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Nonlinear Maxwell modelling of inverse relaxation in yarns and fabrics

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An attempt has been made to fit the derived equation on the experimental inverse relaxation curves by employing Levenberg-Marquardt's method for nonlinear regression and calculation of the constant values involved in the equation. The relaxation curves can be classified into ordinary relaxation, mixed relaxation and inverse relaxation curves. There seems to be good concurrence between the experimental and the fitted inverse relaxation curves.

Keywords: Cotton, Fabric, Inverse relaxation, Mathematical model, Non-linear regression, Visco-elastic recovery, Yarn

1 Introduction

Time dependent mechanical properties such as stress-relaxation, creep and creep recovery in textile materials have been the subjects of detailed study in the past. But a closely associated property, namely inverse relaxation seems to have received little attention. If a fibre of any polymeric material is strained upto some extension level and is then allowed to retract partially so that there still exists some strain in the fibre, (considering some stress present in the fibre), it is observed that the stress in the fibre goes on increasing with time. The rate of increase in stress is high in the beginning, but it decreases progressively, so that the stress almost levels off after a long duration. This particular phenomenon called as inverse relaxation has received very little attention in the field of polymer research.

Inverse relaxation phenomenon is mentioned by Woods[1] although no experimental evidence of it is documented. Vitkauskas and Matukonis[2-4] studied the phenomenon of inverse relaxation by assuming a fibre model consisting of a spring in parallel with two Maxwellian elements, one of the elements having period of relaxation many times shorter than the other. Though the theory is able to explain the phenomenon qualitatively, it does not explain it on the quantitative basis. Inverse relaxation has also been studied in cotton fibres and yarns at the Central Institute for Research on Cotton Technology independently[5-8]. The work carried out till now has been mainly of an exploratory nature. Subsequently, the study was extended to inverse relaxation in spun yarns of different materials[6]. The study included ring-spun yarns of cotton, polyester, viscose and jute. A further investigation was carried out in the fibres of cotton, polyester, viscose, wool and ramie[7]. All these polymeric fibres which included natural as well as synthetic fibres exhibited the phenomenon of inverse relaxation. These preliminary studies revealed that any polymeric substance which shows delayed recovery mechanism active in it, exhibits this phenomenon.

Pociene and Vitkauskas[9] studied the inverse stress relaxation (IR) and viscoelastic recovery (VR) that takes place in acetate and polyester multifilament yarns is dependent on the mechanical pre-history. The above-mentioned time-effects were investigated in two different [stress relaxation (R) and creep (C)] testing cycles. The fact that inverse stress relaxation process takes place in C- test cycle, i.e. after previous sustaining the specimen at constant load, was experimentally confirmed. It was shown that viscoelastic recovery is the slower process than the inverse stress relaxation. At identical elongations of the yarns at the end of loading period the inverse relaxation and viscoelastic recovery processes go on similarly regardless of the character of testing cycle. This phenomenon has been reported by Nachane *et al.*[5-7,10] where they termed it as inverse creep. It is a corollary of inverse relaxation. Nachane gave an equation governing the inverse relaxation in the polymeric materials. The equation was based on Maxwellian model which suggested that the motion of

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the dashpot could be responsible for the occurrence of inverse relaxation. In the present study, an experiment was designed to prove the equation numerically and to obtain the values of constants used in the equation. The other objective of the experiment was to understand the flow of the inverse relaxation from yarn to fabric stage.

2 Inverse Relaxation and its Explanation using Maxwellian Model

To understand the change in stress with time in an inverse relaxation experiment, let us consider a typical curve observed in such an experiment; rate of strain α is assumed to be constant. As can be seen from Fig.1, the portion of stress-time curve *OA* corresponds to continuous straining up to αt_1 over a period of time zero to t_1. The stress developed at *A* is S_a. The portion *AB* corresponds to retraction from time t_1 to t_2 at the same rate of straining. Also, $t_2 < 2t_1$ i.e. the retraction is not complete. At point B, the stress in the fibre is S_b.

Also, the strain in the fibre at this time t_2 is $\alpha(2t_1-t_2)$. The fibre is now constrained to remain at this extension of $\alpha(2t_1-t_2)$. From this point as the time increases, the stress also goes increasing. If the stress is observed at a sufficiently long interval of time (of the order of a few thousand seconds), e.g. at point *C* as shown, then we would have almost reached a steady state point. Let this stress be S_c. Then we have $S_c > S_b$.

Due to variability in different types of fibres studied, in the actual experiments carried out, instead finding out stresses S_a, S_b, S_c developed, the loads W_a, W_b and W_c developed on individual fibre specimen were measured and a ratio $(W_c-W_b)/W_a$ was defined as an inverse relaxation index[10]. This index was experimentally determined for all the fibres and yarns stated above, at different levels of extension and retraction.

It was found that the phenomenon of inverse relaxation is dependent on the fibre material, the extension level upto which it is stretched and the retraction level upto which it is retracted. This is a time-dependent phenomenon much like stress relaxation and creep. At any particular level of extension and retraction, the rate of increase in stress is fast initially, but decreases with time. To understand the behaviour of any polymeric fibre at different retraction levels but with the same extension level, let us consider the curve, as shown in Fig.2. Extension and retraction are at the same constant rate α.

The section *OA* represents increasing stress with time as the strain increases at the rate of extension α. The load in the specimen for the time $t>t_2$, i.e during inverse relaxation, can therefore be written[8,10] as

$$F = K_0\alpha(2t_1-t_2) + \sum \cap_i \alpha \left[\left(2e^{\left(-\frac{K_i}{\cap_i}(t_2-t_1)\right)}\right) - e^{\left(-\frac{K_i}{\cap_i}t_2\right)} - 1\right] e^{-\frac{K_i}{\cap_i}(t-t_2)} \quad \ldots (1)$$

The same equation with relaxation time $\tau_{i=}\frac{\cap_i}{K_i}$ becomes

$$F = K_0\alpha(2t_1-t_2) + \sum \cap_i \alpha \left[\left(2e^{\left(-\frac{t_2-t_1}{\tau_i}\right)}\right) - e^{\left(-\frac{t_2}{\tau_i}\right)} - 1\right] e^{-\frac{t-t_2}{\tau_i}} \quad \ldots (2)$$

While carrying out the inverse relaxation test the specimen is extended by a certain amount and then

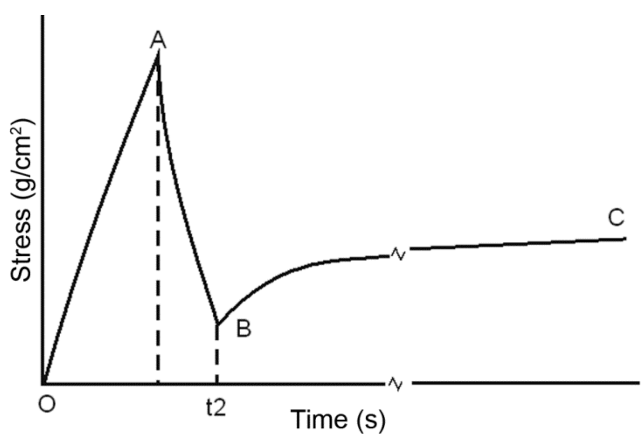

Fig. 1 — A typical stress *vs* time curve in an inverse relaxation experiment

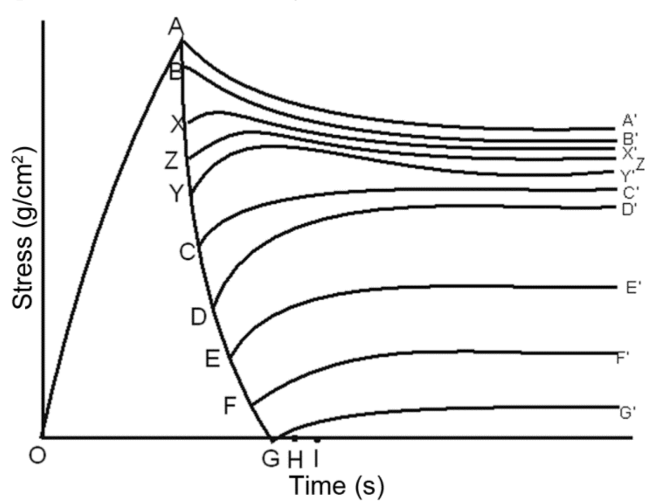

Fig. 2 — Stress *vs* time curve in inverse relaxation experiment at different retraction levels corresponding to a given extension level

retracted to an level with some tension still present in the specimen. The specimen is relaxed at this level of retraction for inverse relaxation to occur. Above equation[5] gives load in the fibre at any time t, after it has been extended upto strain $e_1 = \alpha t_1$ in the time t_1 and retracted by $\alpha(t_2\text{-}t_1)$ during the time interval from t_1 to t_2 to a residual strain $e_2 = \alpha(2t_1\text{-}t_2)$; both the extension and retraction being carried out at the constant rate of straining α.

3 Materials and Methods

In order to cover a range, two types of 100 % cotton yarns [coarse (24s Ne count) and fine (60s Ne count)] were selected. Two fabrics having warp and weft from the same yarns were got woven. One of the fabrics has 24s Ne count as warp and weft yarns while the other has 60s Ne count yarn. Thus, following sets of yarns and fabric were obtained:

Set 1 — (a) Yarn of 24s Ne count
(b) Fabric with warp and weft of 24s Ne count yarn

Set 2 — (a) Yarn of 60s Ne count
(b) Fabric with warp and weft of 60s Ne count yarn

Breaking extensions for yarns and fabric at 200 mm gauge length are given in Table 1. For comparison purpose maximum extension levels in both 24s and 60s count yarns were taken at 8.3 mm. In the case of fabric samples, extension level was so adjusted that the extension in the yarn in the fabric was at the same level as that selected for the individual yarn. Accordingly, three different extension levels were selected for all the samples, namely 8.3, 7.3 and 6.3 mm. Extension values of less than 6.3 mm were not considered, as the relaxation process observed is low and prone to errors. The inverse relaxation of yarn and thus woven fabric was determined at different levels of extension and retractions. The aim of this study was to verify that the curve obtained mathematically, by replacing the values of constants in equation, matches with that obtained experimentally.

Table 1 — Breaking extensions and crimp of fabric samples

Sample	Breaking extension, %	Crimp, %
24s Ne yarn	6.7	-
60s Ne yarn	5.0	-
24s Fabric warp	10.9	3.8
24s Fabric weft	11.9	4.3
60s Fabric warp	11.1	4.4
60s Fabric weft	10.1	5.9

The Instron Universal tester (Model No. 4400R) along with the X9 software was used to determine Inverse relaxation Index of yarn and fabric. There was no readymade test method available in the methods of X9 software. A new test method was created using method editor. The control of machine was kept at relaxation and different limits were entered at respective columns. The first limit was kept at the required extension of the sample from where it was to be retracted. The reversing of cross head was achieved by using the up, down control available on the retrofitted panel of the Instron tester. Second limit was set at which the retraction was to be stopped and then sample was allowed to relax till the load in the specimens was stable. The test method was designed in such a way that after completion of test it has to be prompt for inputting the three points at which reading of load was to be taken. These three points were selected by means of a cross wire which appears on the screen of the monitor. First point selected was at the required extension, second point at which specimen was stopped after retraction, and the third was at the point where load in specimen was stabilized. The X9 software records the load at these points and calculates the inverse relaxation Index of the specimen. The formula of the inverse relaxation was fed into the calculations tab of the newly formed method using method editor.

The software collects 10 points/s while testing a specimen. A tool of software X9 was utilized to export these points to Excel sheet for further processing. The raw data was converted to Excel format and then fed to a statistical software to calculate the K_0 Spring constant of non-linear spring in the Maxwellian element, Dashpot (Viscosity) constant ($\cap_i$) of i^{th} Maxwellian element, and relaxation time constant (τ_i) of i^{th} Maxwellian element. Nonlinear regression analysis was utilized to fit the inverse relaxation curve and calculation of aforementioned constants. For estimation, Levenberg-Marquardt[11] method was utilized.

4 Results and Discussion

In the present study one of the aims is to calculate all the three constants, viz. (i) K_0— Spring constant of non-linear spring in the Maxwellian element, (ii) $\cap_i$ —Dashpot (viscosity) constant of i^{th} Maxwellian element, and (iii) τ_i —relaxation time of i^{th} Maxwellian element. As per the theory given by Nachane[5-8], a specimen is represented (Fig.3) by a

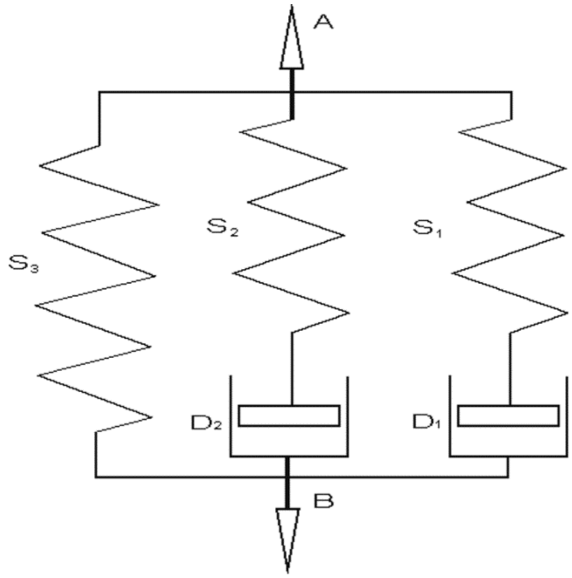

Fig. 3 — Textile specimen represented by a spring in parallel with two Maxwellian elements

spring in parallel with two Maxwellian elements. Element S_1D_1 has a relaxation period much shorter than the element S_2D_2. Force in the specimen at any instant of time is $F = F_1+F_2+F_3$, where F_1, F_2 and F_3 are tensions in the springs S_1, S_2 and S_3 respectively. In the process of continuous deformation of the system, the extensions in the springs S_1 and S_2 do not occur at the same rate as that in S_3 because the plungers in the dashpots are also in motion. The movement of dashpots gives a much lower rate of extension for S_1 as compared to S_2, since the relaxation period of S_1D_1 is small as compared to that of S_2D_2. When the extension is stopped and specimen is kept in this extended state, movements of plungers of dashpots tend to make F_1 and F_2 zero. This corresponds to the stress relaxation.

Now, if the specimen is retracted, the reverse movement of point B is superimposed on the upward movements of pistons in the dashpot D_1 and D_2. If the retraction is stopped at a level where S_1 and S_2 are still under tension, stress-relaxation would result. But, if the retraction is allowed up to a level where S_1 has already crossed its equilibrium position of zero tension and is instead compressed while S_2 is still in the extended condition, the load in S_1 would oppose that in S_2 and S_3. Thus, the total load now would become $F' = F'_3+F'_2-F'_1$. But F'_1 on account of differences in relaxation times, gives rise to inverse relaxation in the beginning followed by stress-relaxation, the latter being due to the fact that tension F'_2 in S_2 reduces to zero as the time passes.

As mentioned, it is the motion of the plunger of the dashpot which gives rise to Inverse relaxation in the system. The plunger is supposed to move in the opposite direction to which whole system is moving, so that the tension is developed in the system and thereafter it relaxes, resulting in the stress relaxation. In order to verify this theory and the equation representing the load in the system at any given time *t* during the inverse relaxation, the constants involved in the equation are calculated using nonlinear regression analysis. The raw data file generated by the software is then exported to the Excel sheet and the portion representing the inverse relaxation is extracted. Using this experimental data and following Eq (2), the constants K_0, $\cap_i$, τ_i have been calculated.

The other quantities such as rate of traverse, time of extension and time of retraction are known from the experiment. Thereafter, the values of the constants are put in the equation and the predicted values are obtained and then compared with experimental values.

The values of constants are tabulated in Table 2 for 60s Ne yarn. Similar results are also obtained for 24s Ne yarn. Let us consider the relaxation time constant (τ) for the yarn. The value of this constant is dependent on extension of the yarn. For a different extension value but same retraction value, the constant is different. As seen from the table the relaxation time constant increases as the Inverse relaxation index increases but it decreases as the IR Index decreases. It is known that the higher the relaxation time constant the higher is the time taken by the system to reach equilibrium. At higher value of inverse relaxation index, the tension in the system is higher and hence it takes longer time to reach equilibrium which results in higher value of relaxation constant.

The spring constant (K_0) of non-linear spring in Maxwellian element increases as the difference between extension and retraction reduces. In other words, as the amount of retraction increases the value of K_0 increases. During the determination of inverse relaxation index when the system is extended with certain rate it develops a tension. The system is retracted to achieve the inverse relaxation which results in the decrease of the tension in the system. This results in reducing the tension in the components of the Maxwellian elements. The linear spring, being a part of the Maxwellian element, will also experience the same and being non-linear in nature a reduction in

Table 2 — Values of constants calculated using non-linear regression analysis for 60s Ne count yarn

Extension mm	Difference between extension and retraction, mm	Relaxation time (τ)of Maxwellian element, s	Dashpot (viscosity) constant (∩) of Maxwellian element, g s	Spring constant (K_0) of non-linear spring in Maxwellian elementg	IR Index
			60s Count Yarn		
6.3	1.8	4.6	-29.6	4.0	2.8
6.3	1.3	9.7	-56.7	5.1	4.0
6.3	0.8	7.8	-9.0	7.4	1.7
7.3	2.8	14.6	-48.6	2.6	1.8
7.3	2.3	24.0	-79.1	3.2	6.7
7.3	1.8	18.4	-51.3	4.3	6.4
7.3	1.3	9.5	-66.4	5.6	5.1
7.3	0.8	0.1	753.8	7.2	1.1
8.3	3.3	21.7	-85.6	1.1	7.1
8.3	2.8	37.1	-95.0	2.7	7.8
8.3	2.3	34.9	-88.7	3.9	8.2
8.3	1.8	16.1	-39.5	4.0	8.1
8.3	1.3	9.2	-23.0	4.9	4.0
8.3	0.8	2.0	0.8	5.4	0.0
			60s Count Fabric Warp		
6.3	1.8	2765.7	-91363	191.6	1.9
6.3	1.3	3969.3	-138220	363.1	2.7
6.3	0.8	3056.7	-106582	468.3	2
7.3	2.8	20.6	-3756.2	182.6	1.2
7.3	2.3	33.6	-4280.9	295.1	5.1
7.3	1.8	40.6	-4769.2	336.2	5.5
7.3	1.3	14.4	-2735.4	458.8	4.6
7.3	0.8	11.0	-100.6	589.2	0.8
8.3	3.3	28.5	-6041.5	171.7	5.9
8.3	2.8	35.0	-7956.3	252.9	6.4
8.3	2.3	16.7	-9743.6	271.9	5.7
8.3	1.8	20.0	-4345.6	385.9	5.3
8.3	1.3	15.3	-102.6	436.3	0.1
8.3	0.8	-	Not Applicable		-0.5
			60s Count Fabric Weft		
6.3	1.8	3.9	-16568	107.7	1.9
6.3	1.3	7.6	-20123	206.8	3.3
6.3	0.8	3.7	-10569	247	1.9
7.3	2.8	2.7	-17956	186.5	2.0
7.3	2.3	2	-42366	204.6	6.5
7.3	1.8	1.7	-39865	245.6	5.2
7.3	1.3	1.2	-32656	295.6	4.1
7.3	0.8	1	-2566	315.6	1
8.3	3.3	9.7	-38964	215.6	5.7
8.3	2.8	11.3	-40956	255.9	6.1
8.3	2.3	12.1	-42694	285.9	6
8.3	1.8	13.7	-50136	302.6	6.7
8.3	1.3	8.7	-35964	312.2	3.1
8.3	0.8	2.3	-40	325.6	0.1

the tension of complete system will result in reduction in the tension of non-linear spring linearly. Therefore, when the difference between the extension and retraction is low there is less reduction of tension in the system and when the difference is high there is higher reduction in the tension in the system. The values of the spring constant K_0 follow the same analogy. The value of K_0 decreases with the increase in the level of retraction. There seems to be no relation between the values of K_0 and the amount of inverse relaxation index. It has direct relation with the amount of retraction.

The dashpot constant (∩) is very important from the inverse relaxation index point of view. The sign of dashpot constant is found to be negative, which signifies that the movement of the plunger in the dashpot is in the opposite direction than that of the whole system, thus proving the presumption made while giving an explanation for the inverse relaxation. As seen from Table 1 the values of dashpot constant follow the same trend as that of inverse relaxation index. The amount of constant is higher near the region where the inverse relaxation index is higher which further proves that the movement of plunger in dashpot gives rise to inverse relaxation. The higher the values of inverse relaxation index the higher is the amount of dashpot constant.

The values of all the three constants are much higher in case of fabric when a comparison is done between yarn and fabric. In case of fabric the values are observed to be much higher in both ways i.e. warp and weft. The determination of inverse relaxation index for yarn is carried out by clamping a single yarn between the grips and then subjected to tension, whereas in case of fabric a strip of 25 mm x 200 mm was clamped between the grips and a tension is applied to it. There are a higher number of yarns in a strip which are subjected to the tension, resulting in a much higher registration of load while straining to same level as that of yarn. Since the amount of load is higher than that of yarn, the values of the constants are also higher compared to yarn. The trends for all the three constants are observed for the fabric woven with 24s Ne count yarn and 60s Ne count yarn individually same as that yarn.

As discussed above, the values of all the three constants have been calculated. The raw data file of series IX software is exported to Excel and the inverse relaxation curves are reproduced. The values of constants are inserted in the Eq. (2) and predicted data points corresponding to that of experimental data points are determined. The predicted data points are then plotted alongside the experimental data points.

Time-force curve (Fig. 4) is plotted with the data points of inverse relaxation test of weft strips of fabric woven from 60s yarn at 6.3 mm extension and 1.3 mm retraction. The graph started moving upward from the 52s of the experiment giving rise to inverse relaxation. The inverse relaxation portion is extracted and the results of experimental values of force and their calculated values are plotted (Fig. 5) with respect to time. It is found that the calculated values more or less follow the experimental values. Similar results are also obtained for other sets of yarn and fabric of 24s and 60s count.

Some of the results of comparison of calculated values, obtained by putting the values of constants in the equation [Eq.(2)], and the experimental values are discussed above. The data points as well as graphs have shown that the data points obtained by equation and actual data points, more or less, follow the same path. It is evident that calculated data points will have

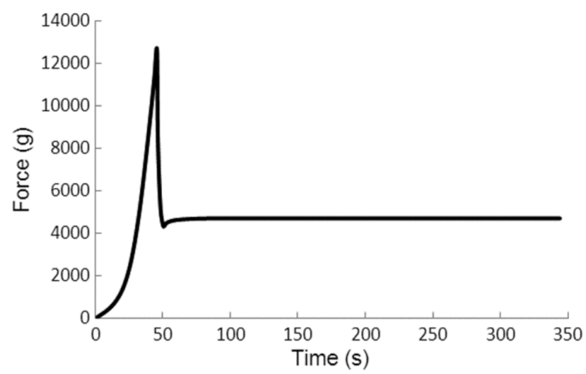

Fig. 4 — Inverse relaxation curve for 60s Ne fabric (weft) [extension 6.3 mm, retraction 1.3 mm]

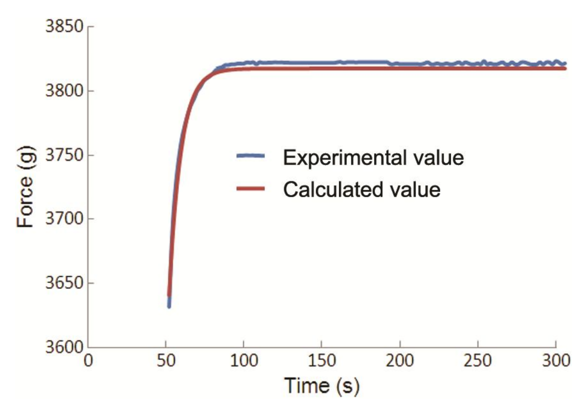

Fig. 5 — Comparison of experimental and calculated data points of inverse relaxation portion of experiment for 60s Ne fabric (weft) [extension 6.3 mm retraction 1.3 mm]

smoother curves compared to that of actual curves. This proves that the inverse relaxation is represented by the Eq. (1).

5 Conclusion

The basic premise of the treatment is that a polymeric fibre can be represented by a mechanical model consisting of a large number of Maxwellian elements in parallel with each other and also with an elastic spring. It is assumed that the relaxation times of the Maxwellian vary from very small values to very large values. The equation representing the inverse relaxation has been verified and compared with experimental data. A method to calculate the values of constants is also evolved.

5.1 The value of dashpot constant ($\cap$) is found negative in all the cases wherever the IR index is positive. It increases as the IR index increases and decreases as the IR index decreases. Therefore, it conforms the theory that it is the dashpot which exerts the force in reverse direction giving rise to inverse relaxation whenever the system is retracted from an extended position and then allowed to rest at a point.

5.2 The relaxation time is a measure of the time required for the energy stored in the spring to shift to the dashpot and dissipate. The relaxation time constant (τ) is positive in all the cases. It is higher in the vicinity of the position where IR is higher.

5.3 The non-linear spring constant K_0 is also positive in all the cases. For a given extension, it increases as the difference between extension and retraction decreases. This is because the force retained in the system is higher when retraction is low, whereas the force retained in the system is lower when the retraction is high.

5.4 The values of dashpot constant ($\cap$) is much higher (negative) in the case of fabric compared to yarn from which it is woven. Similar is the case with K_0 the non-linear spring constant. This may be due to higher force required to extend the fabric to the same level as that of the yarn.

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