Indian Journal of Pure & Applied Physics
Vol. 59, March 2021, pp. 271-276

# Implementation of Monte Carlo Simulation in Evaluation of Uncertainty of Measurement of a Force Transducer

Harshvardhan Choudhary[a*], Girija Moona[b], D Vaithiyanathan[a] & Harish Kumar[a]
[a]National Institute of Technology Delhi, Delhi – 110 040, India
[b]CSIR- National Physical Laboratory, Dr K S Krishnan Marg, New Delhi 110 012, India

*Received 27 January 2021; accepted 17 February 2021*

Force transducers have prominently been utilized in numerous scientific, industrial and metrological applications since decades. They have been developed for the measurement of force in lower as well as the higher capacity to fulfil the industrial and technological requirements. Generally, a force transducer is calibrated as per standard process. For the better perceptive, there is a need of well defined calibration procedure in an organized structure according to standard ISO 376-2011. Also, factors affecting the uncertainty of measurement using law of propagation (LPU) method and their analysis have been discussed. The relative uncertainty contributions due to different factors have been explained with suitable mathematical expression. An alternative approach for evaluating uncertainty of measurement is Monte Carlo Simulation Method (MCS) that assigns probability distribution function (PDF) to input quantities and output. Efforts have been accomplished to determine uncertainty of measurement of force transducer using both methods.

**Keywords:** force transducer, uncertainty of measurement, Monte Carlo Method

## 1 Introduction

Force transducers are constructive measuring devices that are frequently used in many engineering and scientific applications, including electronic weighing machine, thrust measurement and aircrafts[1-2]. They comprise of different types, including analog or digital, strain gauged, dial gauged, tuning fork *etc*. Among them, strain gauged force transducers fulfil force measurement with practical capability and trustworthiness. Strain gauges, usually made of metallic foils are arranged over the sensing part of the force transducer in wheat stone bridge configuration. With the application of force, the circuit that is in the form of bridge becomes disturbed due to changes in resistances of strain gauges and corresponding output is acquired in electrical energy. In some applications, the shape of the force transducers is ring type due to easy design and fabrication process. Also shapes can be altered, such as square or elliptical ring, octagonal etc as per requirement[3-6]. The force measurement may lie in the range of few Newton (lower capacity) to mega Newton (higher capacity). Lower capacity force measurement is gaining attention in food and pharmaceutical industries to fulfil human safety and sensibility. For this particular measurement, usually 50 N tuning fork force transducers have been developed that provide better stability than conventional type strain gauged based force transducers [7-8].

*Corresponding author (E-mail: harshvardhanchoudhary@nitdelhi.ac.in)

## 2 Uncertainty of measurement (UoM)

Measurement of uncertainty is applicable to various metrology laboratories and testing devices to give better quality. Generally, measurement can be termed as a comparison between unknown quantity (measurand or final output) and nominal value. There is always a small amount of imperfection or doubt in any measurement. In other words, when the value of error in a result is unknown, it is termed as uncertainty. Several internal and external factors, including user, procedure, instrument and environmental circumstances put uncertainty in measurement [9, 10]. Table 1 lists sources of uncertainty in a measurement.

### 2.1 A conventional approach: Guide to the expression of uncertainty in measurement (GUM)/ Law of propagation of uncertainties (LPU)

The GUM approach provides a structure for estimating uncertainty of measurement. Generally, it requires a deep knowledge of measurand and the measurement. It is calculated using a combination that is achieved from uncertainties of input quantities.

Table 1 — Different Sources[11,12]

| S. No. | Various Sources of uncertainty in measurement |
|---|---|
| 1. | Unreliable nature of the measurand, variations in repeated observations |
| 2. | Improper behaviour of measuring instrument due to ageing, wear, parallax, poor readings etc |
| 3. | Unknown environmental conditions |
| 4. | Various approximations in the measurement procedure. |
| 5. | Unskilled users, inaccurate values of parameters obtained from external sources |

Broadly speaking, evaluation of uncertainty can be categorised into following ways- type A and type B. Type A uncertainties depend on repeated measurements (statistical analysis) from a process and type B are achieved by the sources that are different from statistical analysis of observations.

The output Y depends on input quantities ($X_1$, $X_2$...........$X_{n)}$ and has a functional relationship with them. So uncertainty of final result depends on the input quantities. Here, relative uncertainty of each input quantity is calculated. The propagation of standard and expanded uncertainties is given as per following Equation [9-10]

$$u_y^2 = \sum_{i=1}^{N} \left( \frac{\partial f}{\partial x_i} \right)^2 u_{x_i}^2 + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left( \frac{\partial f}{\partial x_i} \right) \left( \frac{\partial f}{\partial x_j} \right) \text{cov}(x_i, x_j) \quad \text{... (1)}$$

Where $u_y$ denotes the combined uncertainty for the output Y and $u_{x_i}$ denote the uncertainty for the $i^{th}$ input quantity. Figure 1 shows various steps to calculate uncertainty of measurement using LPU method. A multiplication of combined uncertainty with coverage factor k is made to achieve expanded uncertainty (U) and equation becomes[11-12],

$$U = k.u_y \quad \text{... (2)}$$

and measurand equation becomes,

$$Y = y \pm U \quad \text{... (3)}$$

**2.2 An alternative approach : Monte Carlo simulation**

A feasible approach, termed as Monte Carlo simulation (MCS) has been developed to evaluate uncertainty of measurement. It can be called as a statistic technique that has been used to validate the some theoretical results experimentally for the evaluation of uncertainty. Input quantities have been assigned appropriate probability distribution function (PDF) and a model is used to give the distribution of the final output. In this method, random values have been generated using an algorithm and follow predetermined distribution. For all inputs, numeric values are drawn from their respective PDFs. Furthermore, these values are produced to known functional relationships. As a result, single numeric value of output is generated. The whole process is repeated a number of times so that a set of simulated results is obtained. It can be termed as a procedure that is used for propagation of PDFs. In addition, it conducts random sampling from the PDFs of input quantities[9-10]. The implementation of MCS method has been organized in Fig. 2 and table 2 lists comparison between LPU and MCS method.

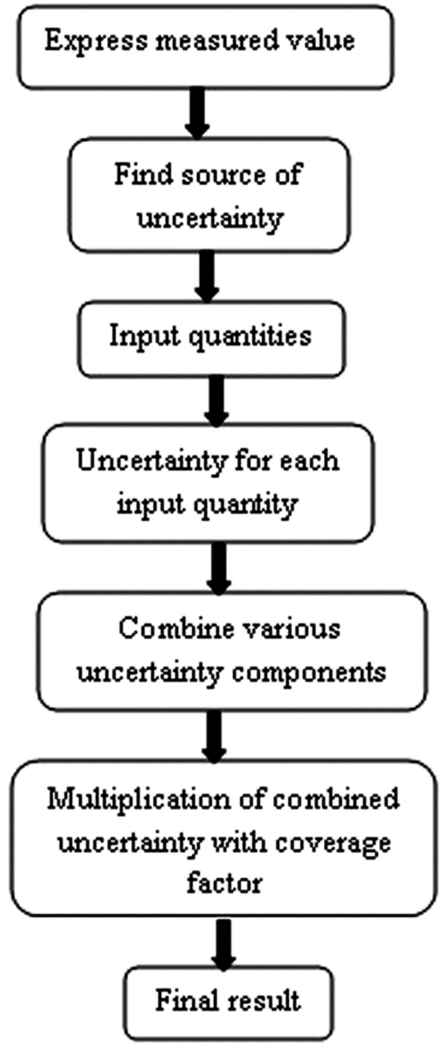


Fig. 1 — Steps to calculate uncertainty of measurement using LPU/ GUM [9-12]

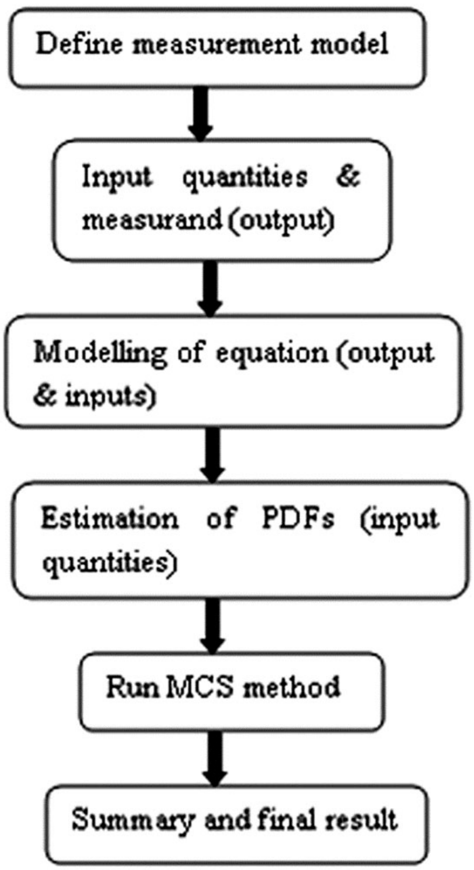


Fig. 2 — Steps to implement MCS Method to evaluate uncertainty of measurement[9-12]

## 3 Calibration Process of Force Transducer

Force transducer is calibrated according to the calibration procedures based on the ISO 376-2011. A dead weight force machine, force transducer and the indicating device (according to type of transducer) complete the set up for calibration. Software helps in operating of dead weight force machine. It is operated through the computer so that there is a minimum amount of human interference during the entire procedure. The calibration procedure has already been developed in past by researchers earlier[15-19]. In general, the relative errors due to different factors as mentioned in Table 3, may seem to be higher than the overall uncertainty of measurement. Furthermore, uncertainty of measurement is calculated from division of the each contributing uncertainty value with the corresponding factor. All these factors and probability distributions are shown in Table 3 (as per calibration procedure, ISO 376:2011).

With different relative uncertainty contributions that are achieved from all above mentioned factors (table 3) and uncertainty of the force machine, the uncertainty of measurement of the force transducer is computed as follows

$$w_{c(tra)} = \left(w_{rep}^2 + w_{rpr}^2 + w_{\text{int}}^2 + w_{rev}^2 + w_{zero}^2 + w_{res}^2\right)^{0.5} \quad \text{... (4)}$$

Table 2 — Comparison between GUM and MCS method[13-14]

| S. No. | GUM | MCS |
|---|---|---|
| 1 | It follows a normal distribution for output quantity | It provides a PDF for output quantity and consistency with PDFs of different inputs |
| 2 | Supports mathematical statistics and probability | Generation of random numbers that represent PDFs on the inputs |
| 3 | It requires certain amount of mathematical skills | Significant reduction in mathematical efforts |
| 4. | Use of partial derivatives, degree of freedom etc | No need of such task |

Table 3 — Different factors with corresponding probability distribution[15-19]

| S. No. | Different factors | Force proving instrument | Force transducer | Different forms of distribution, | Uncertainty type and factor of division |
|---|---|---|---|---|---|
| 1 | Zero Offset | Yes | Yes | Rectangular | Type B, $\sqrt{3}$ |
| 2 | Resolution | Yes | Yes | Rectangular | Type B, $\sqrt{3}$ |
| 3 | Repeatability | Yes | Yes | Rectangular | Type B, $\sqrt{3}$ |
| 4 | Reproducibility | Yes | Yes | U shaped | Type B, $\sqrt{2}$ |
| 5 | Creep | Yes | Optional | Rectangular, | Type B, $\sqrt{3}$ |
| 6 | Reversibility | Yes | Yes | Rectangular, | Type B, $\sqrt{3}$ |
| 7 | Interpolation | Yes | Yes | Triangular | Type B, $\sqrt{6}$ |
| 8 | Applied Force | Yes | Yes | Rectangular, | Type B, 1 |

$$W_{tra} = k.w_{c(tra)} \quad ... (5)$$

$$W = (W_{tra}^2 + W_{cmc}^2)^{0.5} \quad ... (6)$$

Equation 4 represents combined uncertainty of measurement and Eq.6 called as expanded uncertainty of measurement.[9-10,15].

## 4 Results and discussions

Efforts have been made to evaluate uncertainty of measurement of force transducer using LPU and Monte Carlo method.

### 4.1 Evaluation of uncertainty of measurement using LPU method

An experimental set up has been established and a force transducer has been calibrated as per defined procedure (section 3). Table 4 represents various calibration series. An uncertainty budget contains sources and various uncertainty components. The uncertainty budget for strain gauged force transducer with capacity 5 kN has been prepared using LPU method. Table 5 represents uncertainty budget.

### 4.2 Evaluation of uncertainty of measurement using MCS method

MCS method has also been implemented to evaluate the uncertainty of measurement of force transducer.For uncertainty budget of force transducer with capacity 5 kN, random numbers have been generated using Microsoft Excel tool. This procedure is carried out for each relative error, such as repeatability, reproducibility etc. A part of this whole process has been shown in table 6. A Gaussian shaped histogram has also been obtained using MCS method and shown in figure 3. The summary of uncertainty of measurement using MCS method for force transducer has been shown in table 7. An assessment of evaluated uncertainties using both methods has been represented in table 8. Around 200, 000 iterations have been done for MCS and evaluated uncertainty of measurement, obtained from both the methods has been found to be in a good agreement. In MCS method, exercises and calculations related to partial derivatives, sensitivity coefficients and degree of freedom have been eliminated. So in this approach, mathematical efforts are significantly reduced. This approach is more reliable and convenient as compared to LPU method.

Table 4 — Different series of calibration forces

| S. No. | Force (kN) | Series 1 | Series 2 | Series 3 | Series 3' | Series 4 | Series 4' |
|---|---|---|---|---|---|---|---|
| 1 | 0.5 | 10143 | 10149 | 10140 | 10153 | 10150 | 10162 |
| 2 | 1.0 | 20290 | 20301 | 20281 | 20304 | 20300 | 20328 |
| 3 | 1.5 | 30441 | 30456 | 30432 | 30458 | 30456 | 30484 |
| 4 | 2.0 | 40558 | 40602 | 40577 | 40607 | 40602 | 40636 |
| 5 | 2.5 | 50736 | 50749 | 50738 | 50764 | 50768 | 50797 |
| 6 | 3.0 | 60883 | 60900 | 60870 | 60898 | 60909 | 60931 |
| 7 | 3.5 | 71131 | 71148 | 71122 | 71141 | 71162 | 71183 |
| 8 | 4.0 | 81280 | 81293 | 81271 | 81287 | 81304 | 81323 |
| 9 | 4.5 | 91429 | 91444 | 91419 | 91429 | 91458 | 91471 |
| 10 | 5.0 | 101573 | 101590 | 101559 | 101559 | 101605 | 101605 |

Table 5 — An uncertainty budget (5 kN)

| S. No. | Relative error factor (%) | Type | Relative error | Probability distribution | factor | Standard uncertainty |
|---|---|---|---|---|---|---|
| 1. | Repeatability | B | 0.0085 | Rectangular | 1.732 | 0.0049 |
| 2. | Reproducibility | B | 0.0225 | U shaped | 1.414 | 0.0159 |
| 3. | Zero offset | B | 0.0065 | Rectangular | 1.732 | 0.0038 |
| 4. | Resolution | B | 0.0005 | Rectangular | 1.732 | 0.0003 |
| 5. | Reversibility | B | 0.0010 | Rectangular | 1.732 | 0.0006 |
| 6. | Interpolation | B | 0.0120 | Triangular | 2.449 | 0.0049 |
| 7. | Machine uncertainty (cmc) | B | 0.0075 | Normal | 1 | 0.0075 |
| 8. | Combined uncertainty (%), $k$=1 | | | | | 0.019 |
| 9. | Expanded uncertainty (%), $k$=2 | | | | | 0.039 |

Table 6 — Force transducer (5 kN): Sample sheet of evaluation of uncertainty of measurement using MCS method (%)

| S. No. | Repeatability | Reproducibility | Zero offset | Resolution | Reversibility | Interpolation | cmc | Combined uncertainty |
|---|---|---|---|---|---|---|---|---|
| 1. | 0.0064 | 0.0047 | -0.0015 | 0.0005 | -0.0001 | 0.0139 | 0.0122 | 0.0256 |
| 2. | 0.0040 | 0.0011 | -0.0046 | 0.0004 | 0.0001 | 0.0216 | 0.0026 | 0.0263 |
| 3. | 0.0052 | 0.0223 | 0.0063 | 0.0007 | 0.0009 | 0.0102 | 0.0008 | 0.0228 |
| 4. | 0.0042 | 0.0160 | 0.0060 | 0.0006 | -0.0001 | 0.0069 | 0.0036 | 0.0220 |
| 5. | 0.0053 | 0.0157 | -0.0030 | 0.0009 | 0.0007 | 0.0091 | 0.0169 | 0.0345 |
| 6. | 0.0061 | 0.0111 | -0.0012 | 0.0003 | -0.0007 | 0.0159 | 0.0124 | 0.0237 |
| 7. | 0.0059 | 0.0006 | -0.0038 | 0.0007 | 0.0004 | 0.0134 | 0.0122 | 0.0255 |
| 8. | 0.0032 | 0.0045 | -0.0022 | 0.0002 | 0.0007 | 0.0195 | -0.0019 | 0.0223 |
| 9. | 0.0058 | 0.0153 | -0.0024 | 0.0001 | -0.0007 | 0.0077 | 0.0015 | 0.0238 |
| 10. | 0.0015 | 0.0133 | 0.0003 | 0.0003 | -0.0009 | 0.0140 | -0.0016 | 0.0255 |

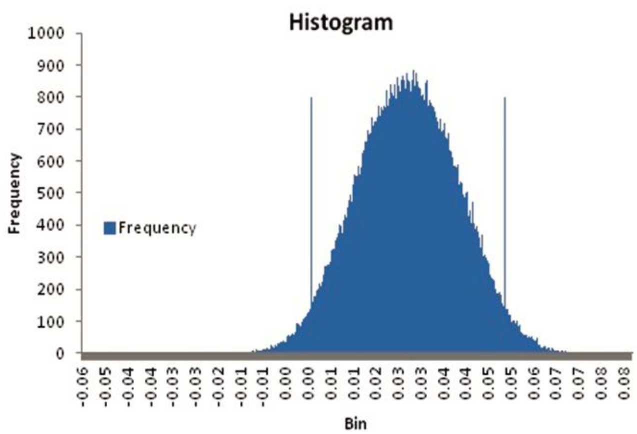


Fig. 3 — A Gaussian shaped histogram from MCS with low and high point.

## 5 Conclusions

- In this paper, two different approaches LPU and MCS methods have been discussed regarding force transducer.
- In LPU method, the uncertainty of measurement of force transducer has been obtained as ± 0.039% ($k$=2).
- In MCS method, the uncertainty of measurement of force transducer has been obtained as ± 0.026% ($k$=2).
- From the observed data, it can be concluded that there is a good agreement of results for both the methods. Furthermore, the final results of uncertainty of measurement from both the methods have been found as comparable and consistent but, MCS seems to be a better method with flexible approach.
- Uncertainty of measurement can be obtained using MCS method with less mathematical efforts but there is long computational time as compared to other conventional approaches.

Table 7 — Summary of MCS method

| S. No. | Parameter (measurand) | Force transducer (5kN) |
|---|---|---|
| 1. | Mean value (%) | 0.028 |
| 2. | Combined uncertainty ($k$=1)(%) | 0.013 |
| 3 | Expanded uncertainty ($k$=2)(%) | 0.026 |

Table 8 -Comparison between LPU and MCS method

| S. No. | Method | Combined uncertainty (%) | Expanded uncertainty (%) |
|---|---|---|---|
| 1. | LPU | 0.019 | 0.039 |
| 2. | MCS | 0.013 | 0.026 |

- Although there is a tight cluster between the results of uncertainty of measurement with both the methods yet MCS method is found to be more suitable, reliable and convenient.

## References


1 Kumar R, Pant B D & Maji S, *Mapan* 32 (2017) 167.

2 Kumar H, Kaushik M & Kumar A, *Mapan* 30 (2015) 37.

3 Kumar R & Maji S, *Eng Solid Mechanics*, 4 (2016) 81.

4 Ştefănescu D M & Anghel M A, *Measurement*, 46 (2013) 949.

5 Uddin M S & Songyi D, *Measurement*, 90 (2016) 168.

6 Soliman E, *Alexandria Eng J*, 54 (2015) 155.

7 Hayashi T & Ueda K, *Measurement*, 114 (2018) 203.

8 Hayashi T, Yoshihisa K, Ueda K, Hoshino T, Suzawa H & Kobayashi M, *Measurement*, 41 (2008) 941.

9 JCGM 100: 2008, *Evaluation of Measurement Data – Guide to the Expression of Uncertainty in Measurement, Bureau International Des Poids Et Mesures, France*, 2008.

10 JCGM 101: 2008, *Evaluation of Measurement Data – Supplement to the 'Guide to the Expression of Uncertainty in Measurement– Propagation of Distributions using a Monte Carlo method,Bureau International Des Poids Et Mesures, France*, 2008.

11 Garg N, Yadav S & Aswal D K, *Mapan*, 34 (2019) 299.

12 Moona G, Sharma R & Kumar H, *Transactions of the Institute of Measurement and Control*, 40 (2018) 2428.
13 Kumar H, Moona G, Arora P K, Haleem A, Singh J, Kumar R & Kumar A, *Indian J Pure Appl Phys*, 55 (2017) 445.
14 Rab S, Yadav S, Zafer A, Haleem A, Dubey P K, Singh J, Kumar R, Sharma R & Kumar L, *Mapan*, 34 (2019) 305.
15 Kumar H, Pardeep, Kaushik M & Kumar A, *Mapan*, 30 (2015) 37.
16 Kumar H , Sharma C, Kumar A, Arora P K & Kumar S, *ISA transactions*, 58 (2015) 659.
17 Singh A P, Ghoshal S K & Kumar H, *Mapan*, 35 (2020) 165.
18 Saxena R, Ghoshal S K & Kumar H, *Mapan*, 34 (2019) 511.
19 Saxena R, Ghoshal S & Kumar H, *Indian J Pure Appl Phys*, 57 (2019) 42.