textile Composites*, Nasa CR 4750* (Langley Research Center, Hampton, Virginia)*, 1997.*
5 Poe C C & Harris C E, *Proceedings, Mechanics of Textile Composites Conference*, NASA Conference Publication 3311, Part 1&2 (Langley Research Center, Hampton, VA), 1995, 555.
6 Chou T W & Ishikawa T, *Textile Structural Composites*, Vol. 3 (Elsevier Science Publishers, Amsterdam), 1989, 209.
7 Ishikawa T & Chou T W, *J Mater Sci*, 17 (1982) 3211.
8 Ishikawa T & Chou T W, *J Compos Mater*, 16 (1982) 2.
9 Naik R A, *Analysis of Woven and Braided Fabric Reinforced Composites*, NASA CR 194930 (Langley Research Center, Hampton, Virginia), 1994.
10 Naik R A, Ifju P G & Masters J E, *Proceedings, 4th NASA/DOD Advanced Composites Technology Conference*, Vol. 1 (Langley Research Center, Hampton, VA), 1993, 525.
11 Rosen B W, Chatterjee S N & Kibler J J, *Proceedings, 4th Conference on Composite Material: Testing and Design* (ASTM, Philadelphia, PA), 1977, 243.
12 Naik N K & Ganesh V K, *Composites*, 26 (4) (1995) 281.
13 Scida D, Aboura Z, Benzeggagh M L & Bocherens E, *Compos Sci Technol*, 59 (1999) 505.
14 Masters J E, Foye R L, Pastore C M & Gowayed Y A, *J Compos Tech Res*, 15 (2) (1993) 112.
15 Byun J H, *Compos Sci Technol*, 60 (2000) 705.
16 Ko F K & Pastore C M, in *Recent Advances in Composites in the United States and Japan, ASTM STP 864*, edited by J R Vinson and M Taya (ASTM, Philadelphia, PA), 1985, 428.
17 Dow N F & Ramnath V, *Analysis of Woven Fabrics for Reinforced Composite Materials*, NASA CR 178275 (Langley Research Center, Hampton, Virginia), 1987.
18 Naik R A, *J Compos Mater*, 29 (17) (1995) 2334.
19 Dems K, *J Theor Appl Mech*, 4 (38) (2000) 807.
20 Dems K, *Fibres Text East Eur*, 28 (2000) 52.
21 Wisniewski J, *Fibres Text East Eur*, 47 (2004) 58.
22 Dems K & Turant J, *Engineering Optimization*, 41 (4) (2009) 351.
23 Szablewski P, *J Ind Text*, 38 (4) (2009) 277.
24 Hyer M W, *Stress Analysis of Fiber-Reinforced Composite Materials* (WCB McGraw-Hill, Boston, MA), 1998, 161.
25 Wolfram S, *The Mathematica Book*, 5th edn (Wolfram Media, Inc., Champaign, IL), 2003.