# Systematic synthesis of OTA-based Wien oscillators 

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#### Abstract

Nodal admittance matrix (NAM) expansion method for systematic synthesis of seven-OTA Wien oscillators is given. The paper presents 32 Wien oscillators using OTAs by means of NAM expansion method. Each oscillator employs seven OTAs and two grounded capacitors, it is easy to be integrated and the oscillation condition and frequency can be tuned electronically, linearly and independently through tuning bias currents of OTAs. The circuit analysis and computer simulation results are in agreement with theory and then the realizability of the derived circuits is verified.


Keywords: Wien oscillator, OTA, Nodal admittance matrix expansion

## 1 Introduction

Recently, nodal admittance matrix (NAM) expansion method for systematic synthesis of linear active circuits has been reported in the literature ${ }^{1-17}$. According to this method, the literature ${ }^{17}$ presents not only 32 NAM equations for Wien oscillators, but also four different classes of the oscillators, namely the class I oscillators employing three OTAs, the class II oscillators employing four OTAs, the class III oscillators employing five OTAs, and the class IV oscillators employing six OTAs. Unfortunately, the class V oscillators have been missed. Therefore, the work of this paper is an addendum to the literature ${ }^{17}$. Based on 32 NAM equations of Wien oscillators, 32 OTA-based Wien oscillators are obtained by using the NAM expansion method. Each oscillator has 128 different forms of expanded matrixes and 128 nullormirror realizations, but employs only seven OTAs and two grounded capacitors. Having used canonic number of components, the circuits are easy to be integrated and the oscillation condition and frequency can be tuned electronically, linearly and independently through tuning bias currents of OTAs. Finally, the realizability of the derived oscillators has been verified through the use of circuit analysis and MULTISIM 11.0 software simulation and the results are in agreement with theory.

## 2 Systematic Synthesis of Seven-OTA Wien oscillators

Starting from the port admittance matrix Eq. (2a) in the literature ${ }^{17}$ and taking into account the oscillators
with ten nodes, the first step in the NAM expansion method is to add seven blank rows and columns, and then use a first nullator to link columns 1 and 4 to move $G_{1}$ to the position 1, 4, then a first norator is connected between rows 1 and 4 to move $G_{1}$ from the position 1, 4 to the position 4,4 , as described next:
$Y=\left[\begin{array}{cccccccccc}\begin{array}{ccccccccc}0 & -G_{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array} & 0 \\ 0 & G_{2} & -G_{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G_{1} & G_{1} & -G_{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array}\right]$

A second nullator is connected between columns 2 and 5 to move $-G_{1}$ to the position 1,5 , then a first current mirror is connected between rows 1 and 5 to move $-G_{1}$ to be $G_{1}$ at the position 5,5 , as shown in Eq. (2).
$\left.Y=\left[\begin{array}{cccccccccc}\begin{array}{ccccccccc}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array} & 0 \\ 0 & G_{2} & -G_{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G_{1} & G_{1} & -G_{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array}\right]\right\}$
...(2)
A third nullator is connected between columns 1 and 6 to move $-G_{1}$ to the position 3,5 , then a second current mirror is connected between rows 3 and 6 to move $-G_{1}$ to be $G_{1}$ at the position 6,6 , as shown in Eq. (3).
$Y=\left[\begin{array}{cccccccccc}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_{2} & -G_{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & G_{1} & -G_{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array}\right]$

A fourth nullator is connected between columns 2 and 7 to move $G_{1}$ to the position 3,7 , then a second norator is connected between rows 3 and 7 to move $G_{1}$ to the position 7, 7, as shown in Eq. (4).

A fifth nullator is connected between columns 2 and 8 to move $G_{2}$ to the position 2,8 , then a third norator is connected between rows 2 and 8 to move $G_{2}$ to the position 8, 8 , as shown in Eq. (5).

A sixth nullator is connected between columns 3 and 9 to move $-G_{3}$ to the position 2 , 9 , then a third current mirror is connected between rows 2 and 9 to move $-G_{3}$ to be $G_{3}$ at the position 9,9 , as shown in Eq. (6).




At last, a seventh nullator is connected between columns 3 and 10 to move $-G_{4}$ to the position 3,10 , then a fourth current mirror is connected between rows 3 and 10 to move $-G_{4}$ to be $G_{4}$ at the position 10,10 , it follows that the NAM with the added nullormirror elements represented by bracket notation is obtained, as shown in Eq. (7).


In Eq. (7), $G_{1}$ denotes the admittance between nodes $4,5,6,7$ and ground, $G_{2}, G_{3}$, and $G_{4}$ are the admittance connected to nodes 8,9 , and 10 and ground, respectively.

After adding the two capacitors $C_{1}$ and $C_{2}$ at nodes 1 and 2, respectively, Eq. (7) is represented in Fig. 1, which is a nullor-mirror equivalent circuit of the oscillators described by the NAM. It can be seen that this equivalent circuit contains seven different pairs of pathological elements, two grounded capacitors, and seven grounded admittances.

Again, starting from the port admittance matrices in Eq. (2a) in the literature ${ }^{17}$, and applying all possible combinations of the added nullor-mirror elements will yield 128 different forms of the expanded matrix,
resulting in 128 different forms of the equivalent nullor-mirror realizations for the oscillator. At last, using the mullor-mirror descriptions for OTA ${ }^{11-12, ~ 14-17}$, only one equivalent OTA-based realization can be achieved, as shown in Fig. 2.
It is clear that the circuit of Fig. 2 has a larger and a smaller feedback loops, the $d c$ gain of the larger feedback loop is negative, whereas one of the smaller feedback loop is positive.

Similarly, starting from the port admittance matrices in Eq. 2 (b-d) and Table 1 in the literature ${ }^{17}$, 31 equivalent OTA-based realizations for oscillators can be obtained by means of NAM expansion, the remaining implementations are omitted. Of course, readers can also obtain them by changing local feedback polarity of OTA 2, 4, 6 and amplifier polarity of OTA $1,3,5,7$ with the aim to provide negative larger feedback gains and positive smaller feedback gains, as shown in Table 1.

In Table 1, if OTA 1, 3, 5 and 7 take positive polarity, the corresponding amplifiers are noninverting ones; on the contrary, the corresponding ones are inverting ones. If OTA $2,4,6$ take positive polarity, the corresponding OTAs form local positive feedback loops; on the contrary, ones form local negative feedback loops. Note, however, that 32 possible polarity combinations for OTAs must ensure that the $d c$ gain of the larger feedback loop is negative, whereas one of the smaller feedback loop is positive.

## 3 Circuit Analysis

For Fig. 2 circuit, routine circuit analysis leads to the following characteristic equation:

$$
\begin{align*}
& s^{2}+\frac{G_{2} C_{1}+G_{6} C_{2}-G_{1} G_{3} C_{1} / G_{4}}{C_{1} C_{2}} s  \tag{8}\\
& +\frac{G_{2} G_{6}-G_{1} G_{3} G_{6} / G_{4}+G_{3} G_{5} G_{7} / G_{4}}{C_{1} C_{2}}=0
\end{align*}
$$



Fig. 1 - Nullor-mirror equivalent circuit described by the NAM in Eq. (7)


Fig. 2 - One of 32 equivalent OTA-based realizations for Wien oscillators, where $\mathrm{G}_{7}=G_{6}=G_{5}=G_{1}, I_{\mathrm{B} 7}=I_{\mathrm{B} 6}=I_{\mathrm{B} 5}=I_{\mathrm{B} 1}$

| Table 1 - Polarity of OTAs in 32 Wien oscillators |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| OTA number |  |  |  |  |  |  | DC loop gain |  |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | $\mathrm{LG}_{1}$ | $\mathrm{LG}_{2}$ |
| - | - | $+$ | + | $+$ | - | $+$ | - | $+$ |
| - | - | - | - | $+$ | - | $+$ | - | $+$ |
| $+$ | $+$ | $+$ | + | - | - | $+$ | - | $+$ |
| - | $+$ | $+$ | - | $+$ | - | $+$ | - | $+$ |
| $+$ | $+$ | - | - | - | - | $+$ | - | $+$ |
| $+$ | - | - | $+$ | - | - | $+$ | - | $+$ |
| $+$ | - | $+$ | - | - | - | $+$ | - | $+$ |
| - | $+$ | - | $+$ | $+$ | - | $+$ | - | $+$ |
| - | - | $+$ | $+$ | $+$ | $+$ | - | - | $+$ |
| - | - | - | $-$ | $+$ | $+$ | - | - | $+$ |
| $+$ | $+$ | $+$ | + | - | $+$ | - | - | $+$ |
| - | $+$ | $+$ | - | $+$ | $+$ | - | - | $+$ |
| $+$ | $+$ | - | - | - | $+$ | - | - | + |
| $+$ | - | $-$ | $+$ | - | $+$ | - | - | $+$ |
| $+$ | - | $+$ | - | - | $+$ | - | - | $+$ |
| - | $+$ | - | $+$ | $+$ | $+$ | - | - | $+$ |
| - | - | $+$ | $+$ | - | $+$ | $+$ | - | $+$ |
| - | - | - | - | $-$ | $+$ | $+$ | - | $+$ |
| + | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ | - | $+$ |
| - | $+$ | $+$ | - | - | $+$ | $+$ | - | + |
| $+$ | $+$ | - | - | $+$ | $+$ | $+$ | - | $+$ |
| $+$ | - | $-$ | $+$ | $+$ | $+$ | $+$ | - | $+$ |
| $+$ | - | $+$ | - | $+$ | $+$ | $+$ | - | $+$ |
| - | $+$ | $-$ | $+$ | - | $+$ | + | - | $+$ |
| - | - | $+$ | $+$ | - | - | - | - | $+$ |
| - | - | - | $-$ | - | - | - | - | $+$ |
| $+$ | $+$ | $+$ | $+$ | $+$ | - | - | - | $+$ |
| - | $+$ | $+$ | - | $-$ | - | - | - | $+$ |
| $+$ | $+$ | - | - | $+$ | - | - | - | $+$ |
| $+$ | - | $-$ | $+$ | $+$ | - | - | - | $+$ |
| $+$ | - | $+$ | - | $+$ | - | - | - | $+$ |
| - | + | - | + | $-$ | - | - | - | + |

From Eq. (8), letting $C_{1}=C_{2}=C$, $G_{7}=G_{6}=G_{5}=G_{2}=G_{1}=G$ and noting that $G_{\mathrm{i}}=I_{\mathrm{Bi}} / 2 V_{\mathrm{T}}$, where $i=1,2,3,4,5,6,7$, the oscillation condition and frequency of the oscillator can be obtained as:


Fig. 3 - Waveforms for nodes 1 and 2 in Fig. 2

$$
\begin{equation*}
I_{\mathrm{B} 3} \geq 2 I_{\mathrm{B} 4} \tag{9}
\end{equation*}
$$

$f_{o}=\frac{I_{\mathrm{B}}}{4 \pi V_{\mathrm{T}} C}$
It is seen that adjusting $I_{\mathrm{B}}$ can turn the oscillation frequency, whereas adjusting $I_{\mathrm{B} 3}$ or $I_{\mathrm{B} 4}$ can turn the oscillation condition. Thus, an attractive feature of this circuit is independent linear current control of the oscillation frequency and the condition.

For sinusoidal steady state, the voltage transfer function from $V_{\text {o1 }}$ to $V_{\mathrm{o} 2}$ can be readily derived as:
$\frac{V_{o 1}}{V_{o 2}}=\frac{G_{5}}{s C_{1}+G_{6}}$
Considering $C_{1}=C_{2}=C, G_{7}=G_{6}=G_{5}=G_{2}=G_{1}=G$, and using Eq. (9), Eq. (11) simplifies to:
$\frac{V_{o 1}}{V_{o 2}}=\frac{1}{\sqrt{2}} e^{-j 45^{\circ}}$
It follows that the circuit can provide two voltageoutputs with a relative phase shift of $45^{\circ}$.

## 4 Simulation Verification

In order to verify the performances of the derived circuits, the OTAs in Fig. 2 employed LM13600 of the analog device library from NI MULTISIM 11.0 software, and then Fig. 2 was simulated with $\pm 5 \mathrm{~V}$ power supplies. Let $C_{1}=C_{2}=1 \mathrm{nF}, \quad I_{\mathrm{B} 7}=I_{\mathrm{B} 6}=I_{\mathrm{B} 5}=I_{\mathrm{B} 4}$ $=I_{\mathrm{B} 2}=I_{\mathrm{B} 1}=100 \mu \mathrm{~A}$, then $I_{\mathrm{B} 3}=204 \mu \mathrm{~A}$, the circuit will oscillate. From Eq. (10), the design value for $f_{\mathrm{o}}$ is 0.306 MHz . The simulation result is shown in Fig. 3. Using the pointer in MULTISIM yields the actual values for $f_{\mathrm{o}}$ as 296 kHz and the corresponding deviation for $f_{\mathrm{o}}$ is $-3.3 \%$. It is seen that MULTISIM simulations have verified the theoretical results.

## 5 Conclusions

This paper deals mainly with synthesis method of Wien oscillators employing seven OTAs by means of the NAM expansion. The derived oscillators include 32 novel circuits using seven OTAs and enjoy not only the features of independent control of the oscillation frequency and condition but also ones of OTA circuits, such as, use of grounded capacitors, no externally connected resistors, etc. This study also shows that a circuit designer who understands NAM expansion method well has great versatility in generating new circuits and controlling their properties.

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