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Dual solutions of three dimensional viscous flow and heat transfer due to a shrinking sheet with slip and suction

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In the present paper, an analysis is presented to study the effect of slip and suction on the dual nature of the solution of the three dimensional boundary layer flow of an incompressible fluid and heat transfer towards a porous axisymmetric shrinking sheet. The governing equations are transformed into self-similar non-linear ordinary differential equations by using suitable similarity transformations and then the transformed equations are solved numerically using the shooting technique with Runge-Kutta forth order method. The numerical results of velocity and temperature profiles as well as skin-friction coefficient and Nusselt number are obtained and displayed graphically with different pertinent parameters to show interesting aspects of the solution. The investigation explores the conditions of the non-existence, the existence and duality of the similarity solutions which depend on the suction parameter S as well as slip parameter λ. The dual solutions exist in a certain domain of suction parameter S and due to an increment in the slip parameter λ, the domain of S where the similarity solution exists, also increases. Also, for increasing values of S and λ, the thickness of the both momentum and thermal boundary layer is decreasing for the first solution while for second solution it is increasing.

Keywords: Axisymmetric shrinking sheet, Heat transfer, Suction, Slip, Dual solutions

1 Introduction

The viscous flow over a stretching sheet is significant due to its enamors applications in engineering processes such as glass-fiber production, wire drawing, paper production, extraction of polymer sheets and many others[1,2]. Crane[3] was the first who investigated the steady viscous flow of an incompressible fluid over a linearly stretching sheet and gave an exact similarity solution in closed analytical form. Crane's work was extended by many researchers such as Gupta and Gupta[4], Pavlov[5], Chakrabarti and Gupta[6], Carragher and Crane[7], Mukhopadhyay and Anderson[8] and Jat *et al*[9]. under various physical conditions. Wang[10] studied the steady three dimensional viscous flow over a plane surface which is stretched in its own plane in two lateral directions at different rates. Further, Ariel[11] gave generalized three-dimensional flow due to a stretching sheet.

Recently, the development of an unusual flow due to a shrinking sheet has attracted considerable interest because the flow induced by the shrinking sheet shows quite distinct physical phenomena from the stretching sheet case. A steady boundary layer flow over a shrinking sheet is not possible as the vorticity generated in this case is not confined within the boundary layer region. To maintain the boundary layer structure, the flow needs a certain amount of external suction at the porous sheet. Miklavcic and Wang[12] investigated both two-dimensional and axisymmetric viscous flow induced by a shrinking sheet in the presence of uniform suction and established the criteria of existence, non-existence, uniqueness and duality of the similarity solutions for both cases. This problem was extended to power-law surface velocity by Fang[13] and then under various physical aspects, the most significant work on shrinking sheet was done by many researchers such as Hayat *et al*[14]., Muhaimin *et al*[15]., Wang[16], Turkyilmazoglu[17], Bhattacharyya and Layek[18] and Jat and Rajotia[19] etc. Ali *et al*[20]. investigated MHD viscous flow and heat transfer with prescribed surface heat flux and gave dual solutions for two dimensional flow and unique solution for axisymmetric flow. Bachok *et al*[21]., extended the idea for unsteady three dimensional boundary flow and found dual solutions in a certain range of suction parameter and unsteadiness parameter. Further, comparison between two dimensional and axisymmetric flow over a shrinking sheet with variable wall temperature was considered by Jat and Rajotia[22]. Recently, Turkyilmazoglu[23] analyzed the two dimensional fluid flow and heat transfer of a micro polar fluid over porous shrinking sheet and obtained dual solutions in closed form. MHD fluid flow and heat transfer due to a shrinking rotating disk were also considered by Turkyilmazoglu[24]. He used a spectral numerical integration scheme to investigate the effects of rotation parameter on the problem.

The assumption that the flow field obeys the conventional no-slip condition at the sheet, i.e., the velocity component parallel to the sheet becomes equal to the sheet velocity at the sheet. In certain situations, however, the assumption of no-slip is not applicable and should be replaced by the partial slip boundary condition. Also, the fluids that exhibit the boundary slip have important technological applications such as in the polishing of the artificial heart valves and internal cavities. Wang[25] has considered the influence of partial slip on the flow of a viscous fluid over a stretching sheet and obtained the solution numerically. Andersson[26] discussed the partial slip effects on the flow characteristics of a viscous fluid by finding an exact analytical solution for the above Wang[25] problem. Ariel[27] has studied the steady, laminar, axisymmetric flow of a Newtonian fluid due to a stretching sheet with a partial slip boundary condition. Further, Wang[28] has revived an interest in the viscous flow due to a stretching sheet with slip and suction. In the present study, he has considered both the two-dimensional and the axisymmetric cases. Sahoo[29] considered the above problem of partial slip with a non-Newtonian second grade fluid past a radially stretching sheet. Recently, Bhattacharyya *et al*[30,31]. gave slip effects on steady and unsteady stagnation-point flow towards a shrinking sheet, respectively. The physically pure exponential type solutions for MHD two dimensional flow of non-Newtonian fluid over a shrinking surface were obtained by Turkyilmazoglu[32] to investigate whether the solutions were unique or multiple under the influence of slip condition. Consequently, Turkyilmazoglu[33] gave the exact analytical solutions for heat and mass transfer of MHD slip 2D flow in nanofluids and then the second order slip flow was also taken by Turkyilmazoglu[34]. Recently, Turkyilmazoglu[35] extended the problem of Miklavcic and Wang[12] in the presence of the velocity slip in the flow field and heat jump in the temperature field, respectively and gave algebraic solutions. He obtained multiple solutions in the case of shrinking sheet for certain values of slip and suction parameter while unique solution for stretching sheet.

Miklavcic and Wang[12] reported that for axisymmetric flow over the shrinking sheet the similarity solution (unique and dual) exists for *S*=1.31175869 and there is no solution for *S*=1.31175869. On the other hand, Turkyilmazoglu[35] extended above problem in the presence of slip condition and obtained that the dual solutions exists for $\lambda \in (-0.27217, 0)$. In the present paper, we have taken a three dimensional boundary layer viscous flow of an incompressible fluid and heat transfer due to a porous axisymmetric shrinking sheet with slip and suction and get numerical results which give comparatively a favourable agreement with the results of Miklavcic and Wang[15] and Turkyilmazoglu[35]. The main result emerging from the study is that the dual solutions exist not only for negative values of slip parameter (mentioned by Turkyilmazoglu[35]) but also for positive values of slip parameter (Not obtained by Turkyilmazoglu[35]).

2 Formulation of the Problem

Consider a three-dimensional viscous slip flow of an incompressible fluid due to a porous axisymmetric shrinking sheet which coincides with the plane $z=0$ while the flow is confined in the plane $z>0$. The x and y axes are taken along the length and breadth of the sheet and z-axis is perpendicular to the sheet, respectively. A uniform suction W is applied normal to sheet to contain the vorticity (Fig. 1).

Under the usual boundary layer approximations, the basic governing boundary layer equations (Miklavcic and Wang[12]) are :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \dots(1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} \qquad \dots(2)$$

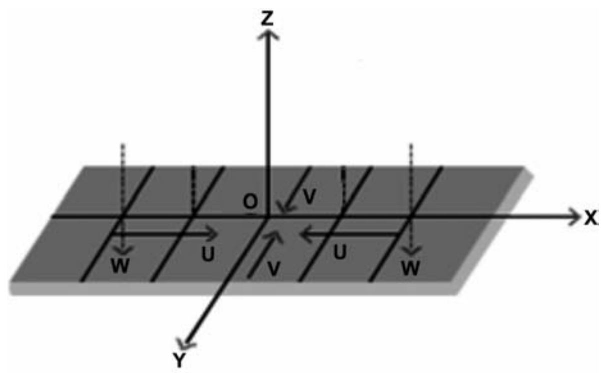

Fig. 1 — Systematic diagram of physical model

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\frac{\partial^2 v}{\partial z^2} \quad \ldots(3)$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\frac{\partial^2 w}{\partial z^2} \quad \ldots(4)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{\kappa}{\rho c_p}\left(\frac{\partial^2 T}{\partial z^2}\right) \quad \ldots(5)$$

where (u,v,w) be the velocity components along the (x,y,z) directions, respectively, p the pressure, ρ the density of the fluid, μ the dynamic viscosity, $\nu=\frac{\mu}{\rho}$ is the kinematic viscosity, κ the thermal conductivity and c_p is the specific heat at constant pressure.The slip boundary conditions applicable to the present flow are:

$$\begin{aligned}&z=0:\ u=-U+\lambda_0\left(\frac{\partial u}{\partial z}\right),\ v=-V+\lambda_0\left(\frac{\partial v}{\partial z}\right),\\&w=-W,\ T=T_w\\&z\to\infty:\ u\to 0,\ v\to 0,\ T\to T_\infty\end{aligned} \quad \ldots(6)$$

where $(a>0)$ is the shrinking constant, $U=ax$ and $V=ay$ are the shrinking velocities, $(W>0)$ is the suction velocity, λ_0 the velocity slip coefficient, T_w the sheet temperature and T_∞ is the free stream temperature.

3 Analysis

Introducing the following similarity transformations:

$$\begin{aligned}&u=axf'(\eta),\ v=ayf'(\eta),\\&w=-2\sqrt{a\nu}\,f(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\\&\eta=\sqrt{\frac{a}{\nu}}z,\end{aligned} \quad \ldots(7)$$

Eq. (1) is identically satisfied by the similarity transformations while Eq. (4) becomes:

$$\frac{p}{\rho}=\nu\frac{\partial w}{\partial z}-\frac{w^2}{2}+\text{constant} \quad \ldots(8)$$

Eqs (2) and (3) reduce to the same Eq. (9), while Eq. (5) reduced Eq. (10) as:

$$f'''+2ff''-f'^2=0 \quad \ldots(9)$$

$$\theta''+2\,\mathrm{Pr}\,f\theta'=0 \quad \ldots(10)$$

Corresponding boundary conditions are:

$$\begin{aligned}&\eta=0:\ f=S,\ f'=-1+\lambda f'',\ \theta=1,\\&\eta\to\infty:\ f'\to 0,\ \theta\to 0,\end{aligned} \quad \ldots(11)$$

where prime (′) denotes differentiation with respect to similarity variable η, $S=\frac{W}{2\sqrt{a\nu}}$ is the Suction parameter, $\mathrm{Pr}=\frac{\mu c_p}{\kappa}$ is the Prandtl number and $\lambda=\lambda_0\sqrt{\frac{a}{\nu}}$ is the Slip parameter.

The physical quantity of interest is the local skin friction coefficient C_f on the surface along the x and y directions, which are denoted by C_{fx} and C_{fy}, respectively and the local Nusselt number Nu i.e. surface heat transfer is given by:

$$C_{fx}=\frac{\tau_{wx}}{\rho U^2/2}=\frac{\mu\left(\frac{\partial u}{\partial z}\right)_{z=0}}{\rho U^2/2}=\frac{2}{\sqrt{\mathrm{Re}_x}}f''(0),$$

$$C_{fy}=\frac{\tau_{wy}}{\rho V^2/2}=\frac{\mu\left(\frac{\partial v}{\partial z}\right)_{z=0}}{\rho V^2/2}=\frac{2}{\sqrt{\mathrm{Re}_y}}f''(0)$$

and

$$Nu=-\frac{x\left(\frac{\partial T}{\partial z}\right)_{z=0}}{(T_w-T_\infty)}=-\sqrt{\mathrm{Re}_x}\,\theta'(0) \quad \ldots(12)$$

where τ_{wx} and τ_{wy} are the wall shear stresses along the x and y-directions, respectively and $\mathrm{Re}_x=\frac{Ux}{\nu}$ and $\mathrm{Re}_y=\frac{Vy}{\nu}$ are the local Reynolds numbers.

4 Method of Solution

The set of nonlinear ordinary differential Eqs (9) and (10) with boundary conditions given in Eq. (11) are solved numerically by converting them to an initial value problem (IVP). We set:

$$f' = p,\quad p' = q,\quad q' = -2fq + p^2 \qquad \ldots(13)$$

$$\theta' = r,\quad r' = -2\,\mathrm{Pr}\, rf \qquad \ldots(14)$$

With boundary conditions:

$$f(0) = S,\quad p(0) = -1 + \lambda q(0),\quad \theta(0) = 1,$$
$$f'(\eta_\infty) = 0,\quad \theta(\eta_\infty) = 0 \qquad \ldots(15)$$

In order to integrate Eqs (13) and (14) as an initial value problem, one requires a value for $q(0)$ i.e. $f''(0)$ and $r(0)$ i.e. $\theta'(0)$ but no such values are given in the boundary conditions given in Eq. (15). The suitable guess for $f''(0)$ and $\theta'(0)$ iis chosen by the shooting technique and then the fourth order Runge-Kutta method is applied to obtain the solution. Then we compare the calculated values for f' and θ at η_∞=10 (say) with the given boundary conditions $f'(10)$=0 and $\theta(10)$=0 and adjust the estimated values of $f''(0)$ and $\theta'(0)$ using the Secant method to give a better approximation for the solution. The step-size is taken as h = 0.001. The above procedure is repeated until we get the converged results within a tolerance limit of 10^{-7}. All the computations are done in the Matlab software which uses a symbolic and computational language.

5 Results and Discussion

Before analyzing the numerical computation for obtaining the condition under which the steady flow is possible over the axisymmetric shrinking sheet in presence of boundary slip condition for various values of the parameters involved such as suction parameter S, slip parameter λ and Prandtl number Pr, we are discussing about previously published analytical results of Miklavcic and Wang[12] and Turkyilmazoglu[35]. First of all Miklavcic and Wang[12] gave algebraic decaying exact solution as $f(\eta) = \dfrac{S^2}{\eta + S}$, for viscous axisymmetric flow due to a shrinking sheet in the presence of uniform suction with no slip boundary condition i.e. $\lambda = 0$ and noticed that for $S<S_0$=1.31175869, there exist no solution and for $S > S_1 = 1.4238297 \approx \sqrt{2}$, there exists only one solution, while dual solutions exist for $S_0 < S < S_1$. Recently, Turkyilmazoglu[35] extended the above problem in the presence of slip boundary condition i.e. $\lambda \neq 0$. He also gave algebraic decaying solutions as $f(\eta) = \dfrac{2S}{2 + \eta S}$, and observed that the dual solutions were simply given only in the range of slip parameter $\lambda \in (-0.27217, 0)$.

Now, for a comparative analysis with Miklavcic and Wang[12] and Turkyilmazoglu[35], we have plotted the Fig. 2(a) of the skin friction coefficient against S for several values of λ. It is noticed from Fig. 2(a) that for no slip boundary condition, there is no similarity solution for S<1.312≈S_0 and consequently, there exist only the dual solutions for all values of $S \geq 1.312$. Hence, in the absence of slip boundary condition, the present numerical investigation explores the conditions of either no solution or dual solutions. Further, for some negative values of λ=–0.1, –0.2, –0.27 (which are taken from the range of $\lambda \in (-0.27217, 0)$ mentioned by Turkyilmazoglu[35] who obtained dual solutions there), there also exist dual solutions. Finally, these results give us a favourable

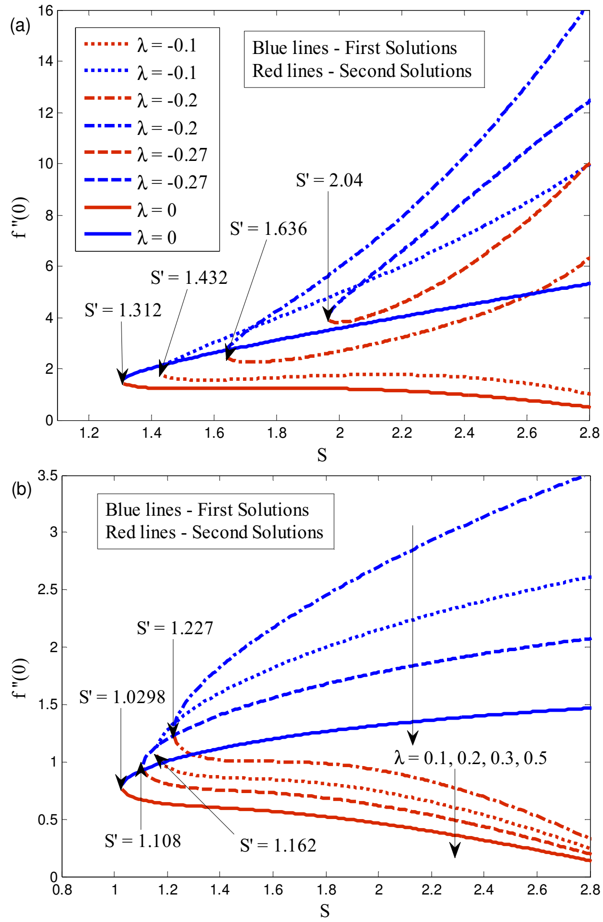

Fig. 2 — Skin friction coefficient against S with several values of slip parameter λ. (S' denotes critical values of S)

agreement with the solutions of Miklavcic and Wang[15] and Turkyilmazoglu[35].

Consequently, a new result is emerged in the further analysis. It is noticed that for positive values of slip parameter i.e. $\lambda>0$, there also exist dual solutions which were not mentioned by Turkyilmazoglu[35]. For this new result, we have plotted Fig. 2(b) of the skin friction coefficient against S for several positive values of λ. It is observed that for increasing values of S, the $f''(0)$ increases for the first solution (upper branch) while for the second solution (lower branch) it decreases with S. Further, for increasing values of λ, $f''(0)$ decreases for both solutions. The range of S for the existence and the non-existence of the dual solutions for different values of λ is given in Table 1. It can be easily seen from Fig. 2 and Table 1 that with an increament in the values of λ, the range of S where the similarity solutions exist, also increases. Here, the enhancement in the existing range of similarity solution due to slip is physically realistic. For increasing slip at the boundary, the generation of vorticity at the sheet is slightly reduced and therefore, to contain the vorticity within the boundary layer the required adequate suction can be taken smaller. Hence, the similarity solution is possible for smaller values of S as well in the presence of the slip.

Dual velocity profile $f'(0)$ for various positive values of λ is shown in Fig. 3 and it is found that with increasing values of λ, the velocity profile increases in the first solution case while showing opposite behaviour in the case of second solution i.e. decreases. Figure 4 shows the effect of on the velocity profile. It is found that the dimensionless velocity profile increases with the increasing value of S for the first solution while for the second solution it decreases with increasing of S.

Table 1 — Range of S for the existence and the non-existence of the dual solutions with several values of λ

Slip parameter λ	Dual solutions (Existence)	No similarity solution
−0.272	$S\geq 2.041$	$S<2.041$
−0.2	$S\geq 1.636$	$S<1.636$
−0.1	$S\geq 1.432$	$S<1.432$
0	$S\geq 1.312$	$S<1.312$
0.1	$S\geq 1.227$	$S<1.227$
0.2	$S\geq 1.162$	$S<1.162$
0.3	$S\geq 1.108$	$S<1.108$
0.5	$S\geq 1.0298$	$S<1.0298$
0.75	$S\geq 0.958$	$S<0.958$
1.0	$S\geq 0.904$	$S<0.904$

Dual temperature profile for various values of λ and S are shown in Figs 5 and 6, respectively. Figure 5 shows that with increase of λ, the temperature profile and thermal boundary layer thickness decrease for the first solution while increase for the second solution. Figure 6 shows that temperature at a point decreases with the increase of S for the first solution while for the second solution it increases. Furthermore, it is very important to note that the temperature profiles with λ and S likely just give reverse phenomena in comparison with those of the velocity profiles.

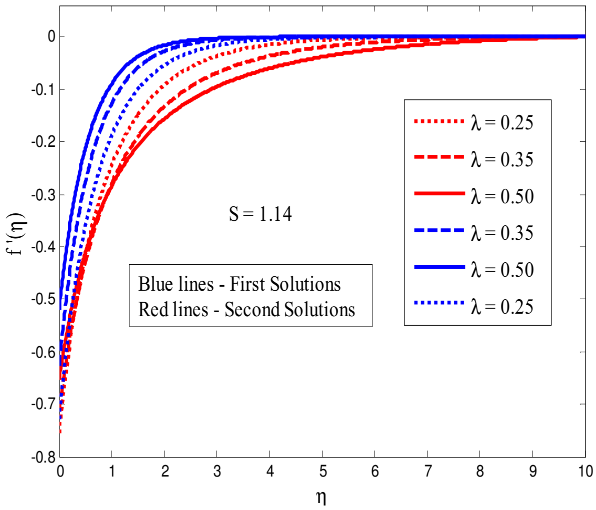

Fig. 3 — Dual velocity profile for various values of λ

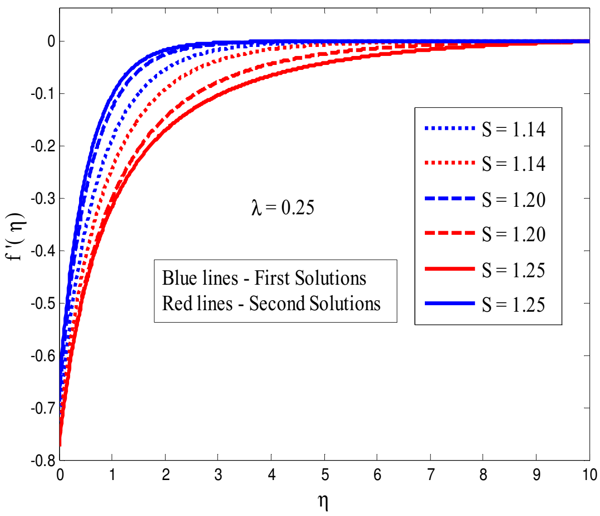

Fig. 4 — Dual velocity profile for various values of S

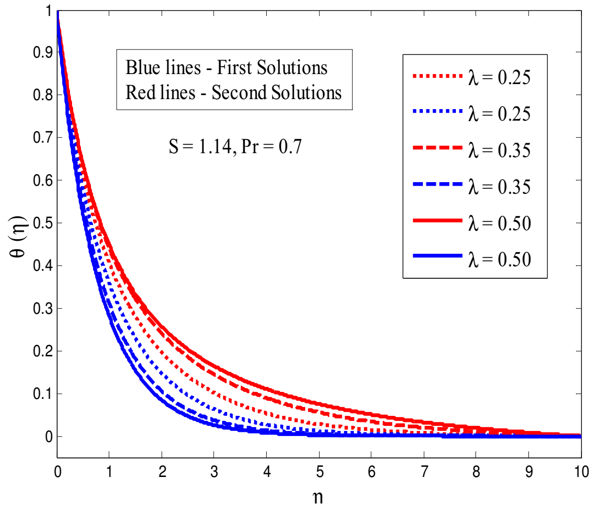

Fig. 5 — Dual temperature profile for various values of λ

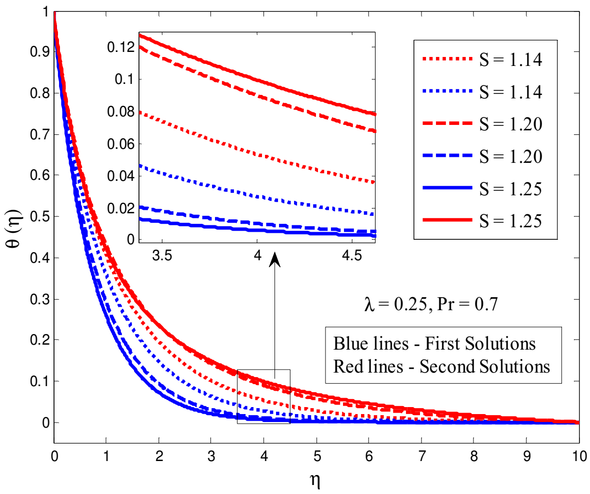

Fig. 6 — Dual temperature profile for various values of S

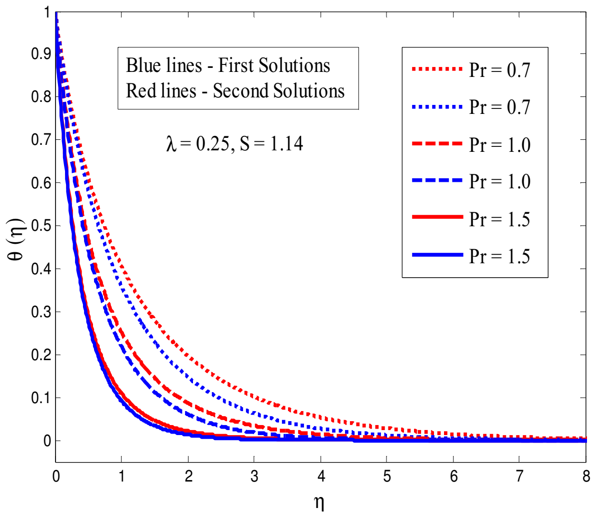

Fig. 7 — Dual temperature profile for various values of Pr

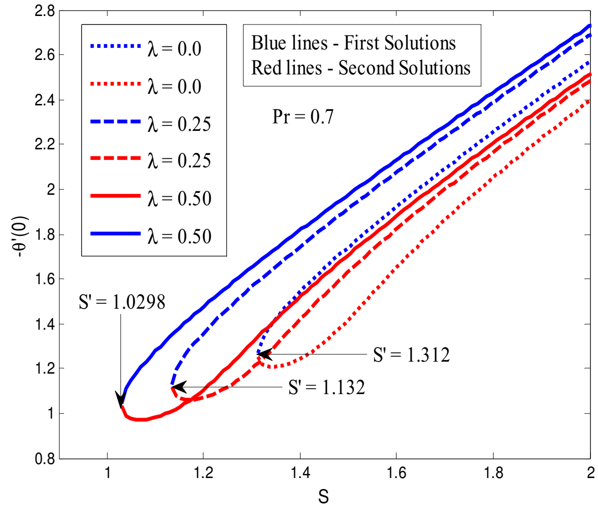

Fig.8 — Dual temperature gradient at the sheet $-\theta'(0)$ with S for various values of λ

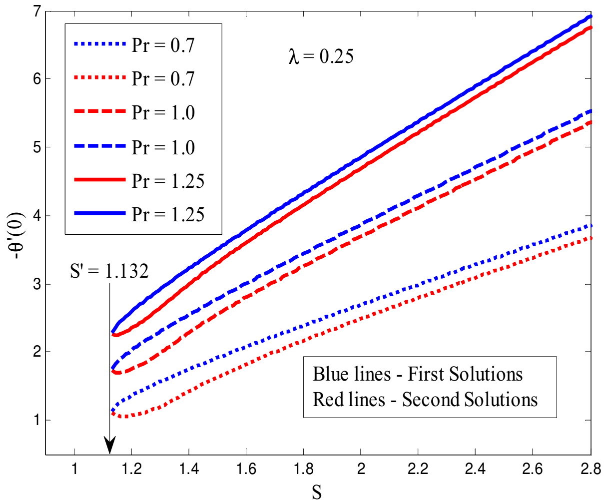

Fig. 9 — Dual temperature gradient at the sheet $-\theta'(0)$ with S for various values of Pr

The dual temperature profile for various values of Pr is shown in Fig. 7. It is observed that in the presence of the slip, the thermal boundary layer thickness of both first and second solutions reduces significantly due to increase of Pr. Since the Prandtl number is inversely proportional to the thermal conductivity, thus the fluids with the lower Pr have higher thermal conductivities and consequently, the heat diffusion is faster in this case. On the other hand, for higher Pr fluids the heat diffusion slows down.

Analysis of the dual nature of the temperature gradient $-\theta'(0)$ at the sheet (i.e. rate of heat transfer from the surface to the fluid) against S for various values of λ and Pr are shown in Figs 8 and 9, respectively. It is observed that in the both cases of the first and second solutions, the rate of heat transfer at the sheet i.e. $-\theta'(0)$ increases with increasing of λ and Pr. Consequently, from both these figures it is found that the rate of heat transfer at the sheet increases with increasing values of *S* in both cases of first and second solutions.

6 Conclusions

The objective of this paper is to analyze the effect of the slip parameter as well as suction parameter on the dual character of the similarity solution of three dimensional boundary layer flow of an incompressible fluid and heat transfer over a permeable axisymmetric shrinking sheet. By using suitable similarity transformations, the governing equations are transformed into non-linear ordinary differential equations and then solved numerically using the shooting technique with Runge-Kutta forth order method. The study reveals that with increasing values of slip parameter, the range of the suction parameter where the similarity solutions exist, also increases. Also, the present investigation explores that the dual solutions not only exist for negative values of slip parameter λ (mentioned in the previous literature) but also for positive values of λ. Moreover, for increasing values of *S* and λ, the thickness of the both momentum and thermal boundary layer is decreasing for the first solution while for second solution it is increasing. The skin friction coefficient decreases with increasing values of slip parameter for both solutions. Whereas for increasing values of suction parameter it increases for first solution and decreases for the second solution. The rate of heat transfer at the sheet increases for both the solutions with increasing values of slip parameter, Prandtl number and suction parameter.

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