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Estimation of uncertainty of effective area of a pneumatic pressure reference standard using Monte Carlo method

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The current paper presents a comparative investigation of the experimental as well as simulated evaluation of effective area and the associated uncertainties, of a pneumatic pressure reference standard (NPLI-4) of CSIR-National Physical Laboratory, India, (NPLI). The experimental evaluation has been compared to the simulated estimation of the effective area obtained through Monte Carlo method (MCM). The Monte Carlo method has been applied by taking fixed number of trials (FMCM) and also by trials chosen adaptively (AMCM). The measurement uncertainties have been calculated using the conventional method, i.e., law of propagation of uncertainty (LPU) as well as MCM. Experimentally, the NPLI-4 has cross-floated against our newly established pneumatic primary pressure standard (NPLI-P10), which is a large diameter piston gauge. An excellent agreement in effective area and measurement uncertainty has been observed between these approaches.

Keywords: Reference standard, Uncertainty, Monte Carlo method, Law of propagation of uncertainty, Pneumatic pressure

1 Introduction

It is well known that metrology plays a very important role in every sphere of life and has a huge economic impact on trade and industry. For the overall development of any country and its industrial performance, metrology undoubtedly plays a significant role. It is now widely accepted that by following diligent metrology, the reliability and quality of products increases and is thus maintained. The harmonization of standards allows free trade among nations and also the costs incurred due to poor quality (COPQ) decrease with the standards metrology. Metrology without the concept of measurement uncertainty is redundant and irrelevant. Uncertainties have conventionally been estimated based largely on the ISO-GUM documents. The joint committee for guides in metrology (JCGM) also provides the guidelines for the evaluation of measurement uncertainty, viz., JCGM 100:2008 guide, Evaluation of measurement data–guide to the expression of uncertainty in measurement[1]. The uncertainty evaluation using this method relies on law of propagation of uncertainty. However, in a few cases, the linear approach to estimation of uncertainty as well as the usual symmetrical probability distribution assumptions, contributes to some limitations faced by this method. In this context, JCGM has introduced an alternative method for the evaluation of measurement uncertainty, i.e., Monte Carlo method (MCM) in its supplement, JCGM 101:2008 (evaluation of measurement data – supplement 1 to the "Guide to the expression of uncertainty in measurement" – Propagation of distributions using a Monte Carlo method)[2].With the advent of GUM supplement-1, JCGM 101:2008 Monte Carlo method is now being used alternatively to evaluate measurement uncertainty in many National Metrology Institutes (NMI). In this approach, the propagation of distributions involves the convolution of the source PDFs, using numerical simulation. In the present work, we have attempted to evaluate the effective area of NPLI-4, a reference standard for pneumatic pressures, and estimated its associated measurement uncertainty using the above mentioned approaches. To evaluate effective area the experimental evaluation is carried out using the conventional cross-floating method between two piston gauges and the associated measurement uncertainty is also computed using the conventional law of propagation of uncertainty method (LPU). The theoretical approach used for the estimation of standard uncertainty is the Monte Carlo simulation

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method (MCM). As mentioned earlier, the major difference in the estimation of uncertainties between both the approaches is that, the former is based on propagation of uncertainty of input parameters of the measurand while the latter is based on propagation of probability distribution of input parameters. JCGM has introduced the Monte Carlo method for uncertainty estimation because of the limitations of LPU method. MCM overcomes the limitations of LPU method and is successfully applied in the field of metrology. Since the consideration of the probability distribution contains richer information of a quantity, it is expected that the propagation of distributions (POD) contains relatively richer information than the consideration of the propagation of uncertainties which in turn leads to better estimation of measurand and their corresponding uncertainty. The main requisite for the MCM simulations is a good pseudo-random number generator. A number of commercial software's are available for such random number generation e.g.Oracle crystal ball, Microsoft excel, IBM SPSS etc., while programming can also be done on the Matlab platform. The number of Monte Carlo trials (M) (or iterations) has to be chosen carefully, so as to accommodate the input variability. As M number increases, the standard deviation will decrease and vice versa. Hence, larger numbers of trials have better probability of the convergence of results. The number of trials can be chosen a prior or can also be determined adaptively. In the recent past, many reports have been published on the implementation of MCM for estimation and uncertainty evaluation of measurand[3-7].Wubbeler *et al.*[8] illustrated a two-stage procedure based on Stein's method[9] for determining the number of trials for uncertainty evaluation. Recently, Farrance and Frenkel[10] implemented Monte Carlo simulation using Microsoft excel for medical laboratory application and also paraphrased various advantages of MCM over the LPU from JCGM 101:2008. In general, about 10^6 numbers of Monte Carlo trials (M) is expected to deliver a 95 % coverage interval for the output quantity such that this output is correct to one or two significant decimal digits[2]. JCGM 101:2008 also recommends adaptive approach wherein the number of trials increases progressively till the various results of interest is stabilized. In the present work, Monte Carlo method was applied using fixed number of trials, i.e., fixed Monte Carlo method (FMCM) and also using adaptive approach, i.e., adaptive Monte Carlo method (AMCM). Hence, in the present study, uncertainty was initially computed using conventional LPU method and was subsequently compared with FMCM and AMCM outcomes.

2 Reference Standard (NPLI-4)

In the current study, NPLI-4, a simple piston-cylinder dead weight tester is taken as the test gauge (GUT). NPLI-4 is our pneumatic pressure reference standard and has been used to establish traceability of our secondary pressure standards via traceability to the ultrasonic interferometer (UIM)[11]. This GUT is a Ruska made piston gauge, model 2465 having full scale pressure range up to 4 MPa and a nominal effective area of 8.39 mm^2. The piston as well as the cylinder are made up of cemented tungsten carbide with 6% CO. NPLI-4 has also been used as a reference standard in the APMP.M.P-K1ccomparison[12] and also participated in bilateral comparison with PTB, Germany in 1988[13] and with NIST, USA, in bilateral comparison APMP.SIM.M.P-K1c[14].

3 Primary Standard (NPLI-P10)

NPLI-P10 is a large diameter gas operated piston gauge having nominal piston diameter of 11 mm, and maximum pressure range of 10 MPa and is DHI, USA make. It is used as a primary standard for the pneumatic pressure gauges. The piston and the cylinder are also made up of tungsten carbide and have a nominal effective area of 98 mm^2. It has an automated mass loading system along with a control terminal. It is interfaced through RS232 with a PC and can be controlled either from the control terminal or from the computer. The automatic mass loading is carried out via the use of pneumatic actuators, which automatically load/unload the weights on the piston according to the desired pressure. The full piston stroke is ± 4.5 mm from the mid stroke position. Various real time conditions like piston rotation rate, fall rate, piston position, the temperature of p-c, ambient conditions etc. can be monitored through a computer. The piston of NPLI-P10 was initially calibrated by NIST, USA and the masses were calibrated at National Research Council, Canada. The uncertainty in effective area as reported by NIST, USA is 13×10^{-6} at k=1[15].

4 Experimental

To experimentally determine the effective area of the Reference standard (NPLI-4), it was cross-floated against the primary standard, NPLI-P10 in the pressure range 4-40 bar. Eleven pressure points were selected, i.e., 4.15, 8.15, 12.15, 16.15, 20.15, 24.15, 28.15, 32.15, 36.15, 40.15 and 42.20 bar covering the whole pressure range of NPLI-4. The experiment was done under controlled atmosphere and all the necessary precautions were taken. The temperature of the room was kept at (23±1) °C and the humidity was maintained at (50±5) %. The gauges were kept in cross-floating condition for at least 10 min at each pressure measurement point. At each point three readings were taken in increasing cycle and three in decreasing cycles. The complete experimental procedure is discussed elsewhere[16]. The experimental effective area was calculated using standard formula as follows:

$$A_p = \frac{\sum_i m_i g_i (1 - \rho_{air} / \rho_m)}{P[1 + (\alpha_p + \alpha_c)(T - T_0)]} \quad \ldots (1)$$

where, P is the standard generated pressure at the bottom of the piston which in present case is NPLI-P10, m_i is the total masses used on NPLI-4, ρ_{air} and ρ_m are the densities of air and the masses of NPLI-4 respectively, α_p and α_c are the thermal expansion coefficients of the piston and cylinder of the NPLI-4, respectively. T is the temperature of the piston-cylinder assembly under experimentation and T_0 is the temperature at which A_0 is specified (T_0= 23 °C). Head correction was also applied which occurs due to the difference in the floating/reference level of the two cross-floating pistons.

4.1 Methodology of uncertainty evaluation

The uncertainty through conventional LPU method was estimated using the guidelines as per the GUM document[1] and NABL document[17] 141. The detailed procedure of conventional method is not discussed in this paper and can be found elsewhere [1, 17]. In short, the uncertainty associated with each of the parameters in Eq. (1) was taken into account as well as the statistical variation of the experimental data. The standard uncertainty was therefore estimated via the determination of type A and type B uncertainty components. The simulated values of the effective area and its associated uncertainty were also generated for the complete characterization of NPLI-4. A comparison between three methodologies, i.e., LPU method, FMCM and AMCM of uncertainty estimation was made. As it is well known, the Monte Carlo method is based on the generation of multiple trials to determine the expected value of a random variable. The MCM uses algorithmically generated pseudo-random numbers which is based on the probability distribution function (PDF) of a quantity. In JCGM 101:2008[2], detailed steps are given for the evaluation of measurement uncertainty. The first two steps in MCM are same as in case of LPU, i.e., defining the output and input quantities and their model equation. In the present work, the output quantity is the effective area and the input quantities are each of the parameter in Eq. (1) and hence the model equation for the calculations is also Eq. (1). The next step is the PDF assignment to all input quantities (x_i) based on the available information; data of calibration reports; empirical data; expert's judgment; and measurement data etc. The PDF for all input quantities has been assigned in the following way. The mass (m) and acceleration due to gravity (g) value have been taken from calibration certificates and hence have normal distribution. The estimated standard value of the density of air (ρ_{air}) has been used and again has a normal distribution. The density of dead weights (ρ_m) and the thermal expansion coefficients of the piston and cylinder (α_p and α_c) are provided by the manufacturer having a rectangular distribution. The temperature of piston-cylinder assembly is measured through a platinum resistance thermometer and the uncertainty of the same is taken from its calibration certificate which has a normal distribution. The line pressure (P) is the pressure generated by the standard and its uncertainty is taken from calibration certificate, hence again has a normal distribution. This PDF assignment of all input quantities is tabulated in Table 1. Various input parameters affecting the uncertainty of measurand or

Table 1 — Probability distribution assignment of all input quantities (x_i)

Input quantities (x_i)	Probability distribution
Mass, m_i (kg)	Normal
Acceleration due to gravity, g (m/s^2)	Normal
Density of air ρ_{air} (kg/m^3)	Normal
Density of Mass, ρ_m (kg/m^3)	Rectangular
Thermal expansion coefficient of piston, α_p (°C^{-1})	Rectangular
Thermal expansion coefficient of cylinder, α_c (°C^{-1})	Rectangular
Temperature of piston-cylinder, T (°C)	Normal
Measured pressure, P (Pa)	Normal
Head correction, ΔP (Pa)	Normal
Deformation coefficient, λ (Pa^{-1})	Normal

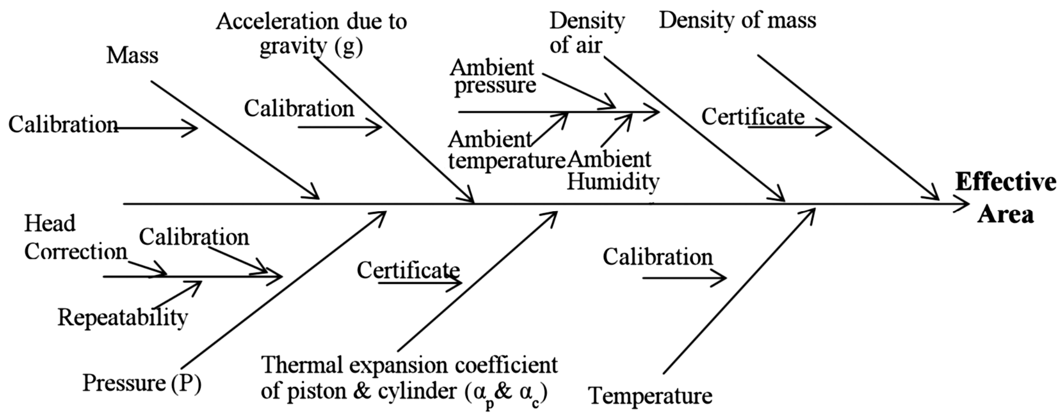

Fig. 1 — Cause and effect diagram for the measurement of effective area

sources of uncertainty is conveniently shown in cause and effect diagram in Fig. 1. Subsequently, the number of Monte Carlo trials (*M*) was selected. As stated above, in the present study, Monte Carlo method was applied using fixed trials in FMCM and also using adaptively in AMCM. In the case of FMCM, the number of trials *M* was kept fixed at 10^6 which are expected to deliver 95% coverage interval for the measure end. The effective area (A_{eff}) is the measure and to begin with. The predefined probability distributions of input quantities as given above are then propagated into the output quantity, i.e., the effective area, according to the model Eq. (1). Thereafter, the required number of runs is carried out and subsequently, a histogram is plotted from the estimate of effective area which is the actual PDF of the measure and. From this obtained PDF, various statistical information can be drawn and finally, the results are summarized viz. estimation of output in the form of average; associated standard uncertainty in the form of standard deviation; and the coverage interval according to the desired coverage probability. The end point of chosen interval according to the coverage probability can be evaluated from the percentiles of the PDF of measured. However, JCGM 101:2008[2] also recommends adaptive methodology where in the MC trials increase progressively till the various quantities have stabilized and therefore, AMCM was also applied to verify and cross check the obtained results. AMCM was implemented as per the procedure laid out in (JCGM101: 2008)[2]. In brief, in the present study, the numbers of significant digits in standard uncertainty n_{dig} were taken as 2 and the coverage probability *p* was taken as 95%. A batch of 10^4 trials was taken which consists of one simulation denoted by *h*. After each simulation, 10^4 values were generated of each of the input quantities and through model Eq. (1), effective area was calculated. Through the *M* trials of the model, the average, standard uncertainty, low and high end points for the h^{th} sequence are calculated as $y^{(h)}$, $u(y^{(h)})$, and $y_{low}^{(h)} y_{high}^{(h)}$, respectively. After the first simulation, *h* was increased by one unit and another simulation was performed. Then the results were checked for stabilization, i.e., twice the standard deviation associated with the results (average, standard deviation, low and high end point) should be less than the numerical tolerance (δ) associated with the standard uncertainty $u(y)$. If the results are not stabilized, then carry out one more simulation till the various results are stabilized. Once the stabilization criterion is fulfilled we stop the run and use all simulation values, i.e., $h \times M$ to plot histogram which is the actual probability distribution of measure and calculate average, standard deviation, low and high end point of coverage interval. The numerical tolerance is calculated as $\delta = ½\ 10^l$ where l is an integer which is obtained from the uncertainty when written in the form of $c \times 10^l$, where c is an integer with number of significant digits of the standard uncertainty. A numerical tolerance of δ/5 was taken as recommended[2]. Contrary to LPU, in MCM sensitivity coefficients and effective degree of freedom are not required.

5 Results and Discussion

Experimentally, NPLI-4 was cross floated with NPLI-P10. At first, the effective area of NPLI-4 was calculated experimentally using Eq. (1) at each pressure point. Head correction was also applied as there is difference between the reference levels of the two standards. Thereafter, MCM was applied using fixed trials and adaptively chosen trials. PDF were assigned for each of the input quantities used in MCM and is tabulated in Table 1. As stated above, in FMCM the number of trials is fixed, i.e., 10^6 hence the average effective area obtained at each pressure point is the average of 10^6 simulated values. In case of AMCM, after applying stabilization criteria, the results were stabilized after performing 0.31×10^6 to 0.46×10^6 trials for the complete pressure range from 4.15 bar to 42.2 bar. Figure 2 shows the variation in effective area obtained experimentally as well as through FMCM and AMCM with increasing pressures. Table 2 shows the detailed values of the effective area obtained from the three approaches along with their agreements at each pressure point. From Fig. 2 as well as Table 2, it is apparent that an excellent agreement in effective area is observed at each pressure point between the three methods. The maximum relative deviation in effective area between experimental and FMCM is only 3.4×10^{-6} while comparing with adaptive method AMCM it is -3.71×10^{-6}. It is observed that the relative deviation between experimental and AMCM values is maximum at lowest pressure, i.e., 4.15 bar, otherwise the maximum deviation at other 10 pressure points is only 1.98×10^{-6} which is an excellent agreement. Further, the comparison of the two methods of MCM, i.e., FMCM and AMCM displays maximum deviation of 1.89×10^{-6}. It can be noted that the trials in FMCM are 10^6 which is huge as compared with AMCM where maximum trials for stabilization were 0.46×10^6, hence it may be surmised that the latter is a better and economical to use adaptive method. The zero pressure effective area (A_0) was also calculated and obtained using first order regression fitting to the pressure vs effective area data in all the three methodologies and is shown in Fig. 3. The experimentally estimated A_0 value thus obtained at 23 °C was found to be 8.392438×10^{-6} m^2. This compares extremely well with the A_0 value obtained by FMCM which is 8.392446×10^{-6} m^2 and by AMCM which is 8.392436×10^{-6} m^2 at 23 °C. The relative agreement

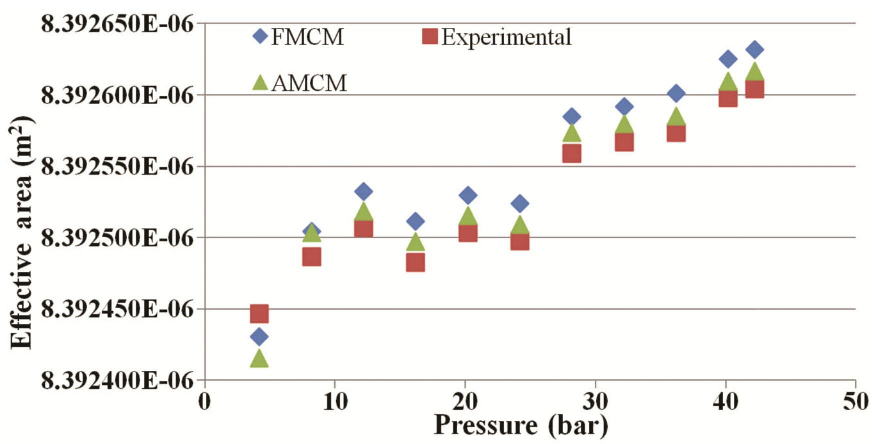

Fig. 2 — Variation of effective area obtained using FMCM, AMCM simulations and via experimental at each pressure point

Table 2 — Effective area of NPLI-4 using experimental, FMCM & AMCM and their relative deviations

Pressure (bar)	Effective Area of NPLI-4 (m^2)			Relative deviations		
	Experimental	FMCM	AMCM	FMCM w.r.t. Experimental	AMCM w.r.t. Experimental	FMCM w.r.t. AMCM
4.15	8.392447E-06	8.392431E-06	8.392416E-06	-1.88E-06	-3.71E-06	1.83E-06
8.15	8.392487E-06	8.392505E-06	8.392504E-06	2.10E-06	1.98E-06	1.21E-07
12.15	8.392507E-06	8.392533E-06	8.392519E-06	3.06E-06	1.40E-06	1.66E-06
16.15	8.392483E-06	8.392512E-06	8.392498E-06	3.40E-06	1.73E-06	1.67E-06
20.15	8.392504E-06	8.392530E-06	8.392516E-06	3.11E-06	1.38E-06	1.72E-06
24.15	8.392498E-06	8.392525E-06	8.392510E-06	3.17E-06	1.39E-06	1.78E-06
28.15	8.392560E-06	8.392585E-06	8.392574E-06	3.05E-06	1.73E-06	1.31E-06
32.15	8.392567E-06	8.392592E-06	8.392580E-06	2.97E-06	1.50E-06	1.46E-06
36.15	8.392574E-06	8.392601E-06	8.392586E-06	3.28E-06	1.39E-06	1.89E-06
40.15	8.392598E-06	8.392626E-06	8.392610E-06	3.24E-06	1.37E-06	1.86E-06
42.2	8.392604E-06	8.392632E-06	8.392617E-06	3.29E-06	1.47E-06	1.82E-06

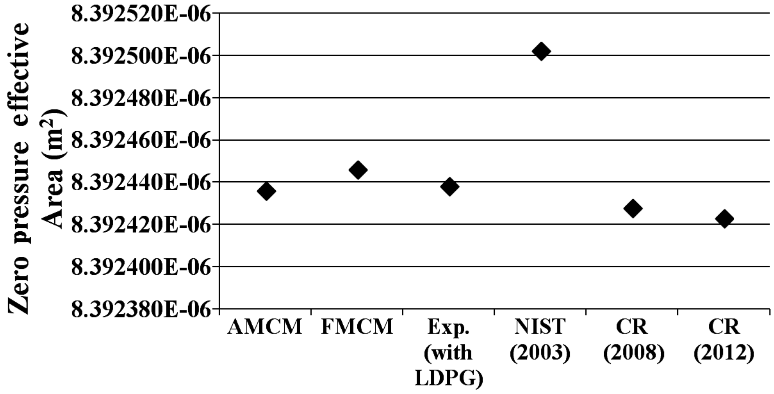

Fig. 3 — Variation of zero pressure effective area

between the experimental and FMCM is less than 1 ppm and stands at a value of only 9.5×10^{-7} and it is even better in when compared with AMCM, i.e., only 2.38×10^{-7}. Thus, the A_0 value obtained from experiment agrees better with AMCM. The A_0 value obtained from the current MCM simulations, i.e., ($A_{0(\mathrm{FMCM})}$) and ($A_{0(\mathrm{AMCM})}$) are also compared with A_0 obtained from characterization of NPLI-4, done in the years 2008 and 2012. The relative deviation of $A_{0(\mathrm{FMCM})}$ and ($A_{0(\mathrm{AMCM})}$) with other experimentally obtained values is tabulated in Table 3. NPLI-4 was also used as a transfer standard in the bilateral comparison with NIST (USA) in the pneumatic pressure region[14] (0.4 to 4.0) MPa. Effective area of NPLI-4 was determined against pneumatic pressure standard of NIST as well as NPLI in 2003. Table 3 shows the value reported by NIST during this bilateral comparison, and a relative agreement of 6.67×10^{-6} is observed in the effective area value reported by NIST as compared to the FMCM value and 7.86×10^{-6} with AMCM value. It is noted that the relative deviation between NIST and the current experimental effective area is also large, i.e., 7.63×10^{-6} which may be reason of large deviation between NIST and MCM values. As the deviations in effective area at each pressure point between AMCM and FMCM are small, the deviation in A_0 is also very small and is only 1.19×10^{-6}. The A_0 value obtained from FMCM and AMCM are in good agreement with the characterization data of year 2008 and 2012 (CR 2008 and CR 2012) and is shown in Table 3. From the table below, it can be observed that the relative deviations between FMCM and AMCM with other methods/characterization are quite small except when compared with NIST (2003) data. The uncertainty in effective area was also estimated using all the three methods. In case of LPU, the estimated uncertainty was evaluated by taking both Type A and Type B components of uncertainties according to the guidelines in literature[1,17]. The uncertainty of all the input quantities have been taken into consideration and the same is propagated in the measurand using partial derivatives of each of the input quantities with respect to the output quantity. The type A uncertainty component estimates the uncertainty occurring due to statistical variation while type B components occur due to the uncertainty in each of the factors in model Eq. (1). The major contributor in type A uncertainty is the standard deviation of effective area while in type B the major factor is the uncertainty of standard used. Further, combined standard uncertainty in effective area was calculated by the root sum square method which is estimated to be 1.18×10^{-10} m^2. In case of FMCM, the standard uncertainty at each pressure point is the standard deviation of effective area obtained through M trials, i.e., it is the standard uncertainty of 10^6 effective area values as obtained after simulation at each pressure point. As can be seen from Table 4, the standard deviation has the same value up to two decimal points at each pressure and is 1.12×10^{-10} m^2. However, the relative deviation in standard uncertainty between LPU method and

Table 3 — Relative deviation of zero pressure effective area (A_0) of NPLI-4

	Zero pressure effective area (A_0) in (m^2)	Relative deviation w.r.t. FMCM	Relative deviation w.r.t. AMCM
AMCM	8.392436E-06	1.19E-06	
FMCM	8.392446E-06		-1.19E-06
Exp. (with LDPG)	8.392438E-06	9.53E-07	-2.38E-07
NIST (2003)	8.392502E-06	-6.67E-06	-7.86E-06
CR 2008	8.392428E-06	2.18E-06	9.89E-07
CR 2012	8.392423E-06	2.78E-06	1.58E-06

Table 4 — Effective area and their respective standard deviation obtained from FMCM and low end point and high end point covering 95 % interval

Pressure (bar)	FMCM Results			
	Effective area (m^2)	Standard deviation(m^2)	Low end point (m^2)	High end point (m^2)
4.15	8.392431E-06	1.12E-10	8.392212E-06	8.392651E-06
8.15	8.392505E-06	1.12E-10	8.392285E-06	8.392725E-06
12.15	8.392533E-06	1.12E-10	8.392314E-06	8.392752E-06
16.15	8.392512E-06	1.12E-10	8.392292E-06	8.392732E-06
20.15	8.392530E-06	1.12E-10	8.392311E-06	8.392750E-06
24.15	8.392525E-06	1.12E-10	8.392305E-06	8.392744E-06
28.15	8.392585E-06	1.12E-10	8.392365E-06	8.392805E-06
32.15	8.392592E-06	1.12E-10	8.392372E-06	8.392812E-06
36.15	8.392601E-06	1.12E-10	8.392382E-06	8.392821E-06
40.15	8.392626E-06	1.12E-10	8.392406E-06	8.392845E-06
42.2	8.392632E-06	1.12E-10	8.392413E-06	8.392852E-06

Table 5 — Effective area and their respective standard deviation obtained from AMCM and low end point and high end point covering 95 % interval

Pressure (bar)	AMCM Results			
	Effective area (m^2)	Standard deviation (m^2)	Low end point (m^2)	High end point (m^2)
4.15	8.392416E-06	1.12E-10	8.392197E-06	8.392635E-06
8.15	8.392504E-06	1.12E-10	8.392284E-06	8.392723E-06
12.15	8.392519E-06	1.12E-10	8.392299E-06	8.392738E-06
16.15	8.392498E-06	1.12E-10	8.392278E-06	8.392718E-06
20.15	8.392516E-06	1.12E-10	8.392296E-06	8.392736E-06
24.15	8.392510E-06	1.12E-10	8.392290E-06	8.392730E-06
28.15	8.392574E-06	1.12E-10	8.392354E-06	8.392794E-06
32.15	8.392580E-06	1.12E-10	8.392360E-06	8.392800E-06
36.15	8.392586E-06	1.12E-10	8.392366E-06	8.392806E-06
40.15	8.392610E-06	1.12E-10	8.392390E-06	8.392830E-06
42.2	8.392617E-06	1.12E-10	8.392397E-06	8.392837E-06

FMCM is 5.1 %. The uncertainty obtained from FMCM is lower than that from LPU which is expected owing to the differences in the approach to calculation as described above and due to the different assumptions adopted in both the methodologies. The low and high end points covering 95% coverage interval is also shown in Table 4. In case of AMCM, after the stabilization of results, standard uncertainty has been calculated, which is the standard deviation of all the simulated values, i.e., $h\times M$ values. The low and high end points in the Table 5 represents the 95 % coverage interval for the effective area results at each pressure points which were also obtained using all the $h\times M$ values. The standard uncertainty thus obtained from AMCM is 1.12×10^{-10} m^2 which is lower than the LPU method 1.18×10^{-10} m^2 and deviation between the two is again 5.1 %. In the present case, the LPU method overestimates the standard uncertainty which may be because of the nonlinear model equation and the assumptions taken in LPU. Hence, it is always better to use AMCM with the LPU approach where LPU method has limitations. And also, as the standard uncertainty obtained from both FMCM and AMCM is same. This observation further strengthened the adoption of adaptive method over FMCM. The probability distributions of the effective area values at each pressure point were obtained from both FMCM and AMCM and were actually observed to follow a normal distribution in both the cases while they were assumed to be normal in case of LPU. These can be conveniently depicted through histograms. Two representative histograms obtained from FMCM and AMCM, one at the lowest pressure point, i.e., at 4.15 bar and another at the highest pressure point, i.e., at 42.2 bar are shown in Figs. 4-7, respectively. The four moments, viz., measure of central tendency or mean, measure of variation, i.e., standard deviation, skewness and kurtosis can be easily derived from

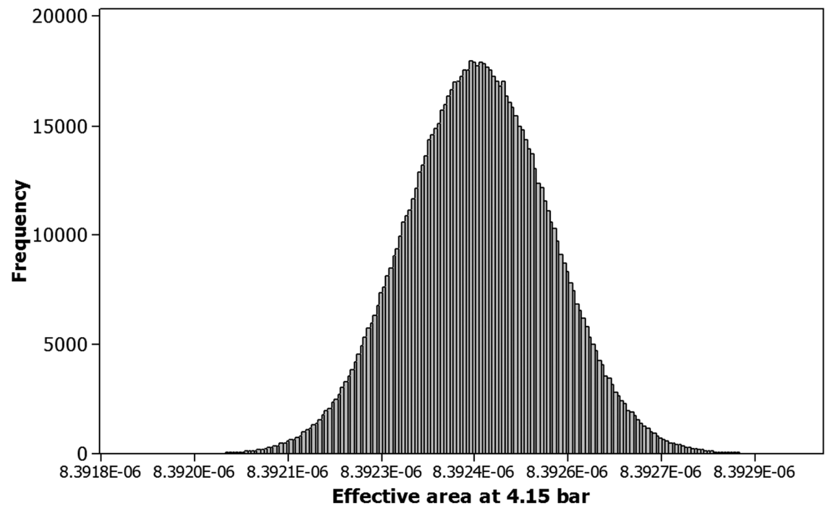

Fig. 4 — Histogram showing effective area of NPLI-4 at 4.15 bar using FMCM

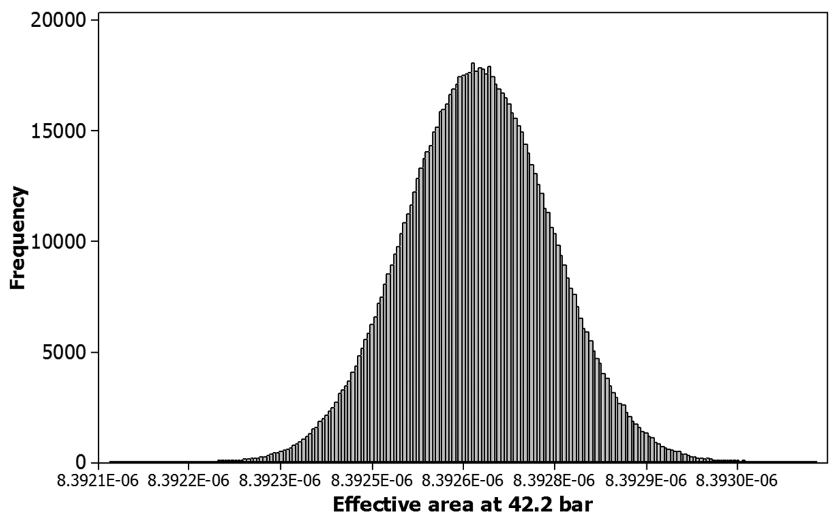

Fig. 5 — Histogram showing effective area of NPLI-4 at 42.2 bar using FMCM

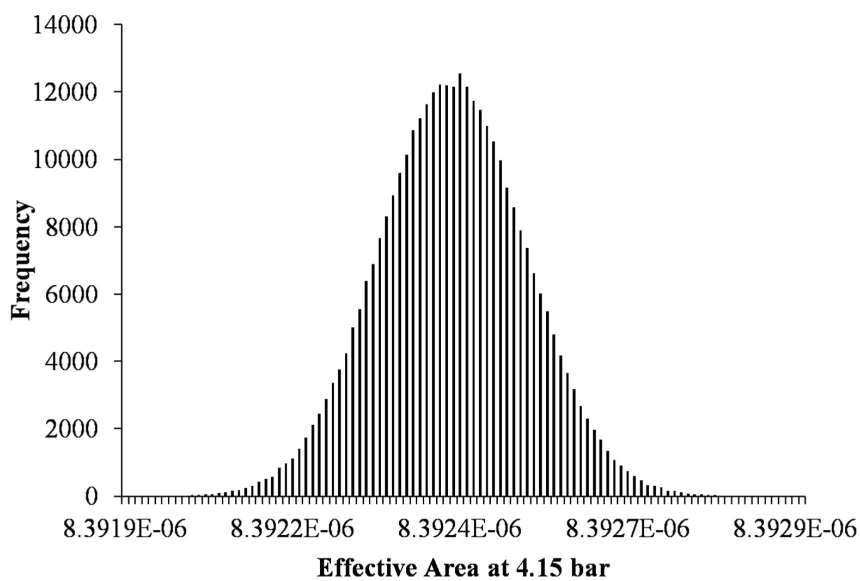

Fig. 6 — Histogram showing effective area of NPLI-4 at 4.15 bar using AMCM

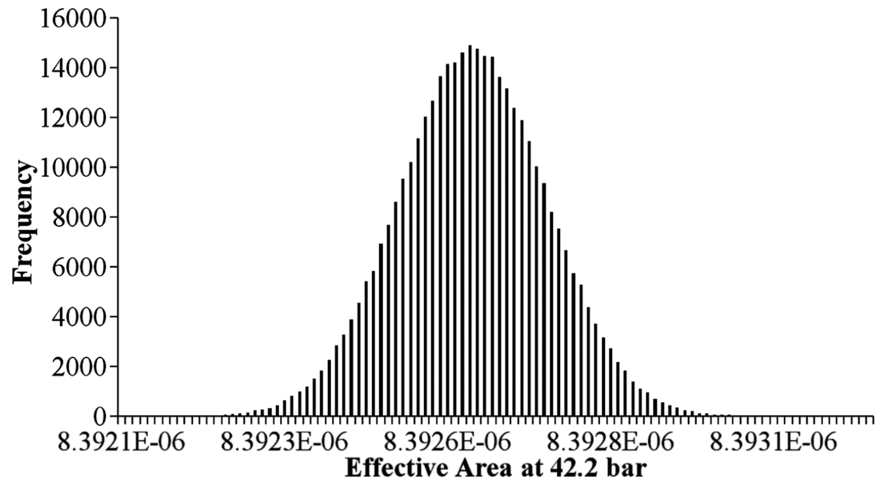

Fig. 7 — Histogram showing effective area of NPLI-4 at 42.2 bar using AMCM

Table 6 — Results of absolute difference of low and high limits of coverage interval obtained from LPU and AMCM.

Pressure (bar)	y (m^2)	$U(y)$ (m^2)	y_{low} (m^2)	y_{high} (m^2)	d_{low} (m^2)	d_{high} (m^2)
4.15	8.392447E-06	2.32E-10	8.392197E-06	8.392635E-06	1.81E-11	4.40E-11
8.15	8.392487E-06	2.32E-10	8.392284E-06	8.392723E-06	2.94E-11	3.91E-12
12.15	8.392507E-06	2.32E-10	8.392299E-06	8.392738E-06	2.39E-11	1.33E-12
16.15	8.392483E-06	2.32E-10	8.392278E-06	8.392718E-06	2.72E-11	2.62E-12
20.15	8.392504E-06	2.32E-10	8.392296E-06	8.392736E-06	2.43E-11	7.50E-13
24.15	8.392498E-06	2.32E-10	8.392290E-06	8.392730E-06	2.43E-11	3.23E-13
28.15	8.392560E-06	2.32E-10	8.392354E-06	8.392794E-06	2.64E-11	2.40E-12
32.15	8.392567E-06	2.32E-10	8.392360E-06	8.392800E-06	2.47E-11	1.46E-13
36.15	8.392574E-06	2.32E-10	8.392366E-06	8.392806E-06	2.43E-11	7.04E-13
40.15	8.392598E-06	2.32E-10	8.392390E-06	8.392830E-06	2.36E-11	7.98E-13
42.2	8.392604E-06	2.32E-10	8.392397E-06	8.392837E-06	2.51E-11	3.48E-13

histogram itself and also percentiles of the PDF are used to estimate the coverage interval. The coverage interval obtained by LPU and AMCM was also compared by comparing the absolute differences of low and high endpoints (d_{low} and d_{high}) of the coverage interval obtained from both LPU and AMCM with the numerical tolerance δ/5. The d_{low} and d_{high} was calculated as:

$$d_{low} = \mathrm{I}y\text{-}U(y)\text{-}y_{low}\mathrm{I} \quad \ldots(2)$$

$$d_{high} = \mathrm{I}y\text{+}U(y)\text{-}y_{high}\mathrm{I} \quad \ldots(3)$$

where y is the estimate of measurand, $U(y)$ is the expanded uncertainty obtained by LPU method and y_{low} and y_{high} are the low and high endpoints respectively obtained by the AMCM for a given coverage probability. As stated earlier, the coverage probability chosen is 95 % and hence the coverage factor used is k =1.96 for estimation of expanded uncertainty. Using the Eqs (2) and (3), the d_{low} and d_{high} values were calculated at each pressure point and compared with the numerical tolerance δ/5 associated with the standard uncertainty. The standard uncertainty obtained from AMCM 1.12×10^{-10} m^2 can be written as 11×10^{-11} m^2, considering two significant digits. Hence, the numerical tolerance δ/5 turns out to be 1×10^{-12}. The coverage intervals of the LPU and AMCM are considered to be equivalent if the absolute difference between their high and low limits (Eqs 2 and 3) is lower than the numerical tolerance associated with the standard uncertainty (δ/5). As can be seen from the Table 6 either the d_{low} or d_{high} are higher than the numerical tolerance and hence in the present case coverage intervals of the two are not equivalents. The reason may be due to the non-linearity of the measurement model and the assumptions taken in LPU. Therefore, in the present case, MCM approach may be more appropriate method.

6 Conclusions

We have determined the effective area uncertainty of NPLI-4 using conventional LPU method and compared it to the values obtained through FMCM and AMCM. The pneumatic pressure reference standard NPLI-4 was cross-floated with the pneumatic pressure primary standard, NPLI-P10. An excellent agreement in zero pressure effective area was found between experimental and FMCM, AMCM, i.e., only 9.5×10^{-7}or 0.95 ppm and 2.38×10^{-7} or 0.238 ppm, respectively. The uncertainty in effective area of NPLI-4 was also calculated which in case of LPU is found to be larger than the uncertainty from FMCM and AMCM. Thus, MCM can be used effectively for evaluating the expectation of the measure and its standard uncertainty and coverage intervals. The standard uncertainty obtained using both FMCM and AMCM is numerically same, so wherever possible AMCM may be preferred over FMCM as in the latter method there is no direct control over the quality of results. Apart from that, MCM explicitly gives PDF of the measure and which in case of LPU is assumed to be normal. The coverage intervals obtained from the two approaches, LPU and AMCM are not equivalents. Owing to various advantages of MCM over LPU e.g. better estimation for non-linear models, no sensitivity coefficients required etc., MCM is an effective and convenient alternative method of uncertainty estimation and should be applied in other fields of metrology as well.

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