Yarn hairiness – Theory about total number of fibre hair

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Received 12 August 2014; revised received and accepted 26 May 2015

Yarn hairiness is determined using projection of hairs perpendicular to the yarn axis. Measurement is done using photo cell which receives light from a small area above the yarn surface. The hairs that project beyond the specific adjusted distance – which is generally 3 mm – intercept the beam of light that falls on the photo cell, causing photo cell to generate a signal. Using this signal, number of hairs in a given length of yarn can be determined. However, the measurement does not cover full circumference of the yarn. A theory is developed in the present work to overcome this problem.

Keywords: Fibre hair length, Staple fibres, Yarn hairiness

Staple fibre yarns are formed by twisting of somewhat parallelized intermingled fibres. Because of overlapping of the staple fibres, continuous linear structure in the form of yarn is produced. Each fibre has two ends distinctly separated over the length of the yarn. Ends of most of the fibres are embedded well within the yarn structure and stay there because of the lateral inward force developed on the twisted structure with the application of even small linear tension. However, for some fibres, portions near one of their ends are near the yarn boundary surface and do not get embedded in the yarn structure. These protruding fibre portions are considered as yarn hair.

Hair on yarn and therefore on the fabric produced from that yarn gives fuzzy appearance to the yarn and the fabric. Lesser the number of hairs, the better is the appearance of the fabric. It is, therefore, desirable to have staple fibre yarn in which there are no hair fibres protruding from the yarn surface. But in reality it is not possible. In fact, it can be theoretically shown that staple fibre yarns will have a certain minimum number of hairs. It is observed that the hairs of varying length are present on any staple fibre yarn. Fibre characteristics and processing parameters affecting yarn hairiness have been reported in the literature. Some studies have shown that hairiness is mainly dependent on the fibre fineness, staple length, yarn diameter and twist.

Many direct and indirect methods have been developed to measure yarn hairiness. With the use of modern technology, hairs are sensed by a miniature photo cell when illuminated yarn is passed across its slit. Figure 1 shows a schematic diagram of a standard yarn hairiness measuring device. Light source and sensor are in a plane perpendicular to the yarn axis. A narrow beam of light travels from source to sensor such as that the beam is at a distance ‘d’ from the yarn surface as shown in the figure. Yarn is moved in one direction at a constant speed. As and when the hairs that project beyond the specific adjusted distance ‘d’ – which is generally 3 mm – intercepts the beam of light that falls on the photo sensor, it causes photo cell to generate a signal. The signal is fed to a digital counter, which goes on counting the number of protruding hair in a given time. Knowing the yarn speed, number of hairs in a given length of yarn can be determined.

Presently, there are units available which can simultaneously measure the number of hairs protruding more than 3 mm to more than 25 mm, at an interval of 1 mm. However, the measurement does not cover the full circumference of the yarn. A theory is therefore developed in the present work to overcome this problem and find out the actual number of hairs with length/protrusion distribution perpendicular to the yarn axis considering the measurement of full circumference of the yarn.

Theory

Let us assume yarn axis to be in the z-direction. We assume that the yarn diameter is very small in comparison with the length of fibres protruding out from the yarn. It is also assumed that the hair is straight and of length l, which is shown as OP in the Fig. 2. Any random hair OP will be at an angle Ø with the z-axis. Let projection of OP on x-y plane make an angle θ with the x-axis. Here x-y plane is perpendicular to the yarn axis at the point of origin of the hair. y-axis is taken in such a way that it is perpendicular to the optical beam used for sensing the
hair and x-axis is in the direct
ion parallel to the
optical beam. Radial coordinates of point P can then
be written as \((l, \varphi, \theta)\). Cartesian coordinates of P can
be written as

\[
\begin{align*}
    x &= l \sin \varphi \cos \theta \\
    y &= l \sin \varphi \sin \theta \\
    z &= l \cos \varphi
\end{align*}
\]  

(1)  
(2)  
(3)

Measurement of hairiness of yarn is carried out by
using optical sensing device such that
the
light beam
is sent perpendicular to the yarn axis and generally at
a distance of 3mm from the yarn axis. This has been
explained earlier with the help of Fig. 1. On the other
hand beam impinges on a photo sensor. Yarn is drawn
at a constant speed along its axis. As and when a hair
fibre crosses the light beam, sensor takes note of the
obstruction and keeps count of the number of such
obstructions. Knowing the yarn speed and the number
of hair counted in a given time, yarn hairiness
measured as number of hair protruding more than
3mm from the yarn axis per unit length of yarn, which
may be hundred meters or one km, is calculated.
Since point P – end of the fibre hair – should lie at a
perpendicular distance of more than 3mm from the
yarn axis, OP’s projection in the z-direction is not
important. Therefore, projection of OP in x-y plane is
considered for determination of hairiness.

Let us assume that hair measurement is done along
y-axis. Whatever may be the actual length of the hair
and its inclination with respect to the z-axis (angle \(\varphi\)),
y component of its projection on x-y plane should be
greater than 3mm to get counted as hair. In other
words, any hair satisfying the condition

\[
y = l \sin \varphi \sin \theta > 3\text{mm},
\]

(4)
will be counted as hair of length greater than 3 mm.
Since the measurement is taking place basically in x-y
plane, individual values of \(l\) and \(\varphi\) do not have any
importance. But the value of \(\theta\) becomes very important.
Let us write projection of hair on x-y plane as \(l_{xy}\), where

\[
l_{xy} = l \sin \varphi
\]

(5)

Thus, amongst all the fibres protruding from the
yarn having \(l_{xy}\) greater than 3mm, only those with
orientation angle \(\theta\) such that

\[
l_{xy} \sin \theta > 3\text{mm}
\]

(6)
will get counted as hair.

Let us assume that there are \(n(l)dl\) fibres of
protrusion length \(l\) such that

\[
l_{xy} < l < l_{xy} + dl
\]

Of these \(n(l)dl\) fibres, only those with angle of
orientation \(\theta > \theta_3\) such that \(l \sin \theta_3 = 3\text{mm}\) will be
counted as hair. This can be clearly seen from Fig. 3,
where \(\theta_3\) is the angle made by projection of the hair
on x-y plane with x-axis. As shown in the figure,
projections OA and OB of length \(l\) make angles \(\theta_3\) and
\(\pi - \theta_3\), respectively, with the x-axis. Of all the \(n(l)dl\)
fibres of length \(l\), only those with projections lying in
the angular zone AOB will be counted as hair. Since
there is no preferred direction for orientation of these
fibres, we assume that they are equally distributed
along the circle (Fig. 3), as indicated below:

\[
LAOB = \pi - 2\theta_3
\]
Therefore, \( dn_3(l) \), the number of fibres getting counted as hair amongst the fibres of length \( l \) to \( l+dl \) can be written as:

\[
 dn_3(l) = \left( \frac{\pi-2 \sin^{-1}\left(\frac{3}{l}\right)}{2\pi} \right) \times n(l)dl 
\]  \( \cdots \) (8)

Total number of hair counted can therefore be written as:

\[
 \int_3^\infty dn_3(l) = \int_3^\infty \left( \frac{\pi-2 \sin^{-1}\left(\frac{3}{l}\right)}{2\pi} \right) \times n(l)dl 
\]  \( \cdots \) (9)

Choosing 3mm or more as protrusion from the yarn axis and perpendicular to it, for consideration of yarn hairiness, is arbitrary. It can be chosen to have any value. Let us assume it to be \( y_o \). Then total number of hair \( N(y_o) \) of length more than \( y_o \) getting counted in the experiment will be given by the following relationship:

\[
 N(y_0) = \int_{y_0}^\infty dn_3(y_o) = \int_{y_0}^\infty \left( \frac{\pi-2 \sin^{-1}\left(\frac{y_o}{l}\right)}{2\pi} \right) \times n(l)dl 
\]  \( \cdots \) (10)

It may be noted here that \( N(y_o) \) is a measured quantity. But the function \( n(l) \) is an unknown. However, from experimental values of \( N(y_o) \) determined for various values of \( y_o \), one can work out the distribution function \( n(l) \) which gives the actual distribution of fibre length in the hairs around the yarn. From this, actual total number of hairs longer than \( y_o \) around the yarn for the given length can be determined. This can be worked out using the following example.

**Example**

Let us assume that there are no hairs protruding more than 12mm perpendicular to the yarn axis. Let us count the number of hairs protruding more than 3mm up to 12mm at an interval of 1mm. These values of measurements on a yarn for the length of 100 m are shown in Table 1.

Using \( dl \) equal to 1mm and \( N(l) \) equal to 10 hairs for protruding length \( l=11\)mm in Eq. (10), we get

\[
 n(11) = \left( \frac{\pi-2 \sin^{-1}\left(\frac{11}{12}\right)}{2\pi} \right) \times n(11)
\]

\( n(11) = 76 \).

Here, the average protruding length of hairs will be 11.5mm for the group lying between 11mm and 12mm. But for simplicity of calculation, length of all the fibres is assumed to 11mm. This does not affect the methodology of calculation, though the actual values may change a little.

It means, in reality there are 76 hairs protruding from the yarn axis having length between 11mm and 12mm which are denoted as \( n(11) \). But of these only 10 are getting counted when measurement is done for hairs of length greater than 11mm. Of these 76 hairs, number of hairs which will be getting counted in the experiment on hairiness measurement at different levels is given in Table 2.

Of the 25 fibres which are counted for length greater than 10mm, 14 fibres are of length greater than 11mm. This means, there are 11 fibres in the length range 10 - 11mm. As has been done for 11mm hairiness level above by using Eq. (10), we find that total number of fibres in the length range 10 - 11mm is 79. For the length range 9 - 10mm, we have 18 fibres of length range 11 - 12mm and 15 fibres of

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**Table 1—Number of protruding hairs counted**

<table>
<thead>
<tr>
<th>Length ( l ), mm</th>
<th>No. of hairs counted ( N(l) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>9</td>
<td>50</td>
</tr>
<tr>
<td>8</td>
<td>90</td>
</tr>
<tr>
<td>7</td>
<td>150</td>
</tr>
<tr>
<td>6</td>
<td>235</td>
</tr>
<tr>
<td>5</td>
<td>350</td>
</tr>
<tr>
<td>4</td>
<td>500</td>
</tr>
<tr>
<td>3</td>
<td>700</td>
</tr>
</tbody>
</table>

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**Fig. 3**—Projection of hair on x-y plane such that it makes an angle of \( \theta_3 \) with x-axis. Its projection in the y direction is 3mm. All the fibres of length \( l \) within angular zone AOB will be counted as hairs of length greater than 3mm.

\[ \theta_3 = \sin^{-1}\left(\frac{3}{l}\right) \]  \( \cdots \) (7)

It may be noted here that \( N(y_o) \) is a measured quantity. But the function \( n(l) \) is an unknown. However, from experimental values of \( N(y_o) \) determined for various values of \( y_o \), one can work out the distribution function \( n(l) \) which gives the actual distribution of fibre length in the hairs around the yarn. From this, actual total number of hairs longer than \( y_o \) around the yarn for the given length can be determined.
length range 10 - 11mm. Thus, amongst 50 fibres counted of length greater than 9 mm, only 17 belong to the length range 9 - 10mm. Using Eq. (10) for this range, we get total number of fibres in the length range 9 - 10mm as 118. Continuing in this manner, we can find out the actual number of fibres in each length range. These calculated \( n(l) \) values for different \( l \) values corresponding to the thought experimental values as given in Table 1, are given in Table 3.

The total number of hairs of length greater than 3mm is therefore 2138 when counted all along the circumference of the yarn for a length of 100 m, though value as would be measured by the instrument is 700.

It may be noted that the calculations shown here are based on assumed readings from the hairiness measuring device. But the same logic is applicable if actual readings from measurement are considered. Procedure remains the same. Only numerical values will change.

**References**