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Analysis of the interaction between magnet and superconducting ring in the mixed state

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The interaction between a small permanent magnet and a superconducting ring in the mixed state has been analyzed using a phenomenological model of the supercurrents known as the $J_s + J_v$ model. The trapped magnetic field has been described by a constant surface current density J_s circulating the inner and outer surfaces of the superconducting ring as well as a volume current density J_v flowing within the entire volume of the ring. Under field-cooled (FC) condition, the results showed that the interaction energy between the magnet and the superconducting ring has the minimum value when the magnet is located at the center of the ring indicating the existence of a stable equilibrium position. The axial force versus the distance of the magnet from the center of the ring showed that the magnitude of the force is zero at the center while maximum when the magnet is just at the borders of the ring. The results are found to be in agreement with the experimental and theoretical results reported previously in literature.

Keywords: Superconducting ring, Supercurrent, Field-cooled condition, Mixed state, Vortex

1 Introduction

The experimental and theoretical studies have been conducted to clarify the potential of the magnet superconducting ring system as a practical low loss non-contact magnetic bearing due to the generated repulsive or attractive forces between a magnet and a superconductor whether the superconductor is in the Meissner state or in the mixed state[1-10]. Ma *et al*[11]. measured the force on a magnet as it moves along the symmetry axis of a superconducting ring (hollow cylinder) and found that the force is zero at the center of the ring while maximum at the borders. These results made the magnet superconducting ring system a good candidate for a low loss magnetic bearing and induced other researchers to subject this system to further extensive theoretical investigations. Alzoubi *et al*[12]. studied the same system making use of the dipole-dipole interactions model to calculate the interaction magnetostatic energy and hence. the force of interaction under the assumption that the superconductor is in the Meissner state in which total expulsion of magnetic field is assumed.

Diaz *et al*[13]. provided a theoretical study based on Maxwell-London model to calculate the force and torque exerted by a small magnet on a superconducting ring in the Meissner state for different positions and orientations. Levitation forces acting on a magnet placed over two dimensional superconducting ring were calculated by Yang[14] using the Meissner effect in superconductors.

In the present paper, the problem of a magnet-superconducting ring interaction has been analyzed when the superconductor is in its mixed state. In this state, the magnetic field penetrates the superconductor in the form of fluxoids with vortex supercurrents circulating around. A theoretical model known as the $J_s + J_v$ model was proposed by Chen *et al*[15]. and employed to describe the magnetic field trapped by a superconducting cylinder. It introduces a constant surface current J_s circulating the round surface of the cylinder and a constant volume current density J_v flowing within the entire volume of the cylinder. In the case of the superconducting ring, we have two surface currents circulating the inner and outer surfaces with opposite directions and a volume current flows in between. The model was used to calculate the interaction energy as well as the force between the magnet and the superconducting ring. The results were compatible with other results obtained using different theoretical models.

2 The Model

A schematic diagram for our proposed magnet-superconducting ring system is shown in Fig. 1 (a). A small magnet treated as a magnetic dipole of magnetic moment $\vec{m}$ aligned parallel to the positive z axis is

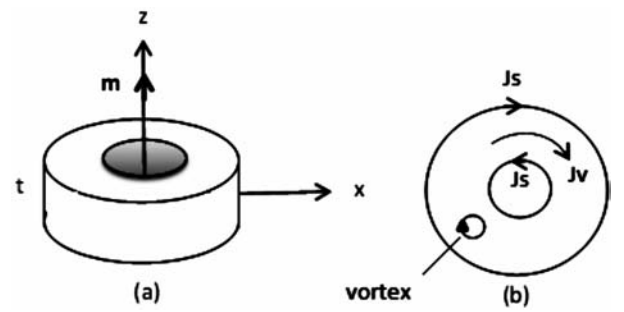

Fig. 1 — (a) Schematic diagram for the magnet-superconducting ring system, (b) Schematic diagram showing vortices and circulating surface and volume supercurrents

constrained to move along the symmetry axis of a superconducting ring of inner radius *a*, outer radius *b* and height *t*. The physical parameters of our proposed system are chosen as follows: a = 0.01 m, b = 0.02 m, t = 0.04 m, and m = 1 Am^2.

The field cooled (FC) conditions are assumed in which the superconducting ring is cooled down in the existence of the magnet at its center so that the field penetrates the superconductor in the form of flux lines surrounded by supercurrents (vortices) as clear in Fig. 1(b). These vortices will result in the formation of two surface currents circulating the inner and outer surfaces of the ring in opposite directions. Due to the decrease in the concentration of vortices towards the outer edge of the ring, the incomplete cancellation of the circulating supercurrents will result in the formation of a volume current density flowing parallel to the outer surface current. Three currents will generate a net magnetic field that interacts with the magnetic dipole. In our calculations, the values of the supercurrents[11,15] are chosen as follows: J_v = 5×10^7 A/m^2 and $J_s = 4\times10^4$A/m.

To calculate the generated magnetic field along the symmetry axis of the ring at any arbitrary position z, the vector potential $\vec{A}$ due to the surface currents has been firstly determined by considering a single current loops of radii a and b located in the xy-plane each carrying a current $J_s dz'$ circulating in opposite directions and then integrating over the full height of superconducting ring for which z' varies from $-t/2$ to $+t/2$. The only non-vanishing component of the vector potential is the azimuthal component[16] A_φ. According to the relation between the magnetic field and the vector potential $\vec{B} = \vec{\nabla}\times\vec{A}$, the z-component of the magnetic field will interact with the magnetic dipole, is given in cylindrical coordinates by:

$$B_z = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\varphi) \quad \ldots(1)$$

According to the schematic diagram shown in Fig. 1, the magnetic field generated by the inner surface will be in the positive z-direction while the field of the outer surface will be in the negative z-direction. It is known that these surface currents allow incorporating the demagnetization effects[17]. Following the same mathematical steps of Ref. 5, the net magnetic field B_{zs} due to the surface currents will be:

$$B_{zs} = \frac{\mu_o J_s}{2}\left(\begin{array}{l}\dfrac{z+t/2}{\sqrt{a^2+(z+t/2)^2}} - \dfrac{z-t/2}{\sqrt{a^2+(z-t/2)^2}} \\ -\dfrac{z+t/2}{\sqrt{b^2+(z+t/2)^2}} + \dfrac{z-t/2}{\sqrt{b^2+(z-t/2)^2}}\end{array}\right) \quad \ldots(2)$$

where μ_o is the permeability of free space and equals to $4\pi\times10^{-7}$ N/A^2.

Beside the surface currents, there exists a volume current flowing within the entire volume of the ring represented by the constant J_v to calculate its contribution to the vector potential we need extra radial integration over ρ from the inner radius a to outer radius b. It is worth mentioning that considering a constant volume current density J_v without any surface currents is equivalent to the assumptions of the Bean's critical state model. Again, following the same steps given in Ref. (15) for evaluating the z-component of the magnetic field, will result in:

$$B_{zv} = \frac{\mu_o J_v}{2}\left(\begin{array}{l}(z+t/2)\log\left(\dfrac{b+\sqrt{b^2+(z+t/2)^2}}{a+\sqrt{a^2+(z+t/2)^2}}\right) \\ -(z-t/2)\log\left(\dfrac{b+\sqrt{b^2+(z-t/2)^2}}{a+\sqrt{a^2+(z-t/2)^2}}\right)\end{array}\right) \quad \ldots(3)$$

According to Fig. 1, it is clear that the direction of magnetic field due to the volume current is in the negative z-direction. Finally, the net magnetic field along the symmetry axis is given by:

$$B_z = B_{zs} + B_{zv} \quad \ldots(4)$$

The magnetostatic interaction energy between the magnetic dipole and the generated magnetic field can be calculated using:

$$U = -\vec{m} \cdot \vec{B} = -mB_z \qquad \ldots(5)$$

and hence the interaction force is given by:

$$\vec{F} = \vec{\nabla}(\vec{m} \cdot \vec{B}) = m\frac{\partial B_z}{\partial z} \qquad \ldots(6)$$

3 Results and Discussion

The interaction energy as a function of the axial position of the magnetic dipole is plotted in Fig. 2. It is clear that the energy has the minimum value when the magnet is located at the center of the ring indicating that this position represents a stable equilibrium position of the permanent magnet. If the magnet is displaced slightly from this position, it tends to oscillate about this equilibrium position. The problem is that the center of the ring being a stable or an unstable equilibrium position depends on the direction of the magnetic dipole relative to the magnetic field there and also depends originally on the way in which we prepared our experiment first. In the case of zero-field-cooled (ZFC) condition, the superconductor is cooled down with the magnet away enough from the superconductor. Experiments showed that in this case the force on the magnet is repulsive as it moves away from the center and no stability can be attained. However, in the field-cooled (FC) condition the superconductor is cooled down in the vicinity of the permanent magnet allowing flux penetration and formation of vortices. In the case of FC condition with the magnet at the center of the ring, experiments by Ma *et al*. showed that the center of the ring is in a position of stable equilibrium and an attractive force pulls the magnet back to the center as we try to move it away. This property makes the magnet-superconducting ring a good candidate for practical magnetic bearings. Our results are in good agreement with these experimental results.

Moreover, one may obtain different results by cooling down a superconducting ring that is subjected to some external magnetic field and as a result the field will be trapped by the superconductor in the form of flux lines surrounded by supercurrents (vortices). Then, the field can be turned off and the permanent magnet can be brought to the axis of the ring. In this case, the magnet will interact with a magnetic field created by vortices that are not belonging to it.

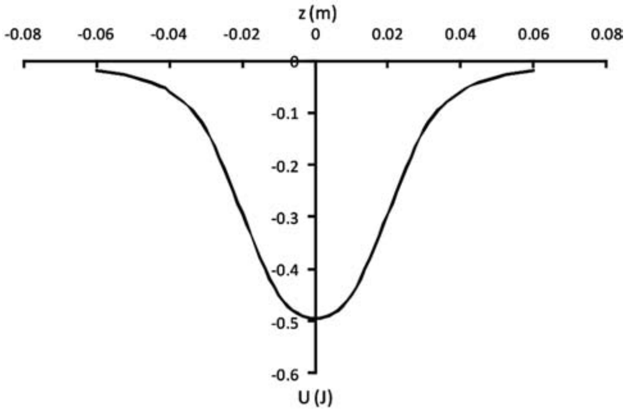

Fig. 2 — Interaction energy between the magnet and the superconducting ring

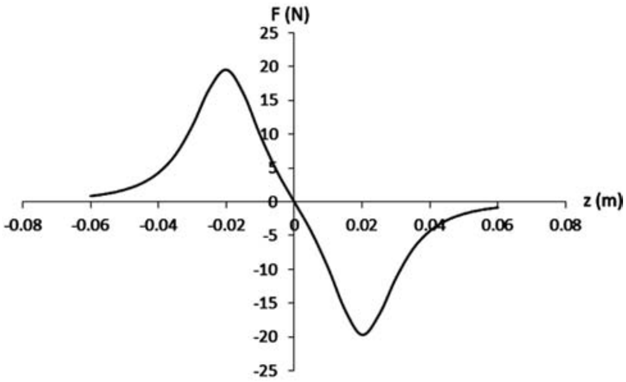

Fig. 3 — Interaction force between the magnet and the superconducting ring

Figure 3 shows the force on the magnet versus the distance from the center of the ring. The magnitude of the force is zero at the center of the ring while maximum at the borders. It is clear that the force is always attractive as we move the magnet away from the center. This result is compatible with results of Ma *et al*[11]. for the FC case.

4 Conclusions

The system composed of a magnet and a superconducting ring in the mixed state was analyzed using $J_s + J_v$ model. Under the field-cooled condition with the magnet initially at the center of the ring to allow the penetration of flux lines within the superconductor, it is found that this axial position is in a stable equilibrium position of the magnet. This indicates a promising future for magnet-superconducting ring system in constructing noncontacting magnetic bearings. Our results are in good agreement with experimental results of Ma *et al*[11]. as well as theoretical results obtained using different mathematical models.

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