$$(\omega^2 - c_1^2)(\omega^2 - c_2^2)(\varpi - l_{\omega_{cb}})^2 = \frac{\omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right)\omega^2(\omega^2-\omega_{L+}^2)\ J_l^2(p_m r_{b0})}{\alpha^1\ p_m^2 J_{l+1}^2(p_m b_1)} \quad \ldots (25)$$

where, $c_1^2 = \omega_{L+}^2 + k^2\alpha^L R c_H^2$ … (26)

and $c_2^2 = \frac{\omega_{L+}^2\alpha^L k_z^2\ R\ c_H^2}{\omega_{L+}^2+k^2\alpha^L R c_H^2}$ … (27)

$R = \frac{M^H}{m^L}$ and $c_H^2 = \frac{T^H}{m^H}$

Here, $\omega \approx c_1$ corresponds to K^+ ion mode and $\omega \approx k_z v_{0b} + l_{\omega_{cb}}$ corresponding to the beam mode. But, we are looking for solutions, when $\omega \approx k_z v_{0b} + l_{\omega_{cb}}$. In this case the two factors on the left-hand side of Eq. (25) are simultaneously zero in the absence of beam. In the presence of beam, we can expand ω as $\omega \approx c_1 + \delta_1 \simeq k_z v_{0b} + l_{\omega_{cb}} + \delta_1$, where δ_1 is the small frequency mismatch due to finite right-hand side of Eq. (25). Then Eq. (25) gives the growth rate of unstable K^+ ion mode as

$$\Gamma = Im\ \delta_1 = \left[\frac{c_1}{2}\frac{(c_1^2-\omega_{L+}^2)\ \omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right) J_l^2(p_m r_{b0})c_L^2}{\omega_{pL}^2\ (\omega^2-c_2^2)\ J_{l+1}^2(p_m b_1)}\right]^{1/3} \frac{\sqrt{3}}{2} \quad \ldots (28)$$

The real frequency of unstable mode in term of beam energy is given by

$$\omega_r = k_z\left(\frac{2eV_b}{m^b}\right)^{1/2} - \frac{1}{2}\left[\frac{c_1}{2}\frac{(c_1^2-\omega_{L+}^2)\ \omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right) J_l^2(p_m r_{b0})c_L^2}{\omega_{pL}^2\ (\omega^2-c_2^2)J_{l+1}^2(p_m b_1)}\right] \ldots (29)$$

where eV_b is the beam energy. However, in the absence of K^+ ions and presence of heavy Ba^+ ions Eq. (22) can be rewrittenas

$$1 - \frac{\omega_{pH}^2\alpha^H k_z^2}{\alpha^1\ \omega^2 p_m^2} - \frac{\alpha^H\omega_{pH}^2}{\alpha^1\ (\omega^2-\omega_{H+}^2)} = \frac{\omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right) J_l^2(p_m r_{b0})}{\alpha^1\ p_m^2\ (\varpi-l_{\omega_{cb}})^2 J_{l+1}^2(p_m b_1)} \quad \ldots (30)$$

Multiplying both sides by $\omega^2(\omega^2 - \omega_{H+}^2)$ and after rearranging the terms, we get

$$\omega^4 - \omega^2\left(\omega_{H+}^2 + \frac{\omega_{pH}^2\,\alpha^H}{\alpha^1} + \frac{\alpha^H\omega_{pH}^2\,k_z^2}{\alpha^1\ p_m^2}\right) + \frac{\alpha^H\omega_{pH}^2\,k_z^2\,\omega_{H+}^2}{\alpha^1\ p_m^2} = \frac{\omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right)\omega^2(\omega^2-\omega_{H+}^2)\ J_l^2(p_m r_{b0})}{\alpha^1\ p_m^2\ (\varpi-l_{\omega_{cb}})^2 J_{l+1}^2(p_m b_1)} \quad \ldots (31)$$

Equation (31) can be rewritten as

$$(\omega^2 - d_1^2)(\omega^2 - d_2^2)(\varpi - l_{\omega_{cb}})^2 = \frac{\omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right)\omega^2(\omega^2-\omega_{H+}^2)\ J_l^2(p_m r_{b0})}{\alpha^1\ p_m^2 J_{l+1}^2(p_m b_1)} \quad \ldots (32)$$

where, $d_1^2 = \omega_{H+}^2 + \frac{k^2\alpha^H c_L^2}{R}$ … (33)

$$d_2^2 = \frac{\alpha^H\ k_z^2\omega_{H+}^2\ c_L^2}{R\ \omega_{H+}^2 + k^2\alpha^H c_L^2} \quad \ldots (34)$$

Here, $\omega \approx d_1$ corresponds to the Ba^+ ion mode and $\omega \approx k_z v_{0b} + l_{\omega_{cb}}$ corresponding to the beam mode. Eq. (32) gives the growth rate of the unstable mode as

$$\Gamma = Im\ \delta_1 = \left[\frac{d_1}{2}\frac{(d_1^2-\omega_{H+}^2)\ \omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right) J_l^2(p_m r_{b0})c_H^2}{\omega_{pH}^2\ (\omega^2-d_2^2)J_{l+1}^2(p_m b_1)}\right]^{1/3} \frac{\sqrt{3}}{2} \ldots (35)$$

The real frequency of unstable mode in terms of beam energy is given by

$$\omega_r = k_z\left(\frac{2eV_b}{m^b}\right)^{1/2} - \frac{1}{2}\left[\frac{d_1}{2}\frac{(d_1^2-\omega_{H+}^2)\ \omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right) J_l^2(p_m r_{b0})c_H^2}{\omega_{pH}^2\ (\omega^2-d_2^2)J_{l+1}^2(p_m b_1)}\right]$$

3 Results

In the numerical calculations, we have used the following typical plasma parameters of the existing experimental paper of Rynn[10] for EIC waves: electron plasma density $n^{0e} = n^{0p} = 10^{10}$ cm^{-3}, temperature of electron, K^+ and heavy positive Ba^+ ions as, $T^e \sim T^L \sim T^H$ =0.2eV, R $(m^H/m^L) = 3.5$, radius of plasma column b_l=2 cm, beam radius of potassium beam r_{b0} = 1.5 cm, beam energy E_b = 10eV, guiding magnetic field Bs ~ 3x 10^3 gauss and mode number n = 1, i.e., the first zero of the Bessel function. EIC waves are the longitudinal oscillations of electrons or ions which propagates nearly perpendicular to the magnetic field. We have studied the EIC waves driven to instability by gyrating ion beam. However, the frequency of any ion beam driven EIC instability is lower than any electron beams driven instability. But temperature of plasma influences its growth rates, as hotter is the plasma, more will be the growth rate. Here, due to an increase in the relative concentration of positive ions, the ion-cyclotron wave exhibits two ion modes, a light K^+ mode and a heavy Ba^+ mode. Using Eqs. (28) and (35), we have plotted Figs. 1 & 2 where it can be observed that the growth rate Γ (in sec^{-1}) of both the unstable electrostatic ion-cyclotron modes increases with their respective relative ion concentrations. In Fig.3, using Eqs. (29) & (36), we have plotted real unstable wave frequency ω_r versus beam energy (eV_b) for K^+ and heavy Ba^+ ions and observed that the increase in beam energy

Fig. 5 — Unstable frequency f (Hz) plots of K^+ and Ba^+ion modes as a function of magnetic field for different values of their relative concentrations.

Fig. 6 — Dispersion relation ω_r/ω_{H+} of a EIC heavy Ba^+ion mode as a function of $d = n^{Ba+}k_z/\omega_{H+}$ for n^{Ba+}=2x10^7 cm^{-3} & $\alpha^H = 0.8$, $\alpha^H = 0.5$ keeping all the other parameters same as taken in Fig 1 or 2.

4 Discussion

Hence, a gyrating ion beam propagating through the collisionless magnetized plasma gives energy to the electrostatic ion cyclotron wave through Cerenkov interaction and excite it into two unstable ion modes; light positive $K^+$and heavy positive Ba^+. The growth rate of both the unstable modes in the presence of heavy positive ions increases with the beam density and scales as the one-third power of the beam density. Also, the parallel phase velocity varies linearly with the beam velocity. The unstable frequencies of both the modes are drastically influenced by the magnetic field also, the frequencies increase with the increase in magnetic field. However, this increase in the frequency is more rapid for the light positive ions as compared to the heavy positive ions.

The theoretical understanding of EIC waves in plasma/dusty plasma is very important in order to understand the theoretical aspects of the phenomenon occurring in the universe. On the other side, positive ion plasmas are found in the earth's ionosphere, magnetosphere and solar winds. Even the interstellar nebulae contain a mixture of several positive ion species, *e.g.*, H^+, O^+, He^+, O_2^+ & NO^+etc., while the concentration of these ions increases with height. Our theoretical model is predicting the excitation of electrostatic ion-cyclotron wave modes by the resonant ion beams and the dependence of growth rate of EIC wave instabilities on the relative concentrations of these positive ions. Our work may find various applications in understanding the space plasma, astronomy, astrophysical plasma as well as terrestrial plasma.

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