



Probing the low-lying level structure of ^{94}Zr through β^- decay

K Mandal^{a,b}, A K Mondal^{a,c}, A Chakraborty^{a,d,e*}, E E Peters^e, B P Crider^{d,f}, C Andreoiu^g, P C Bender^h, D S Cross^g, G A Demandⁱ, A B Garnsworthy^h, P E Garrettⁱ, G Hackman^h, B Hadiniaⁱ, S Ketelhut^h, Ajay Kumar^{d,e,j}, K G Leachⁱ, M T McEllistrem^{d,f}, J Pore^g, F M Prados-Estévez^{d,e}, E T Randⁱ, B Singh^k, E R Tardiff^h, Z-M Wang^{g,h}, J L Wood^l & S W Yates^{d,e}

^aDepartment of Physics, Siksha Bhavana, Visva-Bharati, Santiniketan 731 235, India

^bChandidas Mahavidyalaya, Khujutipara, Birbhum 731 215, India

^cBolpur College, Bolpur, Birbhum 731 204, India

^dDepartment of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506 0055, USA

^eDepartment of Chemistry, University of Kentucky, Lexington, Kentucky 40506-0055, USA

^fDepartment of Physics and Astronomy, Mississippi State University, Mississippi State, Mississippi 39762, USA

^gDepartment of Chemistry, Simon Fraser University, Burnaby, British Columbia V5A 1S6, Canada

^hTRIUMF, University of British Columbia, Vancouver, British Columbia V6T 2A3, Canada

ⁱDepartment of Physics, University of Guelph, Guelph, Ontario, N1G 2W1, Canada

^jDepartment of Physics, Banaras Hindu University, Varanasi 221 005, India

^kDepartment of Physics and Astronomy, McMaster University, Hamilton, Ontario L8S 4L8, Canada

^lSchool of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332-0430, USA [†]deceased

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Low-lying states of ^{94}Zr are populated following β^- decay of ^{94}Y , and the emitted γ rays from ^{94}Zr are detected using the 8π spectrometer composed of 20 Compton-suppressed HPGe detectors. High-statistics coincidence data have been used for the placement of very weak decay branches in the level scheme. Combining the results of level lifetimes from a previous experiment and the precisely measured branching ratio values of the weak decay branches from the present experiment, it is possible to extract the $B(E2)$ values for all the possible decay branches from a given level. These values are helpful for proper identification of the collective and non-collective states of ^{94}Zr . The experimental findings have been compared with predictions from shell-model calculations with a limited valence space; however, these calculations are inadequate in reproducing all of the measured spectroscopic quantities.

Keywords: 8π spectrometer, $\gamma\gamma$ coincidence, Branching ratio, $B(E2)$ value, Shell-model calculation

1 Introduction

The Zr isotopes exhibit a large variety of interesting nuclear structural phenomena and provide a challenging testing ground for the various microscopic and macroscopic nuclear models. It has been observed that the Zr isotopes with $N \leq 58$ display features that characterize a spherical vibrator; the typical $E(4^+) / E(2^+)$ values lie around 2.0 for these isotopes. On the other hand, the isotopes with $N > 58$ exhibit deformed-rotor-like behavior with $E(4^+) / E(2^+)$ values typically around 3.3. This change from spherical to deformed shape is also well established from experimentally measured values of the quadrupole deformation (β) parameter. The value of β is found to lie around 0.1 for the isotopes with $N \leq 58$, whereas a β value

around 0.4 has been measured for the heavier isotopes. Thus, the systematic behavior of the level structure of the Zr isotopes points to a spherical ground state for ^{94}Zr . A large $E0$ strength measured¹ for the $0^+_2 \rightarrow 0^+_1$ decay in ^{94}Zr is suggestive of a deformed 0^+_2 state, which possibly has strong mixing with the spherical 0^+_1 ground state. Shape co-existence prevailing in the low-spin, low-excitation regime of ^{94}Zr was well established in our previous work². We are reporting here a part of our findings for the level structure of ^{94}Zr obtained from an experiment following β^- decay of ^{94}Y .

2 Experimental Procedure and Data Analysis

The experiment was carried out at the TRIUMF Isotope Separator and Accelerator (ISAC) radioactive beam facility. The low-lying excited states of ^{94}Zr were populated following β^- decay of radioactive ^{94}Y

*Corresponding author
(E-mail: anagha.chakraborty@visva-bharati.ac.in)

sources, which were produced as fission products by bombarding a ^{238}UCx target with a 500 MeV proton beam. Following mass separation of the fission products, $A = 94$ activities were deposited on a moving tape collector at the centre of the 8π spectrometer. This spectrometer was composed of an array of 20 Compton-suppressed HPGe detectors along with other ancillary detection devices. The details of the experimental set up can be found elsewhere². For off-line analysis of the acquired data, standard γ -ray spectroscopic techniques were used. A representative coincidence spectrum with a gate set on the 919 ($2_1^+ \rightarrow 0_1^+$) keV γ ray of ^{94}Zr is shown in Fig. 1. All the peaks labelled with their energies belong to ^{94}Zr , and newly observed γ rays have been marked with *. This figure illustrates the very high quality of the

coincidence data. The observation of high-energy γ rays indicates the population of levels very close to the Q_{β^-} value (4.918 MeV) of the decay of the ^{94}Y parent nuclide. From the high statistics of the coincidence data, it was possible to make new placements of very weak decay branches (up to the branching limit of about 0.05%) from some of the previously known levels. The relative branching intensities of the transitions from a given level have been extracted with the following procedures.

Wherever possible, the relative intensities of the γ rays from a given level were extracted through the efficiency corrected coincidence counts with a gate taken on the γ ray feeding the level of interest. The branching of a particular de-exciting γ ray was then obtained using the standard method of dividing the

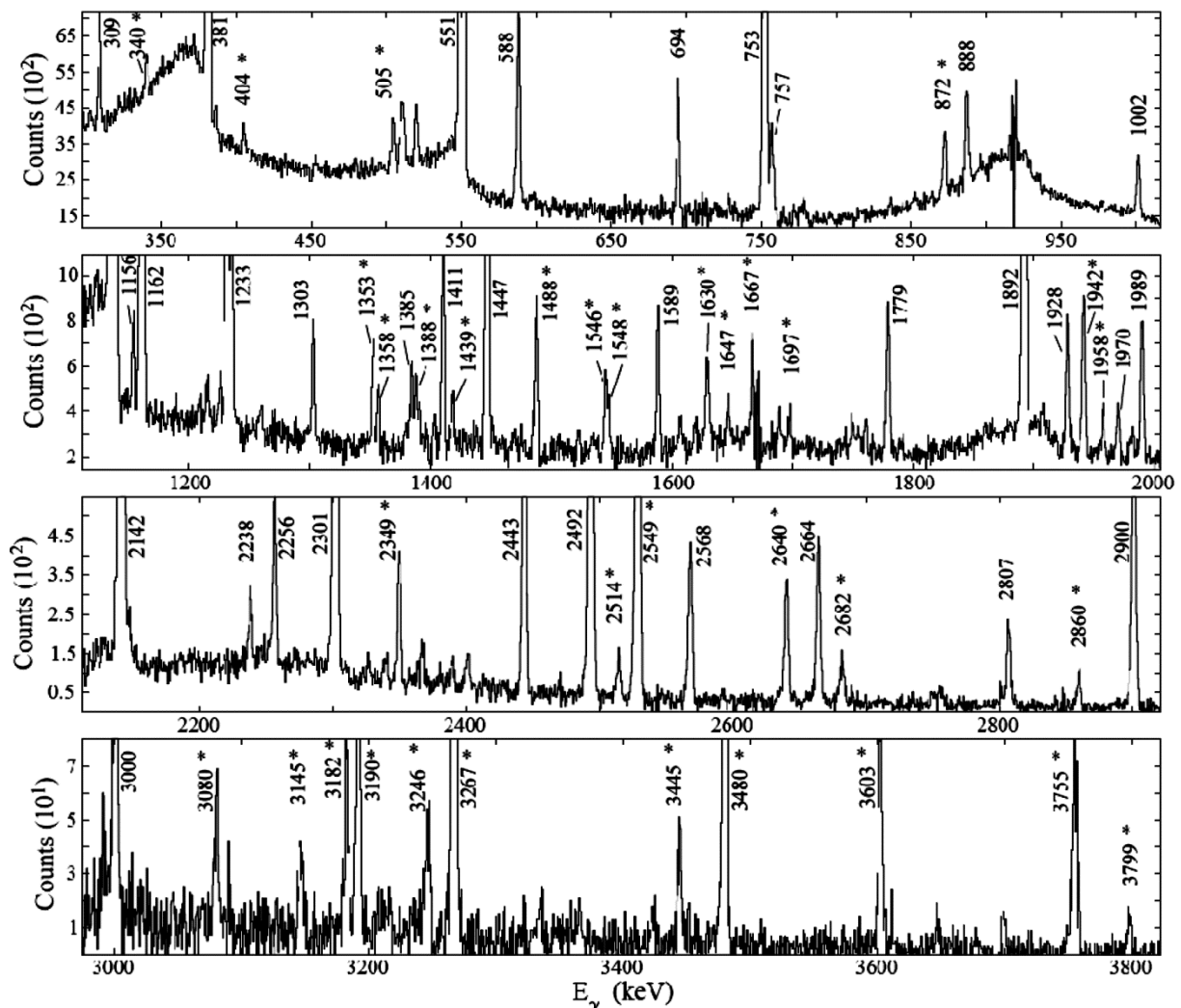


Fig. 1 — Representative $\gamma\gamma$ coincidence spectrum with a gate set on the 919 keV ($2_1^+ \rightarrow 0_1^+$) γ ray of ^{94}Zr . Newly observed γ rays are marked with asterisks (*).

intensity of the γ ray by the total intensity of all the γ rays emitted from that level.

For cases where a particular decay branch from a given level is very weak, it was not possible to obtain the branching of that weaker transition by the aforesaid procedure. In that case, the coincident counts of all the γ rays from a given level must be determined from the respective adjacent gating transitions lying below the level of interest, and the method as described by Kulp *et al.*³ was used. The number of coincidence counts (N_{12}) between the two cascading γ rays, γ_1 and γ_2 (γ_1 is the feeding transition to the level and γ_2 is the decaying transition from the level) is given by the relation:

$$N_{12} = N I_{\gamma_1} \varepsilon_{\gamma_1} B_{\gamma_2} \varepsilon_{\gamma_2} \varepsilon_{12} \omega(\theta_{12}) \quad \dots (1)$$

where, I_{γ_1} is the intensity of γ_1 ; B_{γ_2} is the branching fraction of γ_2 ; ε_{γ_1} and ε_{γ_2} are the singles photo-peak efficiencies of γ_1 and γ_2 , respectively; ε_{12} is the coincidence efficiency; $\omega(\theta_{12})$ is the angular correlation attenuation factor; and N can be considered as the overall normalization constant, which basically characterizes the coincidence data for a given decaying isotope. It is notable that the off-line analysis was carried out with sufficiently wide $\gamma\gamma$ coincidence time gates to take into account all the intensity so that the effect of ε_{12} can be ignored. The term $\omega(\theta_{12})$ is also considered to be negligible due to the geometrical symmetry of the array that was used in the experiment. Thus, for extracting the intensity (I_{γ_1}) of the feeding γ_1 transition, the following simplified relation was used:

$$I_{\gamma_1} = \frac{N_{12}}{\varepsilon_{\gamma_1} B_{\gamma_2} \varepsilon_{\gamma_2}} = \frac{I_{\gamma_1}^{\#}}{B_{\gamma_2} \varepsilon_{\gamma_2}} \quad \dots (2)$$

$$\text{where } I_{\gamma_1}^{\#} = \frac{N_{12}}{\varepsilon_{\gamma_1}}.$$

The branching ratio for the γ_1 transition (BR_{γ_1}) was then extracted using the following relation:

$$BR_{\gamma_1} = \frac{I_{\gamma_1}^{\#}}{B_{\gamma_2} \varepsilon_{\gamma_2}} \bigg/ \sum \frac{I_{\gamma_{1i}}^{\#}}{B_{\gamma_{2i}} \varepsilon_{\gamma_{2i}}} \quad \dots (3)$$

where the summation was taken over all the γ rays from the level of interest.

The branchings from the excited levels of ^{94}Zr were extracted using the aforesaid procedures and a few of them are listed in Table 1. The quoted uncertainties in the branching ratios include statistical uncertainties and the uncertainties due to the efficiency correction. A 3% uncertainty was assumed for making the efficiency correction of the observed transitions and was added in quadrature to the associated statistical uncertainty.

3 Results and Discussion

Combining the revised level lifetimes from previous work⁴ and the newly measured branching ratios from the present work, a comprehensive E2 decay pattern of the transitions up to $E_x \approx 2.3$ MeV has been established and is presented in Fig. 2. The previous g-factor measurements⁵ provide negative and positive values for the 2_1^+ and 2_2^+ states, respectively. These experimental findings indicate the persistence of dominant neutron or proton excitations in the wave functions for these states. It is obvious from Fig. 2 that the 1671 keV, 2_2^+ state preferentially decays to

Table 1 — Comparison of experimental and shell-model results for low-lying states in ^{94}Zr .

Experiment				Theory			
$E_x(\text{keV})$	$E_\gamma(\text{keV})$	Branching (%)	$B(E2)_{\text{Expt}}$ (W.u.)	$J_i^\pi \rightarrow J_f^\pi$	$E_x(\text{keV})$	$B(E2)_{\text{SM1}}$ (W.u.)	$B(E2)_{\text{SM2}}$ (W.u.)
918.77(8)	918.78(16)	100	$4.96_{-0.87}^{+1.34}$	$2_1^+ \rightarrow 0_1^+$	855	1.1	5.0
1300.38(10)	381.59(16)	100	$9.46_{-0.34}^{+0.38}$	$0_2^+ \rightarrow 2_1^+$	1386	0.4	0.3
1469.57(9)	550.76(16)	100	0.88(2)	$4_1^+ \rightarrow 2_1^+$	1509	0.4	1.8
	371.10(16)	0.15(1)	$18.62_{-2.08}^{+2.17}$	$2_2^+ \rightarrow 0_2^+$		4.0	9.6
1671.48(8)	752.61(16)	42.0(16)	$0.06_{-0.06}^{+0.21}$	$2_2^+ \rightarrow 2_1^+$	1736	0.7	1.9
	1671.53(16)	57.8(24)	$3.87_{-0.41}^{+0.43}$	$2_2^+ \rightarrow 0_1^+$		1.5	3.2
	479.87(16)	2.8(1)	$1.64_{-1.59}^{+8.44}$	$2_3^+ \rightarrow 2_2^+$		0.02	0.1
2151.50(8)	851.14(23)	0.79(5)	$0.76_{-0.24}^{+0.27}$	$2_3^+ \rightarrow 0_2^+$		0.7	1.6
	1232.43(16)	94.6(31)	$5.14_{-1.74}^{+2.17}$	$2_3^+ \rightarrow 2_1^+$	2254	0.1	0.7
	2151.32(16)	1.9(2)	$0.018_{-0.006}^{+0.007}$	$2_3^+ \rightarrow 0_1^+$		0.1	0.03
	658.69(20)	5.5(5)	$32.42_{-18.77}^{+25.21}$	$4_2^+ \rightarrow 2_2^+$		5.7	13.2
2329.93(10)	1411.16(17)	94.5(29)	$12.34_{-6.80}^{+8.39}$	$4_2^+ \rightarrow 2_1^+$	2545	1.4	3.0

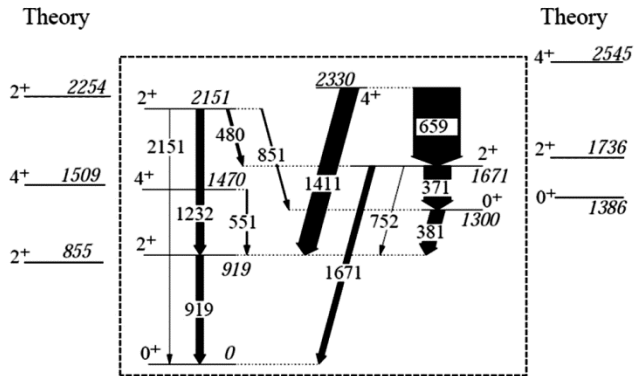


Fig. 2 — E2 strengths of the low-lying excited states of ^{94}Zr obtained from the present investigation. The widths of the arrows are proportional to the E2 strengths of the corresponding transitions. The experimental findings are depicted within the rectangular box. The decay branches are labeled by their energies. The maximum observed E2 strength is 32.42 W.u. for the 659 keV transition. The level energies obtained from the shell-model calculations are also presented for comparison (see text for details).

the 0^+_2 state rather than to the ground state indicating proton dominant character. On the same footing, the 2151 keV, 2^+_3 state seems to have neutron dominant character (see Fig. 2 and Table 1). The 2330 keV, 4^+_2 state appears to be proton dominated as the decay branches from the level exhibit a preference for feeding the 2^+_2 state as compared to the 2^+_1 state. Thus, the experimental findings obtained from the present and previous measurements provide direct evidence for proper identification of the low-lying states of ^{94}Zr in terms of proton and neutron domination. Considering a $B(E2)$ value of about 10 W.u. or more as the benchmark for identifying the collective states, it can be concluded that the 0^+_1 (at 919 keV), 4^+_1 (at 1470 keV), and 2^+_3 (at 2151 keV) states are non-collective in nature, whereas the 0^+_2 (at 1300 keV), 2^+_2 (at 1671 keV), and 4^+_2 (at 2330 keV) states are of collective nature. The wave functions of all the aforesaid collective states have proton dominated character. Thus, it can be surmised that the collectivity in the low excitation regime of ^{94}Zr evolves due to proton excitations.

For interpretation of the observed low-lying structure in ^{94}Zr , shell-model calculations were carried out using the NuShellX code⁶. The two-body interaction matrix file “*gl*” was used for the calculation. The valence space for the calculation was composed of the $\pi(2p_{1/2}, 1g_{9/2})$ and $\nu(3s_{1/2}, 2d_{5/2})$ orbitals outside the $^{88}_{38}\text{Sr}_{50}$ inert core. The calculated and experimental level energies are compared in

Fig. 2 and listed in Table 1. Reasonable agreement was obtained between the theoretical and experimental results. It is observed that the wave functions of the 0^+_1 and 0^+_2 states are quite different. The proton occupation probabilities in the $\pi(2p_{1/2})$ and $\pi(1g_{9/2})$ orbitals are about 65% and 35%, respectively, for the 0^+_1 , 2^+_1 and 4^+_1 states. The almost reverse pattern of proton occupation probability persists for the 0^+_2 , 2^+_2 and 4^+_2 states. This comparison suggests that the dominant contribution to the wave functions of the excited 0^+_2 , 2^+_2 and 4^+_2 states is a result of the excitation of protons [$\pi(2p_{1/2}) \rightarrow \pi(1g_{9/2})$] across the $Z = 40$ subshell closure. As mentioned previously, the 0^+_2 , 2^+_2 and 4^+_2 states are collective in nature, and the calculated wave functions of the states point to the fact that the observed collectivity is due to excitation of protons across the $Z = 40$ subshell closure. The calculated results also show that the neutron occupation probability in the $\nu(3s_{1/2})$ orbital is about 10% for the 0^+_1 , 2^+_1 and 4^+_1 states. On the other hand, there is about a 25% occupancy in the $\nu(3s_{1/2})$ orbital for the 0^+_2 , 2^+_2 and 4^+_2 states. Hence, the $\nu(3s_{1/2}) \rightarrow \nu(2d_{5/2})$ excitations of neutrons provide the major contributions for generating the low-lying 0^+_1 , 2^+_1 and 4^+_1 states.

In order to further test the quality of the wave functions, experimental $B(E2)$ values for a few transitions are compared with the theoretical calculations (see Table 1). The SM1 calculations were performed using the commonly used set of effective charges of $e_p = 1.5$ and $e_n = 0.5$, whereas the SM2 calculations were carried out with larger values for the effective charges ($e_p = 1.8$ and $e_n = 1.2$). It is obvious from Table 1 that SM2 gives better agreement with the experimental results. The need for the larger effective charges is suggestive of the importance of other configurations in the wave functions, which are probably missing in the limited model space used for the calculations. This result points to the need for additional calculations using larger model spaces with the possibility for incorporating the proton excitations from the fp shells to the orbitals above the $1g_{9/2}$ orbital, along with the neutron excitations from $1g_{9/2}$ to the higher-lying orbitals beyond the $3s_{1/2}$ and $2d_{5/2}$ orbitals. The use of ^{56}Ni as an inert core would likely be more appropriate for this purpose, and judicious choices of truncation for the excitation modes of the valence particles would be required in order to overcome the associated computational limitations.

4 Conclusions

Excited states of ^{94}Zr were populated through β^- decay of ^{94}Y . The de-exciting γ rays were detected using an array of 20 Compton-suppressed HPGe detectors and several weak decay branches have been placed through $\gamma\gamma$ coincidence techniques. The branching ratios of the weak decay branches have been extracted and the corresponding B(E2) values have been deduced. Proper identification of the collective and non-collective states of ^{94}Zr was possible. The collective and non-collective states are found to be of proton and neutron dominating nature, respectively. Evidence is presented to support the fact that the onset of collectivity in ^{94}Zr in the low-excitation regime is due to the occurrence of proton excitations across the $Z = 40$ subshell closure. Shell model calculations carried out within the limited valence space of the $\pi(2p_{1/2}, 1g_{9/2})$ and $\nu(3s_{1/2}, 2d_{5/2})$ orbitals are found to be insufficient for reproducing all the measured spectroscopic quantities. Additional calculations using a larger valence space with the possibility for incorporating the $\pi(fp) \rightarrow \pi(1g_{9/2})$ and $\nu(1g_{9/2}) \rightarrow \nu(3s_{1/2}, 2d_{5/2})$ excitations are warranted.

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