Indian Journal of Pure & Applied Physics
Vol. 55, February 2017, pp. 163-167

Exact periodic cross-kink wave solutions for the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation

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Received 26 February 2016; revised 25 August 2016; accepted 26 August 2016

Based on the extended homoclinic test technique and the Hirota's bilinear method, the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation is investigated which describes the fluid propagating and can be considered as a model for an incompressible fluid. With the aid of symbolic computation, we introduce two new Ansätz functions to discuss the multiple periodic-soliton solutions of the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation. Some entirely new periodic-soliton solutions are presented. The figures corresponding to these solutions are illustrated to show abundant physics structures.

Keywords: Hirota's bilinear form, Extended homoclinic test technique, Boiti-Leon-Manna-Pempinelli equation, Symbolic computation

1 Introduction

Many significant phenomena in physics, chemistry, biology and mechanics are described by nonlinear partial differential equations (NPDEs)[1]. Solving exact solutions of NLEEs has been attractive in nonlinear physical phenomena. With the aid of symbolic computation[2-10], many methods have been discussed, such as Hirota's bilinear method[11], homogeneous balance method[12-14], *F*-expansion method[15], the similarity transformation method[16], three-wave approach[17-22] and etc. In this paper, with the help of the extended homoclinic test technique, the Hirota's bilinear method and symbolic computation, we will research the following (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation[19]:

$$u_{yt} + u_{xxxy} - 3u_x u_{xy} - 3u_y u_{xx} = 0, \qquad \text{… (1)}$$

where $u = u(x, y, t)$. Equation (1) was proposed by Gilson *et al.*[23] and recently discussed by Luo[24]. This equation was employed to describe the (2+1)-dimensional interaction of the Riemann wave propagated along the y-axis with a long wave propagated along the *x*-axis. By using the binary Bell polynomials, the bilinear form for the (2+1)-dimensional BLMP equation is presented in[24]. The variable separable solutions and some novel localized excitations for the (2+1)-dimensional BLMP were got in[25]. Based on Wronskian formalism and the Hirota method, new solutions for the (2+1)-dimensional BLMP equation are obtained in earlier studies[26,27]. Some exact solutions including kinky periodic solitary-wave solutions, periodic-soliton solutions and kink solutions are obtained in earlier study[19]. In this paper, by using two new Ansätz functions, we obtain new multiple periodic-soliton solutions of the (2+1)-dimensional BLMP equation that is not presented in other references.

2 New Exact Periodic Cross-Kink Wave Solutions for the (2+1)-Dimensional BLMP Equation

By using Painlevé analysis[28] we suppose:

$$u(x, y, t) = -2[\ln \xi(x, y, t)]_x, \qquad \text{… (2)}$$

where $\xi(x, y, t)$ is an unknown real function. Substituting Eq. (2) into Eq. (1), we can obtain the bilinear form of the (2+1)-dimensional BLMP equation:

$$(\xi_{xyt} + \xi_{xxxxy})\xi - [-2\xi_{xy}\xi_{xxx} + \xi_x(\xi_{yt} + 4\xi_{xxxy}) + \xi_y(\xi_{xt} + \xi_{xxxx})]\xi + \xi_t(2\xi_y\xi_x - \xi\xi_{xy}) + 2\xi_x(-3\xi_{xy}\xi_{xx} + 3\xi_x\xi_{xxy} + \xi_y\xi_{xxx}) = 0. \qquad \text{… (3)}$$

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Supposing the real function $\xi(x,y,t)$ has the following Ansätz:

$$\xi(x,y,t) = k_1 e^{\theta_1} + e^{-\theta_1} + k_2 \cos\theta_2 + k_3 \sin\theta_3, \quad \ldots (4)$$

where $\theta_1 = \alpha_i x + \beta_i y + \delta_i t + \sigma_i, i = 1,2,3$ and $\alpha_i, \beta_i, \delta_i$ and σ_i are constants to be determined later. Substituting Eq. (4) into Eq. (3) and equating all the coefficients of different powers of e^{θ_1}, $e^{-\theta_1}$, $\sin\theta_2$, $\cos\theta_2$, $\sin\theta_3$, $\cos\theta_3$ and constant term to zero, we can obtain a set of algebraic equations for $\alpha_i, \beta_i, \delta_i, \sigma_i (i = 1,2,3)$. Solving the system with the help of symbolic computation, we get:

Case (1): If $k_3 = 0$, the exact periodic cross-kink wave solutions of Eq. (1) have been presented by Dai *et al.*[19]. We will not continue to discuss here.

Case (2):

$$\alpha_3 = \beta_1 = \beta_2 = \delta_3 = 0, \delta_1 = -\alpha_1^3, \delta_2 = \alpha_2^3, \quad \ldots (5)$$

where $\alpha_1, \alpha_2, \beta_3, k_1, k_2, k_3 \neq 0$ and $\sigma_i (i = 1,2,3)$ are free real constants. Substituting these results into Eq. (4), we have:

$$\xi(x,y,t) = k_1 e^{-\alpha_1^3 t + \alpha_1 x + \sigma_1} + e^{\alpha_1^3 t - \alpha_1 x - \sigma_1} + k_2 \cos(t\alpha_2^3 + \alpha_2 x + \sigma_2) + k_3 \sin(\beta_3 y + \sigma_3) \quad \ldots (6)$$

Thus, we derive the following new exact periodic cross-kink wave solutions for Eq. (1) as follows:

$$u_1 = \frac{2[\alpha_1 e^{\alpha_1^3 t - \alpha_1 x - \sigma_1} - k_1 \alpha_1 e^{-\alpha_1^3 t + \alpha_1 x + \sigma_1} + k_2 \alpha_2 \cos(t\alpha_2^3 + \alpha_2 x + \sigma_2)]}{k_1 e^{-\alpha_1^3 t + \alpha_1 x + \sigma_1} + e^{\alpha_1^3 t - \alpha_1 x - \sigma_1} + k_2 \cos(t\alpha_2^3 + \alpha_2 x + \sigma_2) + k_3 \sin(\beta_3 y + \sigma_3)}, \quad \ldots (7)$$

where all parameters are defined by Eq. (5). The evolution and mechanical feature of Eq. (7) is shown in Figs 1 and 2 in $x-t$ and in $x-y$, respectively.

Case (3):

$$\alpha_2 = \beta_1 = \delta_3 = \alpha_3 = 0, \delta_1 = -\alpha_1^3, \delta_2 = \alpha_2^3, \quad \ldots (8)$$

where $\alpha_1, \beta_2, \beta_3, k_1, k_2, k_3 \neq 0$ and $\sigma_i (i = 1,2,3)$ are free real constants. Substituting these results into Eq. (4), we have:

$$\xi(x,y,t) = k_1 e^{-\alpha_1^3 t + \alpha_1 x + \sigma_1} + e^{\alpha_1^3 t - \alpha_1 x - \sigma_1} + k_2 \cos(\beta_2 y + \sigma_2) + k_3 \sin(\beta_3 y + \sigma_3) \quad \ldots (9)$$

Thus, we derive the another new exact periodic cross-kink wave solutions for Eq. (1) as follows:

$$u_2 = \frac{2(\alpha_1 e^{\alpha_1^3 t - \alpha_1 x - \sigma_1} - k_1 \alpha_1 e^{-\alpha_1^3 t + \alpha_1 x + \sigma_1})}{k_1 e^{-\alpha_1^3 t + \alpha_1 x + \sigma_1} + e^{\alpha_1^3 t - \alpha_1 x - \sigma_1} + k_2 \cos(\beta_2 y + \sigma_2) + k_3 \sin(\beta_3 y + \sigma_3)}, \ldots (10)$$

where all parameters are defined by Eq. (8). The evolution and mechanical feature of Eq. (10) is shown in Fig. 3 in $y-t$.

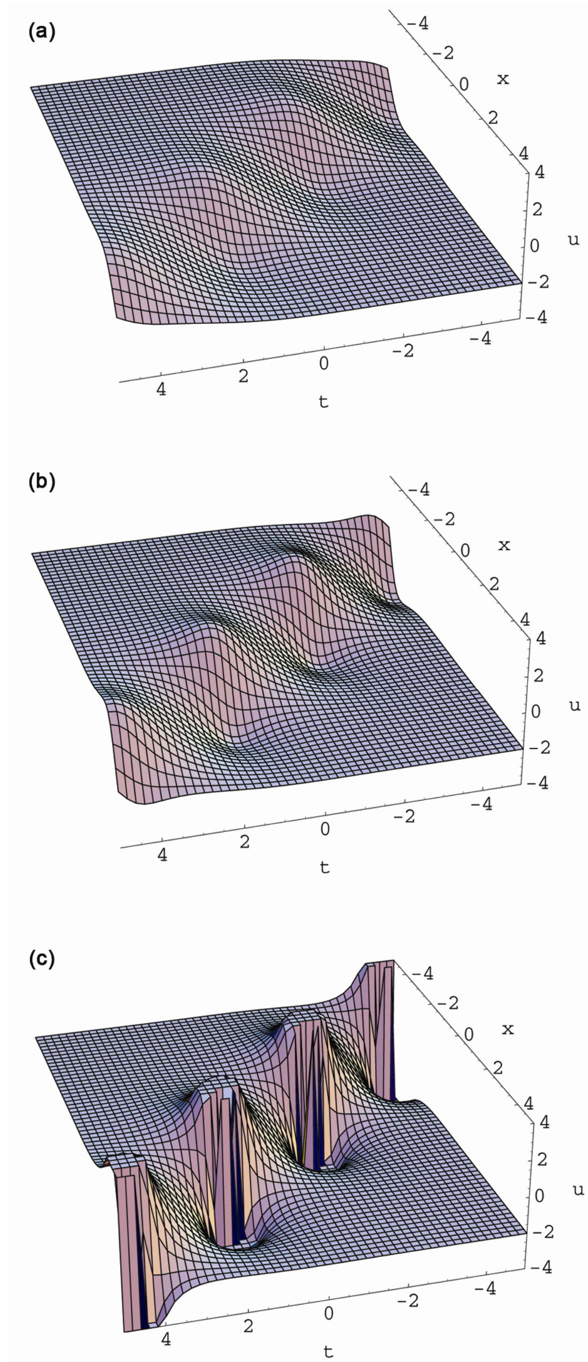

Fig. 1 — Evolution of periodic-soliton solution (Eq. (7)), at $\alpha_1 = \alpha_2 = k_1 = k_2 = 1, k_3 = \beta_3 = -2, \sigma_1, \sigma_2, \sigma_3 = 0$, (a) $y = -5$, (b) $y = 0$ and (c) $y = 5$

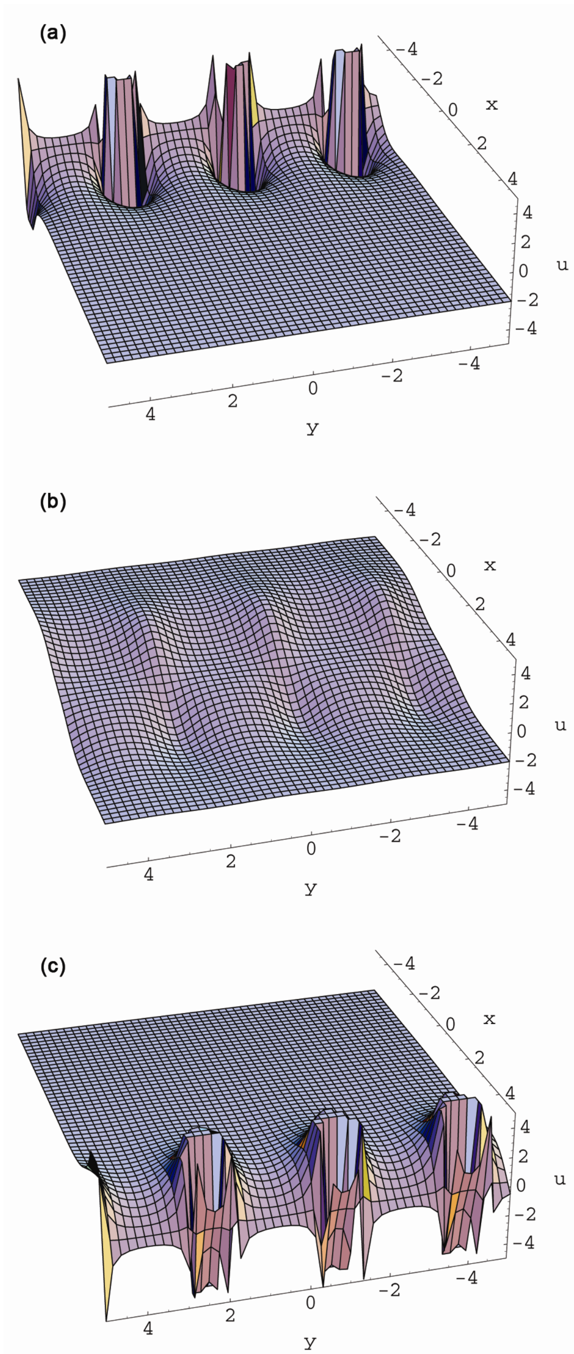

Fig. 2 — Evolution of periodic-soliton solution (Eq. (7)), at $\alpha_1 = \alpha_2 = k_1 = k_2 = 1$, $k_3 = \beta_3 = -2$, $\sigma_1, \sigma_2, \sigma_3 = 0$, (a) $t = -5$, (b) $t = 0$ and (c) $t = 5$

Case (4):

$$\alpha_2 = k_1 = \delta_3 = \alpha_3 = 0, \delta_1 = -\alpha_1^3, \delta_2 = \alpha_2^3, \qquad \ldots (11)$$

where $\beta_1, \beta_2, \beta_3, \alpha_1, k_2, k_3 \neq 0$ and $\sigma_i (i = 1,2,3)$ are free real constants. Substituting these results into Eq. (4), we have:

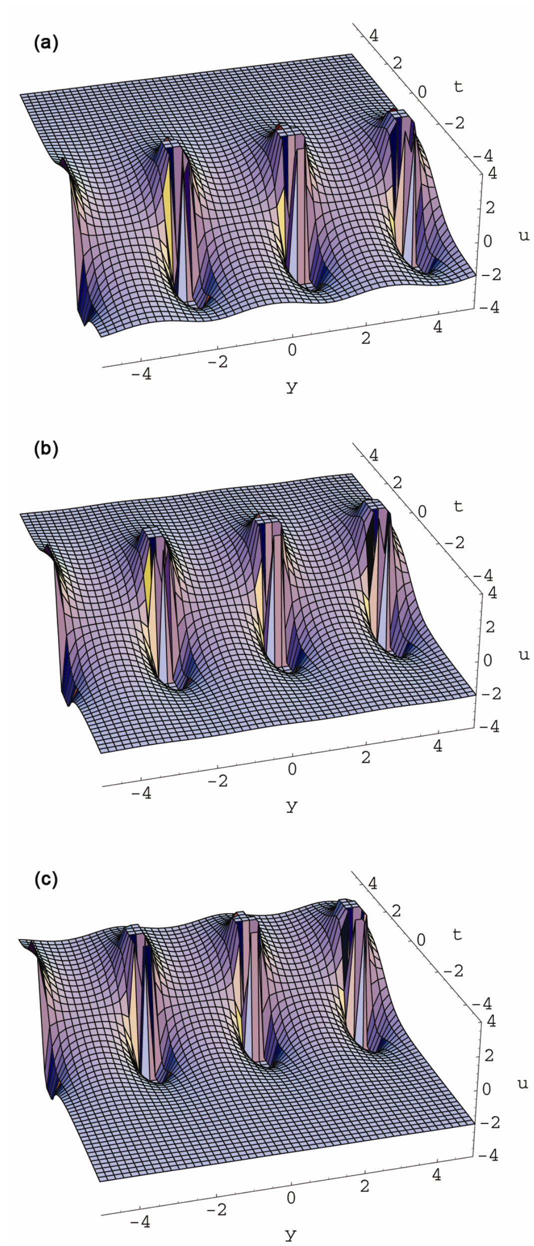

Fig. 3 — Evolution of periodic-soliton solution (Eq. (10)), at $\alpha_1 = k_1 = k_2 = 1$, $\beta_2 = \beta_3 = 2$, $k_3 = -2$, $\sigma_1, \sigma_2, \sigma_3 = 0$, (a) $x = -2$, (b) $x = 0$ and (c) $x = 2$

$$\xi = e^{\alpha_1^3 t - \alpha_1 x - \beta_1 y - \sigma_1} + k_2 \cos(\beta_2 y + \sigma_2) + k_3 \sin(\beta_3 y + \sigma_3) \quad \ldots (12)$$

Thus, we derive the third new exact periodic cross-kink wave solutions for Eq. (1) as follows:

$$u_3 = \frac{2\alpha_1 e^{\alpha_1^3 t - \alpha_1 x - \beta_1 y - \sigma_1}}{e^{\alpha_1^3 t - \alpha_1 x - \beta_1 y - \sigma_1} + k_2 \cos(\beta_2 y + \sigma_2) + k_3 \sin(\beta_3 y + \sigma_3)}, \quad \ldots (13)$$

where all parameters are defined by Eq. (11). The evolution and mechanical feature of Eq. (13) is shown in Fig. 4 in $x - y$.

Case (5):

$$\alpha_2 = i\tau\alpha_1, \alpha_3 = i\varepsilon\alpha_1, \delta_1 = -\alpha_1^3, \delta_2 = -4i\tau\alpha_2^3, \delta_3 = -4i\varepsilon\alpha_2^3, \quad \ldots (14)$$

where $\beta_1, \beta_2, \beta_3, \alpha_1, k_1, k_2, k_3 \neq 0$ and $\sigma_i (i = 1,2,3)$ are free real constants. Substituting these results into Eq. (4), we have:

$$\xi = e^{4\alpha_1^3 t - \alpha_1 x - \beta_1 y - \sigma_1} + k_1 e^{-4\alpha_1^3 t + \alpha_1 x + \beta_1 y + \sigma_1} + k_2 \cosh[-4\tau t\alpha_1^3 + \tau\alpha_1 x - i(\beta_2 y + \sigma_2)] + ik_3 \sinh[-4\varepsilon t\alpha_1^3 + \tau\alpha_1 x - i(\beta_3 y + \sigma_3)] \quad \ldots (15)$$

Thus, we derive the fourth new exact periodic cross-kink wave solutions for Eq. (1) as follows:

$$u_4 = \{-2\alpha_1 e^{4\alpha_1^3 t - \alpha_1 x - \beta_1 y - \sigma_1} + 2k_1\alpha_1 e^{-4\alpha_1^3 t + \alpha_1 x + \beta_1 y + \sigma_1} + 2k_2\tau\alpha_1 \sinh[-4\tau t\alpha_1^3 + \tau\alpha_1 x - i(\beta_2 y + \sigma_2)] + 2ik_3\varepsilon\alpha_1 \cosh[-4\varepsilon t\alpha_1^3 + \tau\alpha_1 x - i(\beta_3 y + \sigma_3)]\} / \{e^{4\alpha_1^3 t - \alpha_1 x - \beta_1 y - \sigma_1} + k_1 e^{-4\alpha_1^3 t + \alpha_1 x + \beta_1 y + \sigma_1} + k_2 \cosh[-4\tau t\alpha_1^3 + \tau\alpha_1 x - i(\beta_2 y + \sigma_2)] + ik_3 \sinh[-4\varepsilon t\alpha_1^3 + \tau\alpha_1 x - i(\beta_3 y + \sigma_3)]\}, \quad \ldots (16)$$

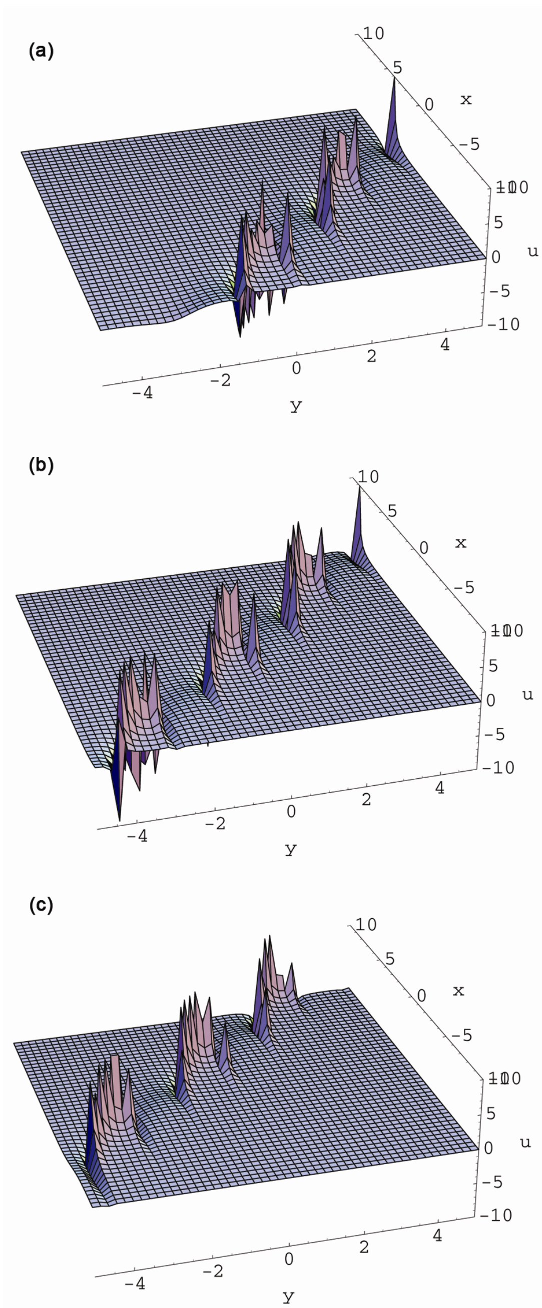

Fig. 4 — Evolution of periodic-soliton solution (Eq. (13)), at $\alpha_1 = k_2 = -1$, $\beta_1 = \beta_2 = 2$, $k_3 = \beta_3 = -2$, $\sigma_1, \sigma_3 = 0$, $\sigma_2 = 5$, (a) $t = -5$, (b) $t = 0$ and (c) $t = 5$

where all parameters are defined by Eq. (14). The evolution and mechanical feature of Eq. (16) is shown in Fig. 5 in $x - y$. Figures 1 and 2 show the shape and motion of the periodic-soliton solution given by Eq. (7) when the values of *y* and *t* are taken to be some different constants. Figure 3 presents the amplitude of the periodic-soliton solution given by Eq. (10) moving with periodic growth and decay with the different value of *x*. Figure 4 describes the propagation of the periodic-soliton solution given by Eq. (13) with periodic oscillation along the distance *t*. In Fig. 5, we can clearly see that the periodic-soliton solution given by Eq. (16) transmits stably without the distortion of the soliton shape and intensity. The variation of the value of *t* affects only the width of the soliton, but the soliton remains its shape.

3 Conclusions

The (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation describes the fluid propagating and can be considered as a model for an incompressible fluid. In this paper, based on the extended homoclinic test technique and the Hirota's bilinear method, the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation is investigated. New exact periodic cross-kink wave solutions for the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equations are obtained. Moreover, the phenomena of soliton interaction are clearly presented in Figs 1-5. These solutions have not been obtained by Dai *e al.*[19]. Of course, the method can also be extended to other nonlinear wave equations.

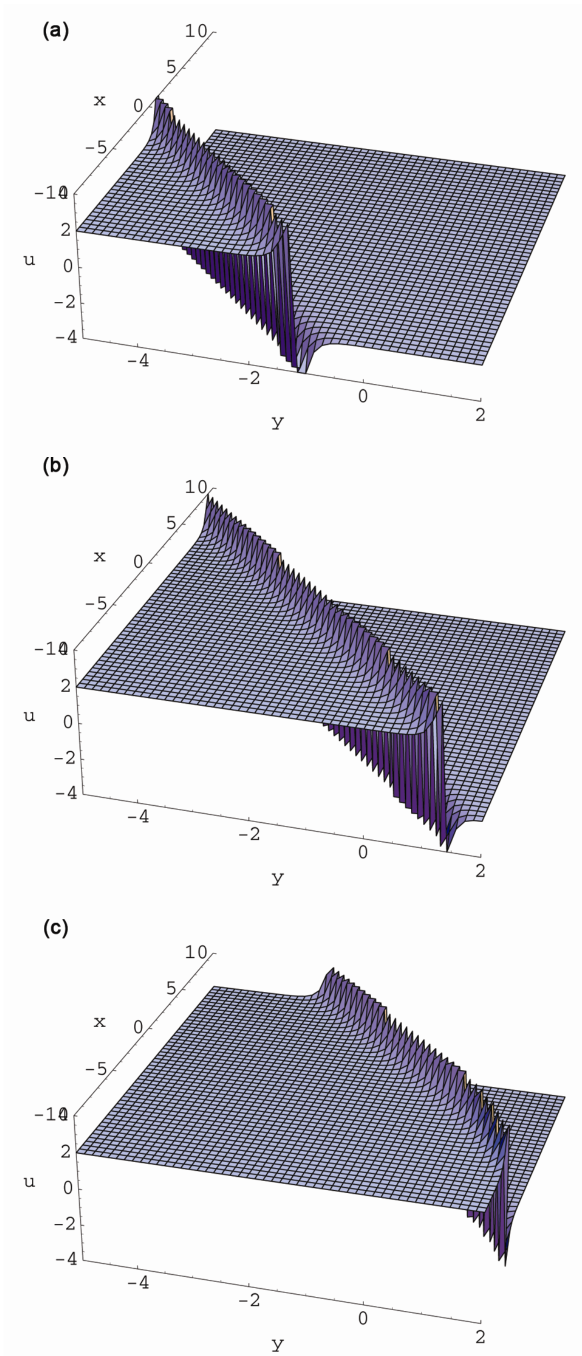

Fig. 5 — Evolution of periodic-soliton solution (16), at $\alpha_1 = k_1 = -1$, $\sigma_1 = 5$, $k_2, \varepsilon, \tau = 1$, $\beta_2 = k_3 = i$, $\beta_1 = -5$, $\sigma_2, \sigma_3 = 0$, $\beta_3 = -i$,, (a) $t = -4$, (b) $t = -2$ and (c) $t = 0$

Acknowledgement

The authors would like to express sincerely thanks to the referees for their useful comments and discussions. Project supported by National Natural Science Foundation of China (Grant No 81160531).

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