Indian Journal of Pure & Applied Physics
Vol. 53, January 2015, pp. 12-17

Dirac-Yukawa problem with Coulomb like tensor interaction via Ansatz method

Nasrin Salehi[1]*, Mona Azizi[2] & Ali Akbar Rajabi[2]

[1]Department of Basic Science, Shahrood Branch, Islamic Azad University, Shahrood, Iran

[2]Physics Department, Shahrood University of Technology, Shahrood, Iran

*E-mail: salehi@shahroodut.ac.ir

Received 22 January 2013; revised 10 April 2014; accepted 10 November 2014

In the present study, the solution of Dirac equation for Yukawa potential including the Coulomb-like potential tensor interaction term has been investigated. In order to obtain the solution of the problem, Yukawa potential has been expanded by using Taylor extension to the power of seventh and brought out its simple from. The energy eigenvalues and the corresponding wave functions are obtained using the Ansatz method. The deuteron mass has also been reported by considering the effects of hyperfine interactions on the relativistic energy spectra of nucleon. The obtained result shows that deuteron mass is found to be in good agreement with the experimental value.

Keywords: Dirac equation, Yukawa potential, Tensor interaction, Hyperfine interaction, Nucleon, Deutron

1 Introduction

Solutions of relativistic wave equations play an important role in many areas of physics. Dirac equation which describes the spin 1/2 particles is solved to obtain complete information about the motion of these particles. In particular, the Dirac equation has been used in solving many problems of nuclear and high-energy physics. It is well known that the exact energy eigenvalues of the boundstate play an important role in quantum mechanics. Recently, there has been an increased interest in searching for analytic solution of the Dirac equation[1-11]. In recent years, tensor couplings have been used widely to study the nuclear properties[12-19] and they were introduced into the Dirac equation[20,21] by substitution of $\vec{P} \to \vec{P} - im\omega\beta.\hat{x}U(r)$.

2 Yukawa Potential

The screened Coulomb potential, also known as the Yukawa potential in atomic physics and the Debye–Huckel potential in plasma physics, is of interest in many areas of physics. It was originally used to model strong nucleon–nucleon interactions due to meson exchange in nuclear physics by Yukawa[19]. It is also used to represent a screened Coulomb potential due to the cloud of electronic charges around the nucleus in atomic physics or to account for the shielding by outer charges of the Coulomb field experienced by an atomic electron in hydrogen plasma. However, the Schrödinger equation for this potential cannot be solved exactly. Hence, various numerical and pertubative methods have been devised to obtain the energy levels and related physical quantities[22]. The generic form of this potential is given by:

$$U(x) = -\frac{g}{x}V(x) \quad g > 0 \qquad \dots(1)$$

that

$$V(x) = e^{-kx} \qquad \dots(2)$$

In this part we expand the Yukawa potential in its meson clouds, $x = a$, by using Teylor extension to the power of seventh.

$$U(x) = -\frac{g}{x}[e^{-ka} - ke^{-ka}(x-a) + \frac{k^2}{2!}e^{-ka}(x-a)^2 - \frac{k^3}{3!}e^{-ka}(x-a)^3 + \frac{k^4}{4!}e^{-ka}(x-a)^4 - \frac{k^5}{5!}e^{-ka}(x-a)^5 + \frac{k^6}{6!}e^{-ka}(x-a)^6 - \frac{k^7}{7!}e^{-ka}(x-a)^7 + ...] \qquad \dots(3)$$

We can reduce Eq. (3) to:

$$U(x) = ax^6 - bx^5 + cx^4 - dx^3 + ex^2 - fx + h - L/x \qquad \dots(4)$$

Now, we have the general Yukawa equation, that *a*, *b*, *c*, *d*, *e*, *f*, *h* and *L* are:

$$a = \frac{k^7}{7!}$$
$$b = -\left(\frac{7}{7!}k^6 + \frac{7}{7!}k^6\right)$$
$$c = \left(\frac{21}{7!}k^5 + \frac{6}{6!}k^5 + \frac{1}{5!}k^5\right)$$
$$d = -\left(\frac{35}{7!}k^4 + \frac{15}{6!}k^4 + \frac{5}{5!}k^4 + \frac{1}{3!}k^4\right)$$
$$e = \left(\frac{35}{7!}k^3 + \frac{20}{6!}k^3 + \frac{10}{5!}k^3 + \frac{6}{4!}k^3 + \frac{3}{3!}k^3 + \frac{1}{2!}k^3\right)$$
$$f = -\left(\frac{21}{7!}k^2 + \frac{15}{6!}k^2 + \frac{10}{5!}k^2 + \frac{6}{4!}k^2 + \frac{3}{3!}k^2 + \frac{1}{2!}k^2\right)$$
$$h = -\left(\frac{7}{7!}k + \frac{6}{6!}k + \frac{5}{5!}k + \frac{4}{4!}k + \frac{3}{3!}k + \frac{1}{2!}k + 2\right)$$
$$L = -\left(\frac{7}{7!} + \frac{6}{6!} + \frac{5}{5!} + \frac{4}{4!} + \frac{3}{3!} + \frac{1}{2!} + 2\right) \quad \ldots(5)$$

3 Exact Analytical Solution of the Dirac Equation for Yukawa Potential

The Dirac equation has been solved under the Yukawa potential in the presence of a Coulomb-like tensor potential (Eq. (4)). We anticipate the answer and then obtain the parameters for highest nuclei, deuteron. A deuteron consists of a neutron and a proton. Let us assume one nucleon is fixed and another nucleon is moving around the center of mass. At last by using single particle model, the effects of one pion exchange for single nucleon have been investigated. Dirac equation under an attractive scalar potential *S*, a repulsive vector potential *V* and a tensor potential *U* in the relativistic unit[23-25] ($\hbar = c = 1$) is given by:

$$H = \vec{\alpha}.\vec{P} + \beta(m+S) + V - i\beta\vec{\alpha}.\hat{x}U \quad \ldots(6)$$

We have $\Delta = V - S$ and $\Sigma = V + S$. The Dirac equation can be solved exactly for the cases $\Delta = 0$ and $\Sigma = 0$. $\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma}_i \\ \vec{\sigma}_i & 0 \end{pmatrix}$ and $\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$ are the usual Dirac matrices where *I* is 2×2 unitary matrix and $\vec{\sigma}_i$ are Pauli three vector matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The upper and lower components of Dirac spinor with Ψ_A and Ψ_B have been indicated.Expressing the Dirac matrices in terms of the Pauli matrices we obtain the following coupled equations[26,27] :

$$\vec{\sigma}.\vec{P}\Psi_A + (V - S - m)\Psi_B + i\vec{\sigma}.xU(x)\Psi_A = E\Psi_B \ldots(7a)$$

$$\vec{\sigma}.\vec{P}\Psi_B + (V + S + m)\Psi_A - i\vec{\sigma}.xU(x)\Psi_B = E\Psi_A \ldots(7b)$$

We assume that *V*, *S* and *U* are radial potentials. Using the identity:

$$\vec{\sigma}.\vec{P} = \frac{(\vec{\sigma}.\vec{x})}{x^2}(\vec{\sigma}.\vec{x})(\vec{\sigma}.\vec{P}) = \frac{(\vec{\sigma}.\vec{x})}{x^2}\left(-ix\frac{\partial}{\partial x} + i\vec{\sigma}.\vec{L}\right) \quad \ldots(8)$$

with Eqs (7a) and (7b) we find the following second order differential equations for Ψ_A and Ψ_B:

$$\vec{P}^2\Psi_A + \left(U^2 + \frac{dU}{dx} + \frac{2U}{x} + \frac{\left(\frac{d\Delta}{dx}\right)}{(E+m-\Delta)}U\right)\Psi_A + \left(4U + 2\frac{\left(\frac{d\Delta}{dx}\right)}{(E+m-\Delta)}\right)\left(\frac{\vec{S}.\vec{L}}{2}\right)\Psi_A - \frac{\left(\frac{d\Delta}{dx}\right)}{(E+m-\Delta)}\frac{\partial\Psi_A}{\partial x} = (E+m-\Delta)(E+m-\Sigma)\Psi_A \quad \ldots(9a)$$

$$\vec{P}^2\Psi_B + \left(U^2 - \frac{dU}{dx} - \frac{2U}{x} - \frac{\left(\frac{d\Sigma}{dx}\right)}{(E-m-\Sigma)}U\right)\Psi_B + \left(-4U + 2\frac{\left(\frac{d\Sigma}{dx}\right)}{(E-m-\Sigma)}\right)\left(\frac{\vec{S}.\vec{L}}{2}\right)\Psi_B + \frac{\left(\frac{d\Sigma}{dx}\right)}{(E-m-\Delta)}\frac{\partial\Psi_B}{\partial x} = (E+m-\Delta)(E-m-\Sigma)\Psi_B \quad \ldots(9b)$$

where $\vec{S}$ stands for the $1/2\vec{\sigma}$ spin operator and $\vec{L}$ for orbital angular momentum operator. The Hamiltonian operator commutes with the total angular momentum operator which is given by $\vec{J} = \vec{L} + \vec{S}$ and with the parity operator. Therefore, the eigenfunctions defined by Hamiltonian can be expressed as:

$$\Psi_{njmk} = \begin{pmatrix} i\dfrac{g_{nk}(x)}{x}\Phi^{l}_{jmk}(\hat{x}) \\ \dfrac{f_{nk}(x)}{x}\Phi^{\bar{l}}_{jm(-k)}(\hat{x}) \end{pmatrix} \quad \ldots(10)$$

where $\Phi^{l}_{jmk}(\hat{x})$ denotes the spin spherical harmonics. The quantum number k is related to l and j as follows:

$$k = \begin{cases} -(l+1) = -\left(j+\dfrac{1}{2}\right) & j = l+\dfrac{1}{2} \\ l = +\left(j+\dfrac{1}{2}\right) & j = l-\dfrac{1}{2} \end{cases} \quad \ldots(11)$$

For spin spherical harmonics, we have:

$$\vec{\sigma}.\hat{x}\Phi^{l}_{jmk} = \Phi^{\bar{l}}_{jm(-k)} \quad \ldots(12)$$

Eqs (9a) and (9b) reduce to a set of coupled equations for the radial wave functions g_{κ} and f_{κ}:

$$\begin{aligned} &\left(\frac{d^2}{dx^2} - \frac{k(k+1)}{x^2} + \frac{2k}{x}U - \frac{dU}{dx} - U^2\right)g_k(x) \\ &+ \frac{\dfrac{d\Delta}{dx}}{(E+m-\Delta)}\left(\frac{d}{dx} + \frac{k}{x} - U\right)g_k(x) \\ &= -(E+m-\Delta)(E-m-\Sigma)g_k(x) \end{aligned} \quad \ldots(13a)$$

$$\begin{aligned} &\left(\frac{d^2}{dx^2} - \frac{k(k-1)}{x^2} + \frac{2k}{x}U + \frac{dU}{dx} - U^2\right)f_k(x) \\ &+ \frac{\dfrac{d\Sigma}{dx}}{(E-m-\Delta)}\left(\frac{d}{dx} - \frac{k}{x} + U\right)f_k(x) \\ &= -(E+m-\Delta)(E-m-\Sigma)f_k(x) \end{aligned} \quad \ldots(13b)$$

Using the following relations given in Eq. (13):

$$\begin{aligned} &2\vec{S}.\vec{L} = \vec{J}^2 - \vec{L}^2 - \vec{S}^2 k \\ &\vec{\sigma}.\vec{L}\Phi^{l}_{jmk} = -(k+1)\Phi^{l}_{jmk} \\ &\vec{\sigma}.\vec{L}\Phi^{\bar{l}}_{jm(-k)} = -(k+1)\Phi^{\bar{l}}_{jm(-k)} \end{aligned} \quad \ldots(14)$$

for particle of m mass and E energy, we have:

$$S = V = \frac{1}{2}\Sigma \quad \ldots(15)$$

$$\Sigma(x) = ax^6 - bx^5 + cx^4 - dx^3 + ex^2 - fx + h - L/x \quad \ldots(16)$$

$$\Delta x = 0 \quad \ldots(17)$$

$$U(x) = -\frac{1}{x} \quad \ldots(18)$$

We assume the case $S(x) = V(x)$ [Refs 28-32] and consider $k(k+1) = l(l+1)$, then the upper component in Eq. (13a) is as follows:

$$\begin{aligned} &\left(\frac{d^2}{dx^2} - \frac{l(l+1)+2+2k}{x^2} - \Sigma(r)(E+m)\right)g_k(x) \\ &= (m^2 - E^2)g_k(x) \end{aligned} \quad \ldots(19)$$

As we know in deuteron, two nucleons have in D-state, so:

$$\begin{aligned} &\left[\frac{d^2}{dx^2} - \frac{l(l+1)+2-2(l+1)}{x^2}\right]g_k(x) \\ &= \left(\begin{pmatrix} a_1x^6 - b_1x^5 + c_1x^4 - d_1x^3 + e_1x^2 \\ -f_1x + h_1 - \dfrac{L_1}{x} \end{pmatrix} - \varepsilon\right)g_k(x) \end{aligned} \quad \ldots(20)$$

Therefore, Schrödinger-like equation[32] as follows:

$$g_1''(x) = \begin{bmatrix} \begin{pmatrix} a_1x^6 - b_1x^5 + c_1x^4 - d_1x^3 + e_1x^2 \\ -f_1x + h_1 - \dfrac{L_1}{x} \end{pmatrix} \\ -\varepsilon + \dfrac{l(l-1)}{x^2} \end{bmatrix} \quad \ldots(21)$$

By setting

$$\begin{aligned} &a(E_1+m) = a_1 && b(E_1+m) = b \\ &c(E_1+m) = c_1 && d(E_1+m) = d_1 \end{aligned} \quad \ldots(22)$$

$$\begin{aligned} &e(E_1+m) = e_1 && f(E_1+m) = f_1 \\ &h(E_1+m) = h_1 && L(E_1+m) = L_1 \\ &(E_1^2 - m^2) = \varepsilon \end{aligned} \quad \ldots(23)$$

In order to solve the differential equation [Eq. (21)], we suppose the following form for the wave function:

$$g(x) = M(x)e^{Z(x)} \quad \ldots(24)$$

Now, for the functions $M(x)$ and $Z(x)$, respectively, we make use of the following ansatz[28,33,34]:

$$M(x)=\begin{cases}1 & \text{if ground state}\\ \prod_{i=1}^{\upsilon}\left(x-a_i^{\upsilon}\right) & \text{if } \upsilon\succ 0\end{cases} \quad \ldots(25)$$

$$Z(x)=-\frac{1}{4}\alpha x^4-\frac{1}{3}\beta x^3-\frac{1}{2}\eta x^2-\tau x+\delta\ln x \quad \ldots(26)$$

For a particular grand angular quantum number γ, there are different solutions which are labeled by υ (υ determines the number of the nodes of the wave function). By substitution of $M(x)$ and $Z(x)$ into Eq. (24) and then taking the second-order derivative of the obtained equation, we can get:

$$g''(x)=\left[Z''(x)+Z'(x)^2+\frac{M''(x)+2M'(x)Z'(x)}{M(x)}\right] \quad \ldots(27)$$

We consider the ground state, which is called the 0 th node solution of the differential equation Eq. (27). By equating Eqs (21) and (27) for $\upsilon=0$, it can be found that:

$$\begin{aligned}\alpha&=\sqrt{a_1}\\ \beta&=\frac{-b}{2\sqrt{a_1}}\\ \eta&=\frac{c_1-\beta_1}{2\alpha}\\ \tau&=\frac{-d_1-2\beta\eta}{2\alpha}\end{aligned} \quad \ldots(28)$$

$$\delta=l-1 \qquad \varepsilon=\eta(1-2\delta)-\tau^2-h_1 \quad \ldots(29)$$

The upper component is as follows:

$$g_1(x)=N_0x^{l-1}\exp\left(-\frac{1}{4}\alpha x^4-\frac{1}{3}\beta x^3-\frac{1}{2}\eta x^2-\tau x\right) \quad \ldots(30)$$

and another component of wave function is:

$$f_1(x)=\frac{\left(\vec{\sigma}.\vec{P}+i\vec{\sigma}.\hat{r}U\right)}{(E+m)}g_1(x) \quad \ldots(31)$$

By substituting Eq. (30) into Eq. (31), we obtained the following equation:

$$f_1(x)=\frac{-i\vec{\sigma}.\hat{\mathrm{x}}}{(E+m)}\left[\frac{d}{dx}-i\vec{\sigma}.\vec{L}-U\right]N_0\,x^{l-1}\times\exp\left(-\frac{1}{4}\alpha x^4-\frac{1}{3}\beta x^3-\frac{1}{2}\eta x^2-\tau x\right) \quad \ldots(32)$$

Therefore, the total wave function is;

$$\psi=N_0\begin{pmatrix}1\\ \frac{-i\vec{\sigma}.\hat{\mathrm{x}}}{(E+m)}\left[\alpha x^3+\beta x^2+\eta x+\tau+\frac{1}{x}\right]\end{pmatrix}\times\exp\left(-\frac{1}{4}\alpha x^4-\frac{1}{3}\beta x^3-\frac{1}{2}\eta x^2-\tau x\right) \quad \ldots(33)$$

Now, we have to obtain parameters *a, b, c, d, e, f, h* and *L* by using of Eq. (5):

$$\begin{aligned}b&=\frac{14}{k}a & c&=\frac{105}{k^2}a\\ d&=\frac{440}{k^3}a & e&=\frac{2555}{k^4}a\\ f&=\frac{6846}{k^5}a & h&=\frac{8659}{k^6}a\end{aligned} \quad \ldots(34)$$

We know that $k=\frac{1}{x}=0.714(fm^{-1})$ by using of Eqs (28 and 29), we can get amount of parameters α, β, η and τ

$$\begin{aligned}\alpha&=\sqrt{a_1}\\ \beta&=-9.8\sqrt{a_1}\\ \eta&=83.08\sqrt{a_1}\\ \tau&=83.08\sqrt{a_1}\end{aligned} \quad \ldots(35)$$

The bonding energy for deuteron[35] is reported $2.224\,\mathrm{MeV}$. This is useful for calculate the numeric values of parameters in Eq. (35).

$$\varepsilon=E^2-m^2=\eta(1-2\delta)-\tau^2-h_1 \quad \ldots(36)$$

By substituting the amounts of Eq. (36) and $l=1$, we obtained the numeric values of parameters in Eq. (35). The parameters are reported in Table 1, while the parameters of Yukawa potential are reported in Table 2.

4 Effect of the Hyperfine Interaction in Structure of Nucleon

In this Section we introduce the hyperfine interaction potential. The standard hyperfine interaction

Table 1 — Best adoption with experimental value is: a_1=3.06136, α=1.74967, β=17.146815, η=145.362999, τ=141.362999, E(MeV)=2.19020

a_1	α	β	η	τ	E (MeV)
3.06131	1.74965	-17.146628	145.361420	-141.02227	2.89658
3.06134	1.74967	-17.146785	145.362749	-141.02356	2.49699
3.06135	1.74967	-17.146805	145.362915	-141.02372	2.34861
3.06136	1.74967	-17.146815	145.362999	-141.0238	2.19020
3.0638	1.74967	-17.147648	145.377006	-141.0234	1.83278

Table 2 — Value of the Yukawa potential parameters

a (fm^{-7})	b (fm^{-6})	c (fm^{-5})	d (fm^{-4})	e (fm^{-3})	f (fm^{-2})	h (fm^{-1})
0.21961	4.30589	45.24045	265.53556	2159.7519	8109.2235	14362.5603

is used in order to reproduce the splittings within the SU(6) − multiplets. As the baryons have spin and isospin, in deuteron neutron and proton can interact together. Therefore, the complete potential is as follows:

$$\langle H_{int}(x)\rangle = V(x) + \langle H_S(x)\rangle + \langle H_I(x)\rangle + \langle H_{SI}(x)\rangle \quad \ldots(37)$$

In the present work, we have added the hyperfine interaction potentials ($H_S(x)$, $H_I(x)$ and $H_{SI}(x)$) which yield properties very close to the experimental results. By regarding $V(x)$ as the non-pertubative potential and the other terms in Eq. (37) as pertubative ones according to this explanation. The spin and isospin potential contains a δ-like term which is an illegal operator. We have modified it by a Gaussian function of the nucleons pair relative distance[36,37]:

$$H_S = A_S \frac{1}{\left(\sqrt{\pi}\sigma_S\right)^3} e^{-x^2/\sigma_S^2} \sum\left(\vec{S}_i.\vec{S}_j\right) \quad \ldots(38)$$

where S_i is the spin operator of the ith nucleon and x is the relative nucleon pair coordinate. A_s and σ_S are constants[38,39]: $\sigma_S = 2.87 fm$ and $A_S = 67.4 fm^2$.We know that the deuteron consists of two nucleons. Where 1 indicates the first nucleon and 2 indicates the second nucleon. We have:

$$H_S = A_S \frac{1}{\left(\sqrt{\pi}\sigma_S\right)^3} e^{-x^2/\sigma_S^2} \left(\frac{1}{2}\right)\left[S^2 - S_1^2 - S_2^2\right] \quad \ldots(39)$$

Furthermore, we add two hyperfine interaction terms to the Hamiltonian nucleon pairs[40] similar to Eq. (38).

$$H_I = A_I \frac{1}{\left(\sqrt{\pi}\sigma_I\right)^3} e^{-x^2/\sigma_I^2} \sum\left(\vec{I}_i.\vec{I}_j\right) \quad \ldots(40)$$

where I_i is the isospin operator of the ith nucleon and the constants[38,39] describe like this $\sigma_I = 3.45 fm$ and $A_I = 51.7 fm^2$. The another one is a spin-isospin interaction[40,41], given by similar Eq.(38).

$$H_{SI} = A_{SI} \frac{1}{\left(\sqrt{\pi}\sigma_{SI}\right)^3} e^{-x^2/\sigma_{SI}^2} \sum\left(\vec{S}_i.\vec{S}_j\right)\left(\vec{I}_i.\vec{I}_j\right) \quad \ldots(41)$$

The fitted parameters are $\sigma_{SI} = 2.31 fm$ and $A_{SI} = -106.2 fm^2$ (Refs 38 and 39), where ψ_γ is the perturbed wave function and we write it as:

$$\psi_\gamma = \psi'_\gamma + \sum_{\gamma'=\gamma} \frac{\psi_{\gamma'}^0 \left|H_{int}\right| \psi_{\gamma'}^0 \psi_{\gamma'}^{\prime 0}}{E_\gamma^0 - E_\gamma^{\prime 0}} \quad \ldots(42)$$

The deuteron mass are given by two nucleons masses and the eigen energies of the Dirac equation E, (E is a function of η, δ, τ, h, m) with the first order energy correction from potential H_{int} can be obtained by using the unperturbed wave function Eqs (38, 40 and 41). The total potential for the ground state as well as the other states[41] can be written as :

$$H_{int} = \frac{\int \psi_\gamma^* H_{int} \psi_\gamma x^2 dx\, d\Omega}{\int \psi_\gamma^* \psi_\gamma x^2 dx\, d\Omega} \quad \ldots(43)$$

We first assume $\gamma = 0$. The potentials can be extracted from Eqs (38, 40) and 41):

$$H_S = 0.10059(MeV)$$
$$H_I = 0.04782(MeV) \qquad \ldots(44)$$
$$H_{SI} = -0.50053(MeV)$$

Now, we can calculate the deuteron mass according to the following formula:

$$M_D = m_n + m_p + E_{\upsilon\gamma} + H_{int} \qquad \ldots(45)$$

By substituting[42] $m_n = m_p = 938(\text{MeV}) = 4.65(fm^{-1})$, energy eigenvalue ($E_{\upsilon\gamma}$) and the expectation values of H_{int} in Eq. (45), we have $M_D = 1877.83808$ (MeV/c^2). By comparing the experimental amount of deuteron mass ($M_D = 1875.612(\text{MeV}) = 9.29(fm^{-1})$) with our calculated mass for deuteron, we found that a good agreement has been obtained by our model[35].

5 Conclusions

In the present work, a new approach is offered for solving of Yukawa potential. We expand this potential around of its mesonic cloud that gets a new form with great powers and inverse exponent. The wave function of the Dirac equation for a new form of Yukawa potential has been calculted, including Coulomb-like tensor potential. By considering the effects of pertubative interaction potential, the theoretical and experimental masses are found to be in complete agreement. These improvement in reproduction of deuteron mass obtained by using a suitable form for confinement potential and exact analytical solution of the Dirac equation for our proposed potential. Finally, one can use this model for another nuclei by relativistic or non-relativistic equation and get the properties of them and amount of g the strength of nuclear force.

References

1 Motavali H, *Mod Phys Lett A*, 24 (2009) 1227.
2 Setare M R & Nazari Z, *Acta Phys Polonica B*, 40 (2009) 2809.
3 Zhang L H, Li X P & Jia C S, *Phys Lett A*, 372 (2008) 2201.
4 Ho C L, *Ann Phys*, 321 (2006) 2170.
5 Panahi H & Jahangiry L, *Int J Theor Phys*, 48 (2009) 3234.
6 Alhaidari A D, *Phys Lett A*, 326 (2004) 58.
7 Adame F D & Gonzalez M A, *Europhys Lett*, 13 (1990) 193.
8 Castro A S de & Hott M, *Phys Lett A*, 342 (2005) 53.
9 Chargui Y, Trabelsi A & Chetouani L, *Phys Lett A*, 374 (2010) 531.
10 Castro A S de, *Ann Phys*, 311 (2004) 70.
11 Guo J Y, Meng J & Xu F X, *Chin Phys Lett*, 20 (2003) 602.
12 Akcay H & Tezcan C, *Int J Mod Phys C*, 20 (2009) 931.
13 Akcay H, *Phys Lett A*, 373 (2009) 616.
14 Aydogdu O & Sever R, *Few-Boby Syst*, 47 (2010) 193.
15 Eshghi M, *Adv Studies Theor Phys*, 5 (2011) 559.
16 Mao G, *Phys Rev C*, 67 (2003) 044318.
17 Alberto P, Lisboa R, Malheiro M & Castro AS de, *Phys Rev C*, 71 (2005) 034313.
18 Furnstahl R F, Rusnak J J & Serot B D, *Nucl Phys A*, 632 (1998) 607.
19 Yukawa H, *Proc Phys Math Soc Jpn*, 17 (1937) 48, 19 (1935) 1084.
20 Moshinsky M & Szczepaniak A, *J Phys A Math Gen*, 22 (1989) 817.
21 Mao G, *Phys Rev C*, 67 (2003) 044318.
22 Bahlouli H, SAbdelmonem M & Nasser I M, *Phys Scr*, 82065005 (2010) 5.
23 Hassanabadi H, Yazarloo B H & Salehi N, *Indian J Phys*, 88 (2014) 405.
24 Lisboa R, Malheiro M, de Castro A S, Alberto P & Fiolhais M, *Phys Rev C*,69 (2004) 024319.
25 Aydogdu O & Sever R, *Phys Lett B*,703 (2011) 379.
26 Shojaei M R & Rajabi A A, *Iranian J of Phys Res*, l7 (2) (2007).
27 Rajabi A A, *Iranian J of Phys Res*, l7 (2) (2005).
28 Salehi N, Rajabi A A, *Modern Phys Lett A*, 24 (32) (2009) 2631.
29 AlhaidariA D, Bahlouli H & Al-hassan, *AwwwarXiv Org*, (2006) 31261.
30 Rajabi A A, *J Sci Islamic Rep of Iran*, 16(1) (2005) 73.
31 Salehi N & Rajabi A A, *Phys Scr*, 85 (2012) 055101.
32 Chen C Y, *Phys Lett A*, 339 (2005) 283.
33 Rajabi A A, *Theor Phys* (Beijing, China) 48 (2007) 151.
34 Salehi N, Rajabi A A & Ghalenovi Zahra, *Acta Physica Polonica B*, 42 (6) (2011) 1247.
35 Wong S M, Prentice Hall International Edition, USA, 1 (1998) 25.
36 Glozman L Ya, Plessas W, Varga K & Wagenbrunn R F, *Phys Rev D*, 58 (1998).
37 laRipelle R M Fabre De, *Nuclear Phys A*, 497 (1989) 595.
38 Sanctis M De, Giannini M M & Santopinto E & Vassallo A, *Eur Phys J A*, 19 (2004) 81.
39 Glozman L Ya & Riska D O, 268 (1996) 263.
40 Gianninia MM, Santopinto E & Vassallo A, *Eur Phys J,A* 12 (2001) 447.
41 Giannini M M, Santopinto E & Vassalo A, *Progress in particle and Nuclear physics*, 50 (2003) 263.
42 Beringer J *et al.* (Particle Data Group), *Phys Rev D,* 86 (2012) 010001.