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Light flavor asymmetry of polarized quark distributions in thermodynamical bag model

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The polarized quark distributions $x\Delta\bar{u}(x)$, $x\Delta\bar{d}(x)$ and $x[\Delta\bar{u}(x)-\Delta\bar{d}(x)]$ are evaluated in the kinematic range 0.01<x<1.0 at an average value of the square of the four momentum transfer Q^2=10 GeV^2 using Thermodynamical Bag Model(TBM). Flavor decomposition of polarized quark distribution are evaluated as a function of x for up $(\Delta u+\Delta\bar{u})/(u+\bar{u})$ and down $(\Delta d+\Delta\bar{d})/(d+\bar{d})$ flavor in the kinematic range 0.033<x<0.6 at Q^2=2.5 GeV^2 using TBM. The calculated values are compared with the theoretical predictions of the Chiral Quark Soliton Model (CQSM) and experimental results of J Lab E99117 and HERMES Collaboration.

Keywords: Polarized distribution functions, Flavor decomposition, Bag model

1 Introduction

The Fermi distribution function is for the non-interacting Fermi particles, we make it to correspond to the interacting quarks emitting gluons and quark-antiquark pairs (sea quarks). The resulting quark distribution function is dependent on both x and Q^2 which determine the invariant mass of the final hadronic state. Semi-inclusive Deep Inelastic Scattering (SIDIS) is a powerful tool to determine the separate contributions Δq_f of the quarks and antiquarks of flavor f to the total spin of the nucleon. The feature[1,2] of the CQSM as compared with many other effective models like the MIT bag model is that it can give reasonable predictions. This TBM model[3,4] offers a clear insight into the transition of static properties of nucleon into its dynamical properties, as observed in DIS of lepton, is highly successful in explaining the wealth of experimental data of both unpolarized and polarized nucleon structure functions. The TBM is used Carlitz-Kaur model[5] only to obtain the quark distribution functions which include the effect of quark interactions. The thermal equilibrium envisaged in the TBM is only for the purpose of deducing the quark distribution functions and not for explaining the mechanism of lepton interaction with the nucleon. For the latter, we still retain the original parton model and the Deep Inelastic Scattering (DIS) of lepton on nucleon is treated as the incoherent sum of elastic scattering of lepton on quarks. In an inelastic event, this allows the energy transfer to a single quark which eventually hadronizes to form a jet or more than one jet if it emits gluons. The spin asymmetries of semi-inclusive cross-sections for the production of positively and negatively charged hadrons have been measured in deep-inelastic scattering. The polarized quark and antiquark distribution is extracted as a function of x for up and down flavors[6]. The highly polarized beam and high pressure polarized ^{3}He target at Jefferson Lab Hall A to investigate the internal spin structure of the neutron in the perturbative and the strong regimes of Quantum Chromo Dynamics (QCD). Thermodynamical Bag Model predictions for spin dependent nuclear quark distributions and the polarized EMC effect for nuclear media such as Li^7 and Al^{27} are evaluated[7], similarly the EMC ratio of iron to deuterium has been performed in our earlier work by using Nachtman variable[8].

2 Thermodynamical Bag Model

The DIS of leptons on nucleons indicates that nucleon consists of three valence quarks, sea quarks and gluons, confined within a small volume. Based on this observation, TBM as a modified form of MIT bag model treats quarks and gluons as fermions and bosons. The invariant mass (W) of the final hadron and the equations of state[3] are:

$$\left[\varepsilon(T)V+BV\right]^2=W^2=M^2+2M\nu-Q^2 \qquad \ldots(1)$$

$$6(n_u - n_{\bar{u}}) = \mu_u T^2 + \frac{\mu_u^3}{\pi^2} \quad \ldots(2)$$

$$6(n_d - n_{\bar{d}}) = \mu_d T^2 + \frac{\mu_d^3}{\pi^2} \quad \ldots(3)$$

where $\varepsilon(T)$ is energy density of the system at a temperature T, V volume of bag, B bag constant, W mass of excited nucleon at temperature T, v energy transfer, Q^2 square of four momentum transfer, M nucleon mass at ground state, $(n_u - n_{\bar{u}})$ number density of u-quark, $(n_d - n_{\bar{d}})$ number density of d-quark, μ_u chemical potential of u-quark and μ_d chemical potential of d-quark. $\varepsilon(T)$ is obtained energy densities of u-quark and d-quark and gluon as follows:

$$\varepsilon_u + \varepsilon_{\bar{u}} = \left(\frac{1}{8\pi^2}\right)\mu_u^4 + \left(\frac{1}{4}\right)\mu_u^2 T^2 + \left(\frac{7\pi^2}{120}\right)T^4 \quad \ldots(4)$$

$$\varepsilon_d + \varepsilon_{\bar{d}} = \left(\frac{1}{8\pi^2}\right)\mu_d^4 + \left(\frac{1}{4}\right)\mu_d^2 T^2 + \left(\frac{7\pi^2}{120}\right)T^4 \quad \ldots(5)$$

$$\varepsilon_g = \frac{\pi^2 T^4}{30} \quad \ldots(6)$$

The chemical potentials of u and d quarks[9] are denoted by μ_u and μ_d and the temperature of the system by T. Since each flavor of quark and antiquark has 6 degrees of freedom and the gluon has 16 degrees of freedom, the energy of the system consisting of u, $\bar{u}$ d, $\bar{d}$ and gluons. The total energy density $\varepsilon(T)$ of the bag comprised of quarks and gluon can be written in terms of the energy densities of the constituents as given by:

$$\varepsilon(T) = 6(\varepsilon_u + \varepsilon_{\bar{u}}) + 6(\varepsilon_d + \varepsilon_{\bar{d}}) + 16\varepsilon_g \quad \ldots(7)$$

The pressure balance condition or the energy minimization condition with respect to the nucleon volume taken into consideration. Eq. (1) relates the invariant mass of the nucleon obtained from DIS kinematic variables and TBM. The invariant mass in TBM is obtained by considering the energy transfer to the nucleon results in heating up the constituents of the nucleon. The temperature and two chemical potentials are not free parameters rather they are evaluated in accordance with x and Q^2 either with fixed Q^2 or with fixed x. At very low Q^2, i.e. as Q^2 tends to zero, temperature of the bag T also tends to zero and only the valence quarks dominates. When $T\approx 0$ MeV, the invariant mass is equal to the mass of the nucleon at rest. As Q^2 increases, temperature of the bag increases and in turn more and more sea quarks and gluons are generated and hence the invariant mass corresponds to the excited state of nucleon for $T>0$ MeV. The unpolarized up and down quark distribution functions[4] are expressed in terms of temperature and chemical potential.

$$u(x,Q^2) = \left(\frac{6V}{4\pi^2}\right)M^2 Tx \ln\left\{1+\exp\left[\left(\frac{1}{T}\right)\left(\mu_u - \frac{Mx}{2}\right)\right]\right\} \quad \ldots(8)$$

$$d(x,Q^2) = \left(\frac{6V}{4\pi^2}\right)M^2 Tx \ln\left\{1+\exp\left[\left(\frac{1}{T}\right)\left(\mu_d - \frac{Mx}{2}\right)\right]\right\} \quad \ldots(9)$$

The anti-quark distribution can be obtained by putting $\mu \rightarrow -\mu$

$$\bar{u}(x,Q^2) = \left(\frac{6V}{4\pi^2}\right)M^2 Tx \ln\left\{1+\exp\left[\left(\frac{1}{T}\right)\left(-\mu_u - \frac{Mx}{2}\right)\right]\right\} \quad \ldots(10)$$

$$\bar{d}(x,Q^2) = \left(\frac{6V}{4\pi^2}\right)M^2 Tx \ln\left\{1+\exp\left[\left(\frac{1}{T}\right)\left(-\mu_d - \frac{Mx}{2}\right)\right]\right\} \quad \ldots(11)$$

μ is the chemical potential of a quark with flavor. $u(x,Q^2)$ and $d(x,Q^2)$ include both valence and intrinsic sea quarks which are identical and not distinguishable in our approach.

3 Polarized Quark Distributions Functions

The nucleon spin structure, it is now widely accepted that the intrinsic quark spin contributes only a small fraction of the total nucleon spin. The spin sum rule indicates that the remaining part is carried by the sea quarks, gluons and orbital angular momentum. Here we present in the kinematic region $0.01<x<1.0$ where the Bjorken Scaling variable x is large. For these kinematics, the valence quark is dominant and ratio's of structure function can be estimated based on our theoretical model calculations. Let $\Delta q(x)$ denotes the difference between the probability of finding a quark or antiquark with positive helicity and the corresponding probability of finding a quark or antiquark with negative helicity.

$$\Delta q(x) = \left[q^{\uparrow}(x) + \overline{q}^{\uparrow}(x) - q^{\downarrow}(x) - \overline{q}^{\downarrow}(x) \right] \quad \ldots(12)$$

The polarized distribution functions Δu and Δd which can be obtained by multiplying the unpolarized distributions[10].

$$\Delta u(x) = \left[[u(x) - \overline{u}(x)] - \frac{2}{3}[d(x) - \overline{d}(x)] \right] \cos 2\theta(x) \quad \ldots(13)$$

$$\Delta d(x) = \left[-\frac{1}{3}[d(x) - \overline{d}(x)] \right] \cos 2\theta(x) \quad \ldots(14)$$

Polarized antiquark distribution is obtained by:

$$\Delta \overline{u}(x) = \left[\overline{u}(x) - \frac{2}{3}[\overline{d}(x)] \right] \cos 2\theta(x) \quad \ldots(15)$$

$$\Delta \overline{d}(x) = \left[-\frac{1}{3}[\overline{d}(x)] \right] \cos 2\theta(x) \quad \ldots(16)$$

where $\cos 2\theta(x) = \dfrac{1}{\left[1 + (\frac{H_0}{\sqrt{x}})(1-x)^2 \right]}$ is known as spin dilution factor which vanishes as $x \to 0$ and becomes unity as $x \to 1$ characterizing the valence quark helicity contribution to the proton. Since the spin dilution factor is not derived from first principles it is adjusted to satisfy the Bjorken sum rule, which is considered as the fundamental test of QCD, this enables to determine the valence quark distributions explicitly. Here H_0 is the only free parameter which is used to satisfy the Bjorken sum rule.

The spin dilution is a factorization process in which it has an inbuilt property of quarks, gluons and sea quarks inside the nucleon. In the low x region, sea quarks varies of the order of reciprocal of Bjorken variable and at higher x region the distribution is proportional to the power values of x. The polarized antiquark sea in the nucleon is also flavor asymmetry[11]. We are considering a bag which contains non-interacting valence quarks, sea quarks in a sea of gluons moving at a thermodynamical temperature T. The exchange of gluons between the constituent quarks may affect quarks helicities and orbital angular momentum.

The gluon distribution can be written as :

$$G(x) = -\left(\frac{16V}{4\pi^2} \right) M^2 T x \ln \left[1 + \exp\left(\frac{-Mx}{2T} \right) \right] \quad \ldots(17)$$

The effect of the gluon contribution increases as the temperature increases. Gluons carry large amount of spin than the valence quarks and the sea quarks. At low excitation, the proton contains only valence quarks. The spin carried by the gluons Δg is obtained by using x for the spin dilution function for the gluons is:

$$\Delta g = xG(x) \quad \ldots(18)$$

The Bjorken sum rule[12] is given by:

$$\int_0^1 \left[g_1^p(x,Q^2) - g_1^n(x,Q^2) \right] dx = \left[\left(\frac{1}{6} \right) \left(\frac{g_A}{g_V} \right) \left(1 - \frac{\alpha_s(Q^2)}{\pi} \right) \right] \quad \ldots(19)$$

where g_A and g_V are axial vector and vector coupling constants of weak interactions and Bjorken sum rule is of fundamental importance for the understanding of elementary particles. The gauge-coupling constant[13] is expressed as:

$$\alpha_s = \left[\left(\frac{4\pi}{11 - (2N_f/3)} \right) \ln \left(\frac{Q^2}{\Lambda^2} \right) \right] \quad \ldots(20)$$

The strong interactions have a typical energy scale Λ_{QCD}=250 MeV at which the coupling constant becomes of order one. The one gluon exchange potential has been studied in detail using semi-relativistic quark model calculations[14]. In a formal way, the nucleon's total spin of $1/2$ can be decomposed into constituents.

$$\begin{aligned} < SN > &= \frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \Delta L \\ &= \frac{1}{2}(\Delta u_v + \Delta d_v + \Delta q_s) + \Delta G + \Delta L_q + \Delta L_G \end{aligned} \quad \ldots(21)$$

where the three terms are the contributions for the quark and gluon spins and from their orbital angular momentum. The total quark helicity $\Delta\Sigma$ is defined as the sum of the helicity distributions of up, down and strange quarks and antiquarks. The polarized quark and antiquark distribution is extracted as a function of x for up and down flavors. The flavor asymmetry of the polarized quark distributions are evaluated within the range 0.01<x1.0 at an average Q^2=10 GeV^2 by using TBM and results have been compared to the predictions[15] of the CQSM.

4 Flavor Decomposition

The DIS of polarized leptons on polarized proton revealed that the spin carried by the quarks is very small and considerable excitement started on 'EMC spin crisis'[16] and the inclusive deep inelastic scattering that only a fraction of the nucleon spin can be attributed to the quark spins and that the strange quark sea seems to be negatively polarized. These conclusions follow from the extraction of the first moments of up, down and strange quark spin distributions from the inclusive data by assuming $SU(3)_f$ flavor asymmetry. These Parton distribution functions (PDFs) depend on whether the Parton's helicity is equal or opposite to that of the nucleon. The flavor decomposition of polarized u-quark distributions are evaluated by using the relation $(\Delta u+\Delta\bar{u})/(u+\bar{u})$ and similarly for d-quarks, the flavor decomposition is $(\Delta d+\Delta\bar{d})/(d+\bar{d})$ where u and d are the unpolarized up and down quark distribution functions, $u\Delta$ and Δd are the polarized distribution functions. We used the up-quark and down-quark distributions[17] obtained in J Lab E99117 along with preliminary results of the HERMES semi-inclusive measurements[18].

5 Results and Discussion

In the present work, the flavor asymmetry has been evaluated by quark distributions based on TBM. Using the quark distribution functions (obtained by transforming the Fermi distribution function to the infinite momentum frame and substituting the bag variables) the nucleon structure functions are calculated. Figure 1 shows similar behaviour in the positive x domain while comparing our evaluated values with CQSM predictions. However, in the negative x domain our estimated values are negative in whole x region. TBM results shows that the $x\Delta\bar{u}(x)$ is positive in whole x region and $x\Delta\bar{d}(x)$ is negative region in 0.15<x0.8 and also almost positive for x>0.8. Thus, our prediction matches well with CQSM predictions. Figure 2 shows $x[\Delta\bar{u}(x)-\Delta\bar{d}(x)]$ is positive in the entire x region which is similar to the CQSM prediction. At very low values of x, the dominance of sea quarks is a natural consequence of this model. In evaluating spin structure function, our efforts to satisfy Bjorken sum rule as defined by Eq. (19). The momentum fraction carried by polarized quarks in nuclear medium. The charge carried by u-quark is positive and hence polarized u-quark distribution is positive distribution. In case of d-quark

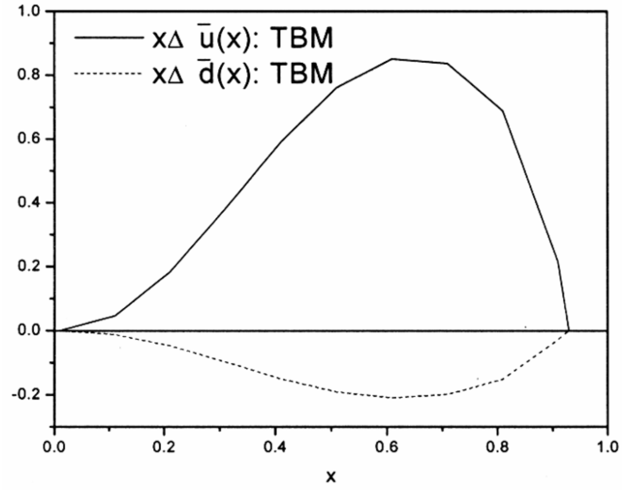

Fig. 1 — Polarized spin distribution function $x\Delta\bar{u}(x)$ and $x\Delta\bar{d}(x)$ as a function of Bjorken variable x at Q^2=2.5 GeV^2

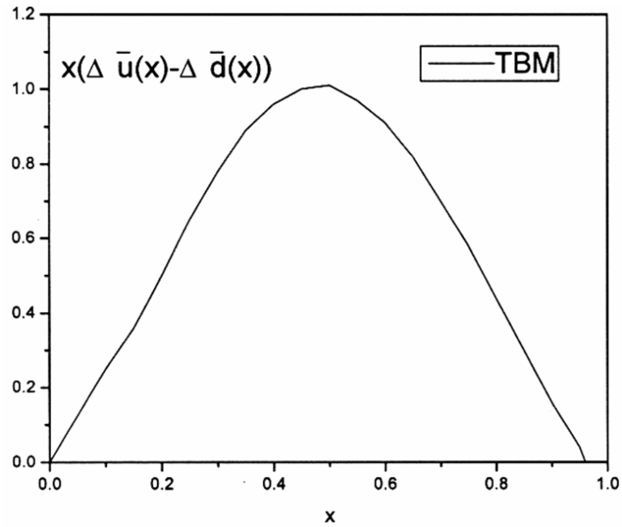

Fig. 2 — Polarized spin distribution function $x[\Delta\bar{u}(x)-\Delta\bar{d}(x)]$ as a function of Bjorken variable x at Q^2=10 GeV^2

distribution is downward negative distribution. As expected, the helicity density of the u-quark is found to be positive and large at x>0.1 and that of the d quark is negative, while the helicity densities of the light sea quarks are found to be compatible with zero.

According to scaling process, the valence quarks dominate more in the higher x region than in the low x region. Since the polarization occurs only in the low x region, the up and down quarks are flipped by the sea quarks and gluon in that region. The distribution functions of the u and d quarks for the proton u-quarks dominate over the d-quarks whereas for the antiquarks the opposite feature is observed. The

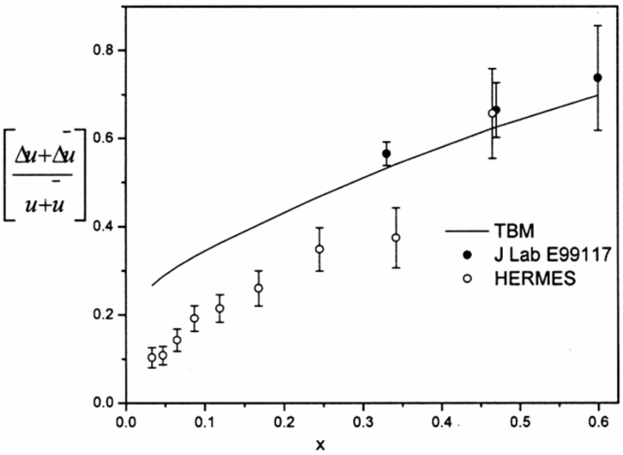

Fig. 3 — Flavor decomposition of the u quark polarization as a function of x at Q^2=2.5 GeV^2 TBM (solid line), J Lab E99117 data (filled circles) and The HERMES Collaboration data (open circles)

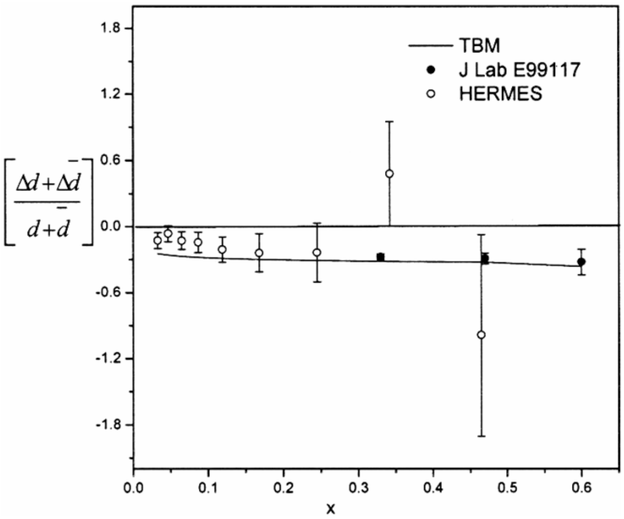

Fig. 4 — Flavor decomposition of the d quark polarization as a function of x at Q^2=2.5 GeV^2 TBM (solid line), J Lab E99117 data (filled circles) and The HERMES Collaboration data (open circles)

dominance of the $\bar{d}$-quarks over the $\bar{u}$-quarks is observed because the chemical potential μ_u is greater than μ_d as a consequence of two valence u-quarks and one valence $\bar{d}$-quarks for the proton.

The flavor decomposition of the u and d quark polarization as a function of x has been studied using TBM and is shown in Figs 3 and 4, respectively. The standard deviation[19] can be carried out for our theoretical calculation with corresponding experimental observations. In the case of inclusive DIS that only a fraction of the nucleon spin can be attributed to the quark spins and that the strange quark sea seems to be negatively polarized. But in the semi-inclusive polarized deep inelastic scattering which separates spin contributions of quark and antiquark flavor to the total spin of the nucleon can be determined as a function of the Bjorken scaling variable x. It is evident that the flavor decomposition of polarized quark distribution in the nucleon in the range $0.033<x0.6$ at Q^2=2.5 GeV^2 by using TBM. The calculated values show close agreement with the experimental data from The J Lab E99117 and HERMES Collaboration in the middle region.

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