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Unsteady MHD oscillatory visco-elastic fluid flow through an inclined channel in presence of chemical reaction with Soret and Dufour effects

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The purpose of present paper is to analyze the Soret and Dufour effects on an unsteady magneto hydrodynamic oscillatory flow of radiative, visco-elastic fluid through an inclined channel filled with saturated porous medium with non-uniform wall temperature in presence of first-order chemical reaction. The governing dimensionless momentum equation coupled with the energy and mass diffusion equations are solved analytically. The expressions for velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number are obtained and are analyzed graphically for various values of the dimensionless flow parameters.

Keywords: chemical reaction, Dufour effect, MHD, Soret effect, visco-elastic fluid

Introduction

The study of the problem of heat and mass transfer of an electrically conducting fluid has many applications in engineering problems. These include MHD generators, plasma studies, nuclear reactors, geothermal energy extraction, hydro-magnetic chromatography, crystal magnetic damping control. In the light of these applications, MHD flow in a channel has been studied by many authors Nigam and Singh[1], Attia and Kotb[2], Soundalgekar and Bhat[3], Raptis *et al*.[4], Vajravelu[5], Makinde and Mhone[6], Choudhury and Das[7].

The study of combined heat and mass transfer problems in presence of magnetic field and chemical reaction has received much attention due to its applications in many branches of engineering science and technology. Several authors Chamkha[8], Cortell[9], Mahdy[10], Sivaiah[11], Reddy[12], Ibrahim *et al.*[13], Jena *et al.*[14], Sreedevi[15], Sharma and Bisht[16] have examined the effect of chemical reaction on different physical situations.

In many cases of simultaneous heat and mass transfer problems the effects of Soret and Dufour are cannot be neglected (Eckert and Drake[17]). Dursunkaya and Worek[18] investigated the Soret and Dufour effects in convective flow from a vertical surface. Many authors have studied the effects of Soret and Dufour in different flow problems. Some of them are Kafoussias and Williams[19], Chamkha and Ben-Nakhi[20], Alam *et al.* [21], Srinivasacharya and Kaladhar[22], EL-Kabeir *et al.* [23], Das [24].

The objective of the present paper is to investigate the influence of visco-elastic parameter, Soret and Dufour effects on an unsteady magneto hydrodynamic oscillatory flow of radiative, visco-elastic fluid characterized by Walters liquid model B' as proposed by Walters[25] through an inclined channel filled with saturated porous medium with non-uniform wall temperature.

Mathematical analysis

Consider an unsteady two dimensional flow of an incompressible, electrically conducting, optically thin, radiative visco-elastic fluid characterized by walter's liquid (model B') in an inclined channel filled with saturated porous medium in presence of Soret and Dofour effects with first-order chemical reaction. A uniform magnetic field of strength B_0 is applied in transverse direction. It is assumed that there exists rate constant Kr' between the diffusing species and the fluid of the homogeneous first order chemical reaction. The two channel walls separated with a distance a are kept at constant temperature T_0 for one wall and T_w for the other wall with $T_w > T_0$. The x'-axis is taken along the centre of the channel and y'-axis in the normal direction (Fig. 1). Under the above assumption and usual Boussinesq's

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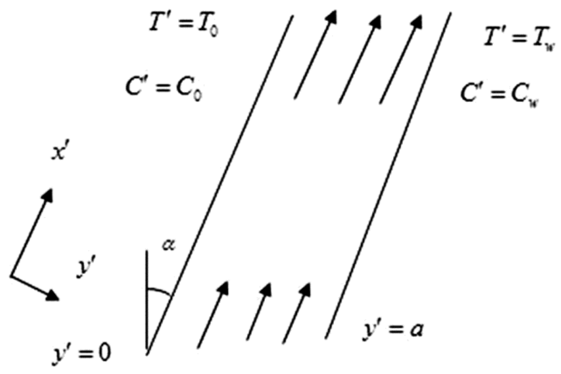

Fig. 1 — Physical model of the problem.

approximation, the governing equations for visco-elastic fluid are given by

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial P'}{\partial x'} + \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\upsilon u'}{K'} - \frac{\sigma B_0 u'}{\rho} + g\beta_T (T' - T_0)\cos\alpha - \frac{K_0}{\rho}\frac{\partial^3 u'}{\partial y'^2 \partial t'} + g\beta_c (C' - C_0)\cos\alpha, \quad (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_P}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_P}\frac{\partial q_r}{\partial y'} + \frac{D_m K_T}{C_s C_P}\frac{\partial^2 C'}{\partial y'^2}, \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - Kr'(C' - C_0) + \frac{D_m K_T}{T_m}\frac{\partial^2 T'}{\partial y'^2}. \quad \dots (3)$$

The boundary conditions are

$$u' = 0, T' = T_0, C' = C_0 \text{ on } y' = 0;$$
$$u' = 0, T' = T_W, C' = C_w \text{ on } y' = a, \quad \dots (4)$$

where u' is the axial velocity, t' is the time, T' is the fluid temperature, P' is the pressure, g is the acceleration due to gravity, K' is the permeability, σ is the conductivity of fluid, β_T is the co-efficient of volume expansion due to temperature, β_c is the co-efficient of volume expansion due to the concentration, C_P is the specific heat at constant pressure, k is the thermal conductivity, α is the inclination of plane with the vertical, D_m is the co-efficient of mass diffusivity, K_T is the coefficient of thermal diffusion, C_s is the concentration susceptibility, C' is the dimensionless concentration, Kr' is the chemical reaction parameter and T_m is the mean temperature of the fluid.

The temperature of the walls T_0 and T_w are assumed to be high enough to induce radiative heat transfer. Using Cogley *et al.*[26], the radiative heat flux q_r is given by:

$$\frac{\partial q_r}{\partial y'} = 4\alpha_1^2 (T_0 - T') \quad \dots (5)$$

We introduce the following dimensionless quantities:

$$x = \frac{x'}{a}, y = \frac{y'}{a}, u = \frac{u'}{U}, t = \frac{t'U}{a},$$
$$\text{Re} = \frac{Ua}{\upsilon}, Sc = \frac{\upsilon}{D}, \theta = \frac{T' - T_0}{T_w - T_0}, \phi = \frac{C' - C_0}{C_w - C_0},$$
$$P = \frac{aP'}{\rho\upsilon U}, Gr = \frac{g\beta_T (T_w - T_0)a^2}{\upsilon U}, Pe = \frac{Ua\rho C_P}{\kappa},$$
$$Gc = \frac{g\beta_c (C_w - C_0)a^2}{\upsilon U}, N^2 = \frac{4\alpha^2 a^2}{\kappa},$$
$$Du = \frac{D_m K_T (C_w - C_0)}{C_S C_P \upsilon (T_w - T_0)}, K_1 = \frac{K_0 \text{Re}}{\rho a^2},$$
$$Sr = \frac{D_m K_T (T_w - T_0)}{T_m \upsilon (C_w - C_0)}, \quad \dots (6)$$

In view of the equation (6), the equations (1) – (3) reduce to the following dimensionless form:

$$\text{Re}\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (s^2 + H^2)u + Gr\theta\cos\alpha + Gc\phi\cos\alpha - K_1\frac{\partial^3 u}{\partial t \partial y^2}, \quad \dots (7)$$

$$Pe\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2\theta + \frac{DuPe}{\text{Re}}\frac{\partial^2 \phi}{\partial y^2}, \quad \dots (8)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{\text{Re}\,Sc}\frac{\partial^2 \phi}{\partial y^2} - Kr\phi + \frac{\partial^2 \theta}{\partial y^2}, \quad \dots (9)$$

with boundary conditions

$$u = 0, \theta = 0, \phi = 0 \text{ on } y = 0;$$
$$u = 0, \theta = 1, \phi = 1 \text{ on } y = 1, \quad \dots (10)$$

where $Gr, Gc, H,, Pe, \text{Re}, Da, s^2 = (1/Da), Sc, Kr, Du, Sr, K_1$ are thermal Grashof number, solutal Grashof number, Hartmann number, radiation parameter, Peclet number, Reynolds number, Darcy number, porous medium shape factor parameter, Schmidt number, chemical reaction parameter, Dufour number, Soret number and viscous-elastic parameter respectively.

Solution of the problem

To solve the equations (7)-(9) for purely oscillatory flow, let us take

$$-\frac{\partial P}{\partial x} = \lambda \exp(i\omega t), u(y,t) = u_0 \exp(i\omega t),$$
$$\theta = \theta_0 \exp(i\omega t), \phi = \phi_0 \exp(i\omega t) \quad \text{... (11)}$$

where λ is a constant and ω is the frequency of the oscillation.

Using (11) into the equations (7) - (9), and comparing the harmonic terms on both sides, we obtain,

$$(1 - iK_1\omega)\frac{\partial^2 u_0}{\partial y^2} - m_3^2 u_0 = -\lambda - Gr\theta_0 \cos\alpha - Gc\phi_0 \cos\alpha, \quad \text{... (12)}$$

$$\frac{\partial^2 \theta_0}{\partial y^2} + m_1^2 \theta_0 + \frac{DuPe}{\text{Re}}\frac{\partial^2 \phi_0}{\partial y^2} = 0, \quad \text{... (13)}$$

$$\frac{\partial^2 \phi_0}{\partial y^2} - m_2^2 \phi_0 + ScSr\frac{\partial^2 \theta_0}{\partial y^2} = 0, \quad \text{... (14)}$$

with boundary conditions

$$u_0 = 0, \theta_0 = 0, \phi_0 = 0, \text{ for } y = 0,$$
$$u_0 = 0, \theta_0 = 1, \phi_0 = 1, \text{ for } y = 1. \quad \text{... (15)}$$

Where,

$$m_1 = \sqrt{N^2 - i\omega Pe}, m_2 = \sqrt{KrSc\,\text{Re} + i\omega\,\text{Re}\,Sc}$$
$$m_3 = \sqrt{s^2 + H^2 + i\omega\,\text{Re}}.$$

Solving the equations (12)-(14) analytically subject to boundary conditions (15), we obtain the fluid velocity, temperature and concentration as follows:

$$u(y,t) = \{C_5 \exp(A_5 y) + C_6 \exp(-A_5 y) - A_{13}\exp(-m_4 y) - A_{13}\exp(-m_4 y) - A_{14}\exp(-m_5 y) + A_{11} y + A_{15}\}\exp(i\omega t). \quad \text{... (16)}$$

$$\theta(y,t) = \{A_1 \exp(-m_4 y) + A_2 \exp(-m_5 y)\}\exp(i\omega t), \quad \text{... (17)}$$

$$\phi(y,t) = \{A_3 \exp(-m_4 y) + A_4 \exp(-m_5 y) + C_3 y + C_4\}\exp(i\omega t) \quad \text{... (18)}$$

The constants are not presented here for the sake of brevity.

The skin friction co-efficient τ at the surface $y = 0$ is given by

$$\tau = -\left[\frac{\partial u}{\partial y} - K_1 \frac{\partial^2 u}{\partial y \partial t}\right]_{y=o} \quad \text{... (19)}$$

The heat flux at the wall $y = 0$ in terms of Nusselt number is given by:

$$Nu = -\left[\frac{\partial \theta}{\partial y}\right]_{y=0} \quad \text{... (20)}$$

The rate of mass transfer at the wall $y = 0$ in terms of Sherwood number is given by

$$Sh = -\left[\frac{\partial \phi}{\partial y}\right]_{y=0} \quad \text{... (21)}$$

Result and discussion

In order to study the flow problem, we have carried out numerical calculations for velocity field, temperature field, species concentration field, skin friction, Nusselt number and Sherwood number at the walls by assigning the following pertinent parameter values for computations unless otherwise indicated in the figure:

$$Gr = 1.0, Gc = 0.5, H = 0.5, Kr = 0.5,$$
$$\text{Re} = 2.0, K_1 = 0.2, Du = 0.5, s = 1,$$
$$Sr = 1.0, Pe = 2.0, N = 1.0, Sc = 0.6,$$
$$\text{Re} = 2.0, t = 0, \lambda = 1, \omega = 1, \alpha = 0.$$

Figures 2-8 depicts the pattern of velocity profiles u against y under the influence of visco-elastic parameter (K_1), porous medium shape factor parameter (s), chemical reaction parameter (Kr), inclination angle (α), radiation parameter (N), Soret

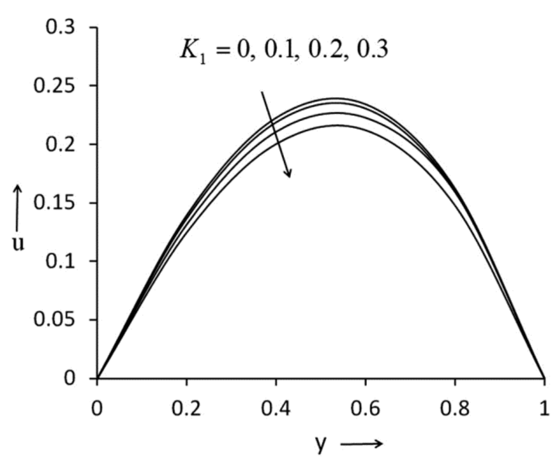

Fig. 2 — Velocity profiles for different values of K_1.

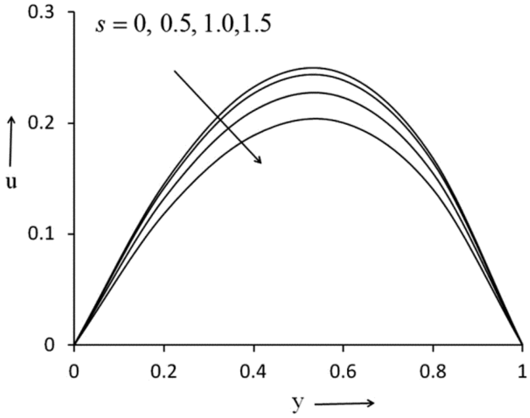

Fig. 3 — Velocity profiles for different values of s.

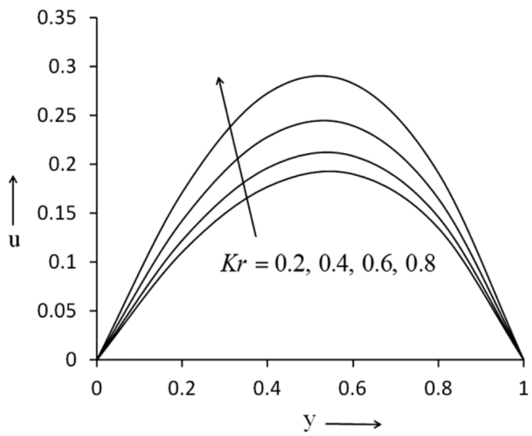

Fig. 4 — Velocity profiles for different values of Kr.

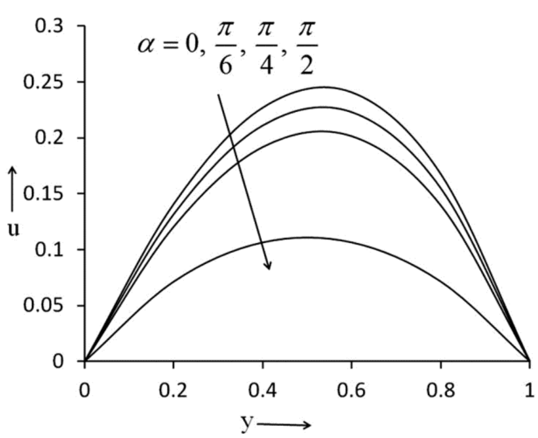

Fig. 5 — Velocity profiles for different values of α.

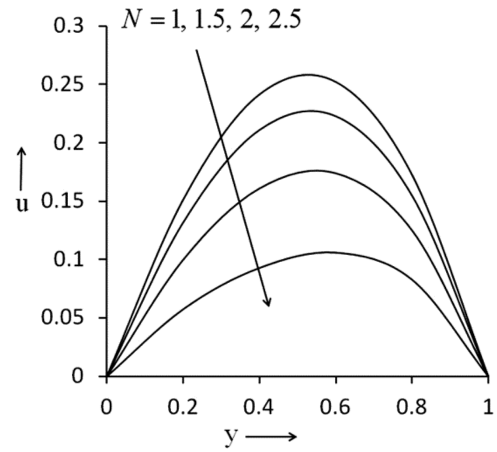

Fig. 6 — Velocity profiles for different values of N.

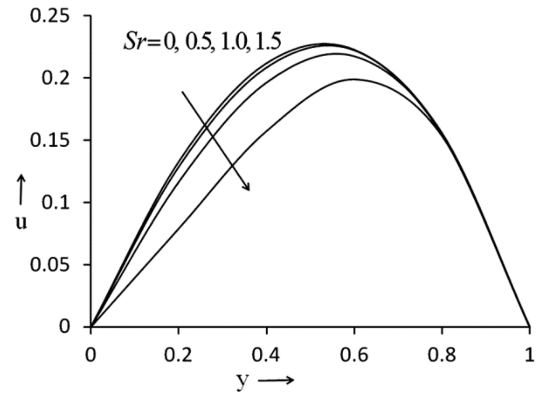

Fig. 7 — Velocity profiles for different values of Sr.

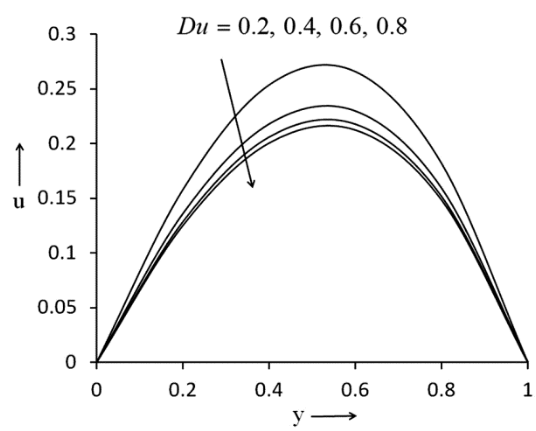

Fig. 8 — Velocity profiles for different values of Du.

number (Sr), and Dufour number (Du), respectively. In all these figures, it is observed that the velocity profiles are parabolic in nature with maximum magnitude along the channel centre line and minimum at the walls. From Fig. 2, it is seen that the velocity decreases with an increase in the visco-elastic parameter from $K_1 = 0$ through $0.1, 0.2$ to 0.3. The Newtonian fluid flow is represented by $K_1 = 0$. From Fig. 3, it is observed that the fluid velocity decrease with an increase in porous medium shape factor parameter. This is physically true because an increase in porous medium shape factor implies an increase in the resistance of the medium. It is observed that an increase in the inclination angle (α) and radiation parameter leads to fall in the fluid velocity whereas reverse trend is seen for chemical reaction parameter as shown in Figs 4-6. The effect of Soret number on temperature and concentration profiles is shown in Figs 9-10. It is observed that an increase in Soret number results to fall in velocity of the fluid (Fig. 7) whereas reverse trend is seen for temperature (Fig. 9). Also, from Fig. 7 it is seen that the effect of Soret number on velocity is more prominent in the region between $y=0$ to $y=0.8$, whereas the effect of Soret number on temperature is least important near the wall $y=0$ and most noteworthy near the wall $y=1$. From the Fig. 10, it is noticed that concentration profile increases with an increase in Soret number but the pattern are reversed when $y<0.58$. Soret number signifies the ratio of temperature difference to concentration. Thus, the bigger Soret number stands for a larger temperature difference and the fluid velocity decreases due to greater thermal diffusion factor. The effect of Dufour number on concentration profile is shown in Fig. 11. It is observed that the velocity (Fig. 8) and concentration decreases with an increase of Dufour number.

Figures 12-14 shows the effects of Soret number, Dufour number and chemical reaction parameter on coefficient of Skin friction with various values of visco-elastic parameter. It is observed that an increase in Soret number, chemical reaction parameter causes a rise in the value of skin friction, whereas reverse trend is seen in case of Dufour number. Also, it is noticed that an increase in visco-elastic parameter causes a rise in skin friction coefficient with the increasing values of Soret number, Dufour number and chemical reaction parameter.

Figures 15-16 depicts the Nusselt number and Sherwood number at the wall $y=0$ for various values of the Soret number. It is noticed that an increase in Soret number causes an increase in the Nusselt

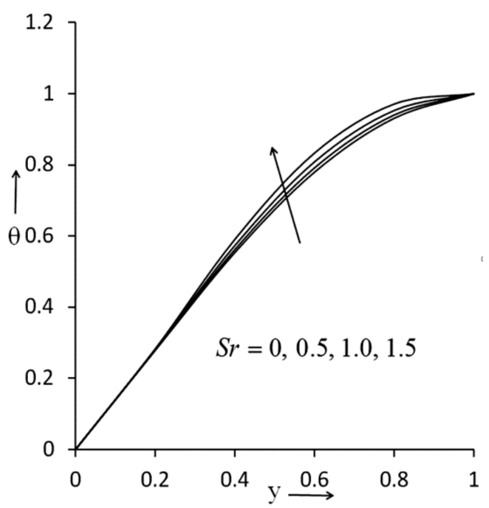

Fig. 9 — Temperature profiles for Sr.

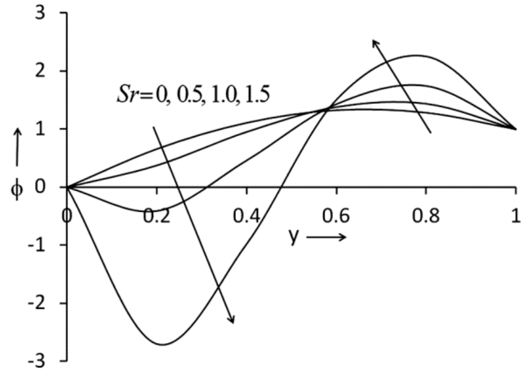

Fig. 10 — Concentration profiles for various Sr.

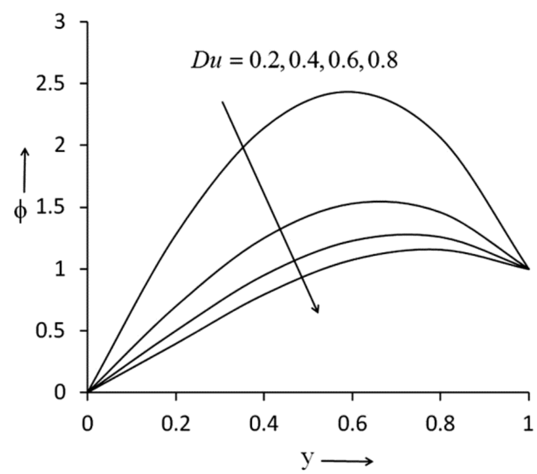

Fig. 11 — Concentration profiles for various Du.

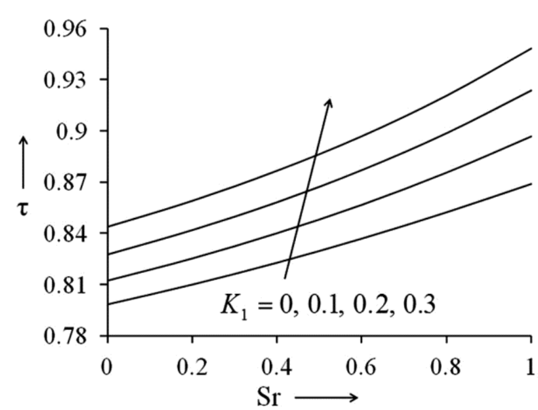

Fig. 12 — Skin friction coefficients for different values of visco-elastic parameter with Sr.

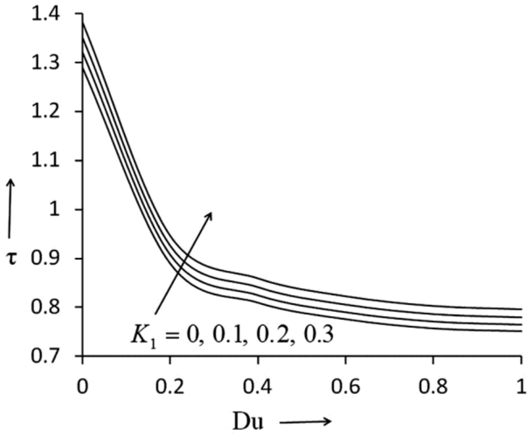

Fig. 13 — Skin friction coefficients for different values of visco-elastic parameter with Du.

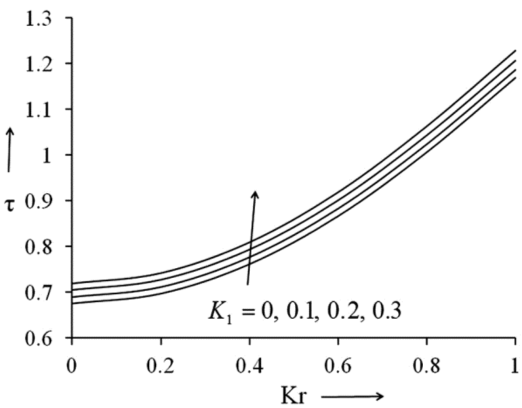

Fig. 14 — Skin friction coefficients for different values of visco-elastic parameter with Kr.

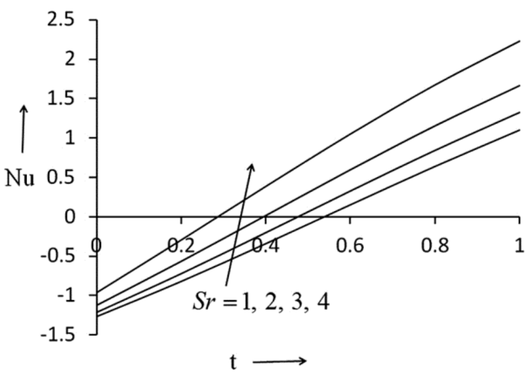

Fig. 15 — Nusselt number for different values of Sr.

number and Sherwood number. Figures 17-18 displays the Nusselt number and Sherwood number at the wall $y = 0$ for various values of the Dufour number. It is observed that an increase in the Dufour number causes a rise in Nusselt number and Sherwood number. Further, the negative values of Nusselt number and Sherwood number physically represent the fact that the heat flows from the wall surface to the ambient fluid.

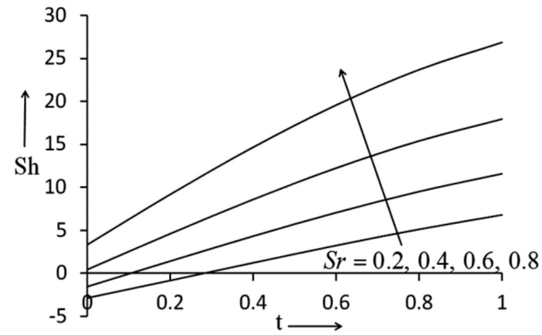

Fig. 16 — Sherwood number for different values of Sr.

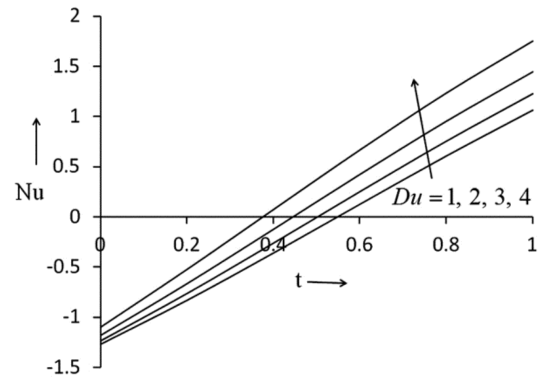

Fig. 17 — Nusselt number for different values of Du.

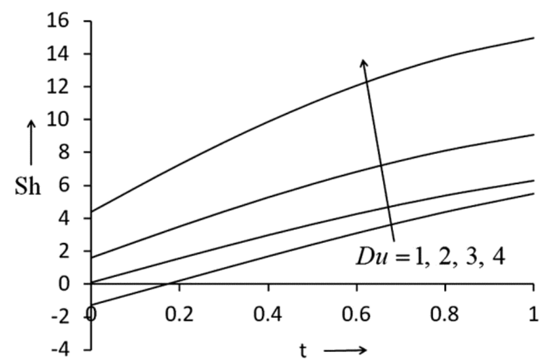

Fig. 18 — Sherwood number for different values of Du.

In absence of Soret and Dufour effects with $K_1 = 0$ and $\alpha = 0$, the results obtained here are compared with those of Ibrahim *et al.*[13] and are found to be in good agreement.

Conclusions

The Soret and Dufour effects on unsteady magneto-hydrodynamic oscillatory flow of radiative, visco-elastic fluid characterized by Walters liquid model B' through an inclined channel filled with saturated porous medium with non-uniform wall temperature in presence of first-order chemical reaction is considered. Employing perturbation technique, we have solved the governing equations. The above study leads to the following conclusions:

1. The fluid velocity increases with the increasing values of chemical reaction parameter whereas decreases with the increase in porous medium shape factor parameter, Soret number, Dufour number, visco-elastic parameter, inclination angle of the channel wall,and radiation parameter.
2. The fluid temperature increases with the increasing values of Soret number.
3. The fluid concentration decreases with the increasing values of Dufour number.
4. Skin friction increases with the increasing values of visco-elastic parameter, Soret number and chemical reaction parameter, but it decrease with an increasing value of Dufour number**.**
5. Nusselt number and Sherwood number increases with the increasing values of Soret number and Dufour number.

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