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Effect of laser-pulse-shape on the transference of laser energy into the energy of fast ions in radiation pressure dominant regime

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The effect of laser-pulse temporal-profile on the transference of laser energy into the energy of fast ions when the radiation pressure is dominant, has been studied. The laser temporal profiles considered are Lorentzian, hyperbolic secant, and Gaussian. The numerical results are compared for different temporal profile of the laser pulse. It is found that the Lorenztian pulse is more effective to impart kinetic energy to the ions in comparison to Gaussian or hyperbolic secant laser pulse.

Keywords: Laser ion acceleration, Laser pulse shape dependence, Radiation pressure acceleration, Relativistic plasma mirror

1 Introduction

Radiation pressure mechanism is most efficient in imparting energy and momentum to the ions of a thin plasma foil. The result of interaction of ultra-intense few-cycle laser pulse with thin plasma foil has led to the possibility to generate energetic charged particles[1-12]. In fact relativistic non-linearities in a plasma allow us to manipulate and concentrate electromagnetic radiation in space and time. One such mechanism is provided by the coherent action of the particles in a plasma foil when they are made to reflect electromagnetic radiation as a mirror. It is assumed that the condition of collectiveness of the plasma is satisfied i.e. $n_e^3 >> 1$. This condition provides a strong enhancement of the plasma response as the radiation emitted by the plasma can in principle scale as the square of the number of particles instead of linearly. Under appropriate conditions, the reflection of ultra-intense electromagnetic radiation from a thin dense plasma foil can be exploited in order to push the mirror to relativistic energies. Vice-versa the energy density of a pulse interacting with a counter-propagating relativistic mirror can be amplified as the pulse frequency gets up-shifted and the pulse length shortened. The so-called radiation pressure acceleration (RPA) of ion bunches has recently attracted much interest as it may provide an efficient way of generating intense quasi-monoenergetic ion beams[4-6,9-12]. Here we are mostly interested in the process of ion acceleration in the interaction between a plasma foil and an ultra-intense laser pulse. Relativistic effects in a high energy plasma provide a matching condition that makes it possible to exchange very effectively ordered kinetic energy and momentum between the electromagnetic (EM) fields and the plasma (provided the plasma remains non-transparent). Suppose radiation momentum is absorbed (or reflected) by matter, then the relation between the (ordered kinetic) energy and the momentum gained by matter scales as v/c. Thus, the efficiency of laser energy conversion into ordered kinetic energy of matter tends to unity when matter moves at relativistic velocities. For direct acceleration of protons, laser intensity of the order of 10^{24}W/cm^2 is required. This corresponds to the normalized vector potential $a_0 = \frac{eE}{m_e \omega c} = \frac{m_p}{m_e} \approx 1836$, where E, ω are the electric field and frequency of the electromagnetic wave, e and m_e are the electron charge and mass, respectively and m_p is the proton mass.

The interaction of a relativistically strong ultra-short laser pulse irradiating a thin foil with thickness l and electron density n_e, is considered.

The laser pulse is assumed to be perfectly reflected from this mirror. In the lab-frame, before the reflection it has the energy ε, and after the reflection its energy becomes $\frac{\varepsilon}{4\gamma^2}$. As a result the energy acquired by the plasma mirror is ~ $\left(1-\frac{1}{4\gamma^2}\right)\varepsilon \approx \varepsilon$, as

$\gamma >> 1$. With the availability of the light pulses with duration comparable to the carrier oscillation cycle, we consider the different types of analytic pulse shape, $E(t) = A(t)\sin(\omega_c t + \psi)$; the envelopes are described by the Gaussian, Lorentzian and hyperbolic secant pulse[13,14]. This paper presents the study of the effect of laser pulse temporal profiles, number of cycles of the laser pulse and plasma density on the transformation of laser energy into the energy acquired by the fast ions when the radiation pressure is dominant. Numerical results show that the ion kinetic energy depends on time asymptotically as $t^{1/3}$.

2 Dynamics of the Thin Plasma-Slab

Under the action of the radiation pressure, the electrons are pushed instantaneously. Due to this, the intense electric field is created that drags ions at a speed almost equal to the speed of light. Because of the higher mass of the ions relative to electrons, the energy of the accelerated ions greatly exceeds that of the electrons. The ion energy $\varepsilon = \sqrt{m_i^2 c^4 + (eE_{II}\, c\, t)^2}$ becomes relativistic in a time duration ~ few-cycle laser pulses.

Under the radiation pressure the equation of motion for a thin plasma foil representing the ion momentum[4] can be written as:

$$\frac{d\,p}{d\,t} = \frac{E_L^2\left[t - x(t)/c\right]}{2\pi\, n_e\, l}\, |\rho(\omega')|^2\, \frac{\sqrt{m_i^2 c^2 + p^2} - p}{\sqrt{m_i^2 c^2 + p^2} + p} \qquad \ldots(1)$$

We obtain numerical results for Gaussian, Lorentzian, and hyperbolic secant temporal profile of the laser pulse.

3 Expression for Ion Kinetic Energy

The expression for ion kinetic energy is $= m_i c^2\left[\left(1 + \frac{c^2 p^2}{m^2 c^4}\right)^{\frac{1}{2}} - 1\right]$. Following the calculations of Ref. (4) we can write the solution of Eq. (1) as:

$$p = m_i\, c\left[\sinh u - \frac{1}{4\sinh u}\right]$$

where $u = (1/3)\times \operatorname{arcsin}h\left(\Omega\, t + h_0^3/2 + 3\, h_0/2\right)$. Thus, the Ion kinetic energy becomes:

$$= m_i\, c^2\left\{\left[\sinh u + \frac{1}{4}\cos ech\, u\right] - 1\right\}$$

$$= m_i\, c^2\left[\frac{4\sinh^2 u + 1 - 4\sinh u}{4\sinh u}\right]$$

$$\approx m_i\, c^2 \sinh u = m_i\, c^2\left(\frac{1}{4}\sinh 3u\right)^{1/3}.$$

Therefore:

$$\mathrm{E}_{i\,kin} \approx m_i\, c^2\left[\frac{1}{4}\Omega\, t\right]^{1/3} \approx m_i\, c^2\left[\frac{3\, E_L^2\, t}{8\,\pi\, n_e\, l\, m_i\, c}\right]^{1/3}.$$

To calculate an upper limit of the ion energy acquired due to the interaction with a laser pulse of finite duration, the dependence of the laser EM field on space and time is included. So we consider the dimensionless variable[5].

$$\psi = \int_{-\infty}^{t - \frac{x(t)}{c}} \frac{E_L^2(\xi)}{4\,\pi\, n_e\, l\, m_i\, c}\, d\xi$$

which can be considered as the normalized energy of the laser pulse that has been interacting with the moving foil by time *t*. Its maximum value is $\max\{\psi\} = \frac{\mathrm{E}_L}{N_i\, m_i\, c^2}$, where E_L is the laser pulse energy, N_i is the number of ions in the foil. The ion maximum energy and the acceleration efficiency in the model of flat foil driven by the E M radiation pressure, have been estimated.

4 Results and Discussion

For obtaining numerical results, we consider laser pulses with duration comparable to carrier oscillation cycles. Following Brabec and Krausz[13] that the concept of envelope can be extended to pulse duration equal to the carrier oscillation period, we consider various analytic pulse shapes, $E(t) = A(t)\sin(\omega_c\, t + \psi)$ the envelopes are described by a Gaussian, $A_g(t) = A_0\exp\left[-\left(\frac{1.67\,t}{\tau}\right)^2\right]$ Lorentzian

$$A_L(t) = A_0\left[\frac{1}{1 + \left(\frac{1.29\,t}{\tau}\right)^2}\right],$$ and hyperbolic secant

temporal profile, $A_h(t) = A_0 \sec h\left(\frac{1.76\,t}{\tau}\right)$. The radiation pressure on the foil depends on the pulse shape and also its reflectance from the foil.

Figure 1 shows the variation of the momentum acquired by the ions as a function of time when the incident laser-pulse is Gaussian. Fig. 1 shows the variation when the two-cycle laser pulse is incident on a thin film of plasma density n=10 n_{cr}, 12 n_{cr}, 15 n_{cr}, where n_{cr}=1.1×10^{21} cm^{-3}. Plasma layer thickness is 1λ and normalized vector potential is a_0=1836. Figure 2 shows the variation of the momentum acquired by the ions as a function of time when the incident laser-pulse is Lorentzian. Figure 2 shows the variation of momentum when the two-cycle laser pulse is incident on a thin film having plasma density n=10 n_{cr}, 12 n_{cr}, 15 n_{cr}, where n_{cr}=1.1×10^{21} cm^{-3}. Figure 3 shows the variation of the momentum acquired by the ions as a function of time when the incident laser-pulse temporal profile is hyperbolic secant. The thin film plasma density is assumed to be n=10 n_{cr}, 12 n_{cr}, 15 n_{cr}, where n_{cr}=1.1×10^{21} cm^{-3}. Figure 4 shows a comparative study of the momentum acquired by the ions when the incident laser pulses are assumed to be Gaussian, Lorentzian, and hyperbolic secant. The parameters are: plasma density n=10 n_{cr}, n_{cr}=1.1×10^{21} cm^{-3}, normalized vector potential is a_0=1836 and plasma layer thickness is 1λ.

Figures 5-7 show the energy gained by the ions and its variation on the laser pulse cycles. Figure 5 shows the variation of the energy acquired by the ions as a function of time when the incident laser-pulse is Gaussian. Figure 5 shows the variation when the incident laser pulse has 1, 2, 3 cycles per laser pulse. Figure 6 shows the variation of the energy acquired by the ion as a function of time when the

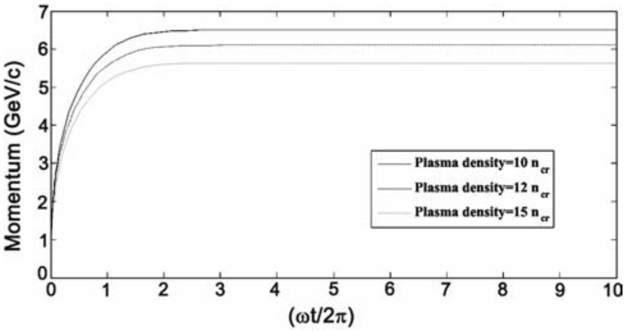

Fig. 1 — Effect of the plasma density on the momentum acquired by the ions as a function of time for a two cycle laser pulse having Gaussian temporal profile. The plasma layer thickness is 1λ, normalized vector potential is a_0=1836, and variation in plasma density n=10 n_{cr}, 12 n_{cr}, 15 n_{cr}, where n_{cr}=1.1×10^{21} cm^{-3}

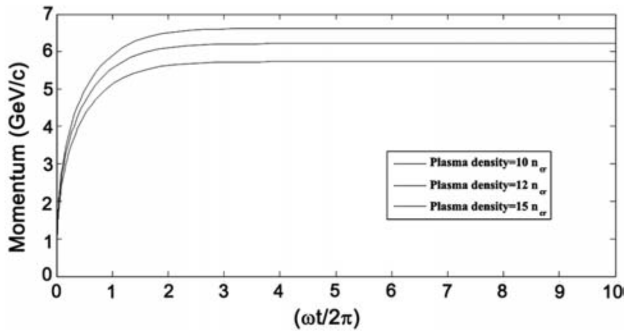

Fig. 3 — Variation of the momentum acquired by the ions as a function of time for two cycle laser pulse having Hyperbolic secant temporal profile. The parameters are: plasma layer thickness is 1λ, normalized vector potential is a_0=1836 and variation in plasma density n=10 n_{cr}, 12 n_{cr}, 15 n_{cr}, where n_{cr}=1.1×10^{21} cm^{-3}

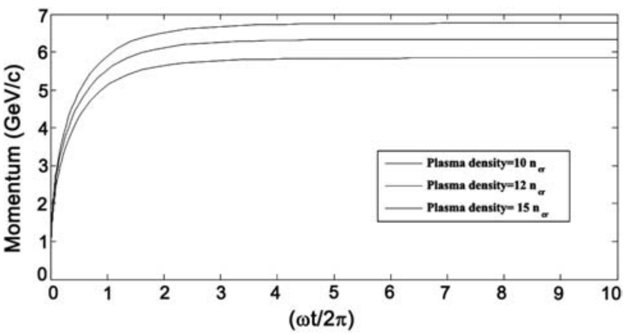

Fig. 2 — Variation of the plasma density on the momentum acquired by the ions as a function of time for a two cycle laser pulse having Lorentzian temporal profile. The parameters are: plasma layer thickness is 1λ, normalized vector potential is a_0=1836 and variation in plasma density n=10 n_{cr}, 12 n_{cr}, 15 n_{cr}, where n_{cr}=1.1×10^{21} cm^{-3}

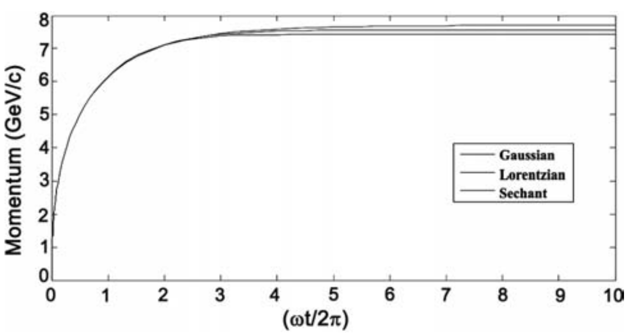

Fig. 4 — Variation of the momentum acquired by the ions as a function of time for a three cycle laser pulse. The temporal profiles of the laser pulse are assumed to be Gaussian, Lorentzian, and Hyperbolic Secant. The parameters are: plasma density n=10 n_{cr}, n_{cr}=1.1×10^{21} cm^{-3}, normalized vector potential is a_0=1836 and plasma layer thickness is 1λ

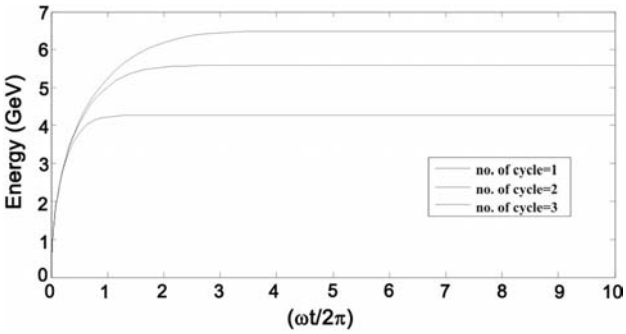

Fig. 5 — Variation of the ion energy as a function of time when the incident laser pulse has 1, 2, 3 cycles per laser pulse. The laser pulse temporal profile is Gaussian. The parameters are: plasma density n=10 n_{cr}, n_{cr}=1.1×10^{21} cm^{-3}, normalized vector potential a_0=1836 and plasma layer thickness 1λ

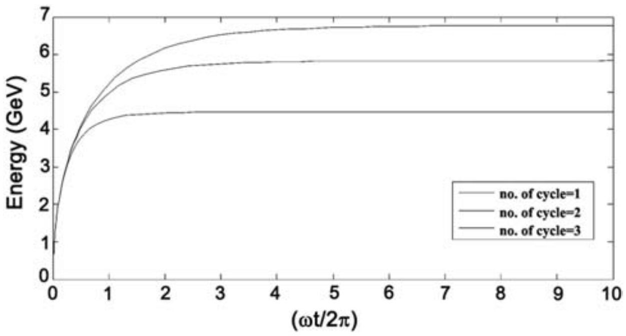

Fig. 6 — Variation of the ion energy as a function of time when the incident laser pulse has 1, 2, 3 cycles per laser pulse. The laser pulse temporal profile is Lorentzian. The parameters are: plasma density n=10 n_{cr}, n_{cr}=1.1×10^{21} cm^{-3}, normalized vector potential is a_0=1836 and plasma layer thickness is 1λ

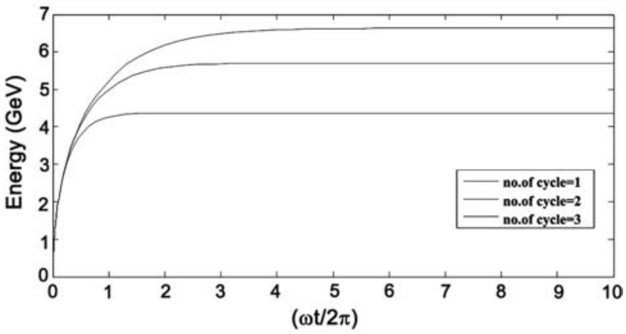

Fig. 7 — Variation of the ion energy as a function of time when the incident laser pulse has 1, 2, 3 cycles per laser pulse. The laser pulse temporal profile is Hyperbolic Secant. The parameters are: plasma density n=10 n_{cr}, n_{cr}=1.1×10^{21} cm^{-3}, normalized vector potential is a_0=1836 and plasma layer thickness is 1λ

incident laser-pulse is Lorentzian. Figure 6 shows the variation when the incident laser pulse has 1, 2, 3 cycles per laser pulse. Figure 7 shows the variation of the energy acquired by the ions as a function of time when the incident laser-pulse is hyperbolic secant.

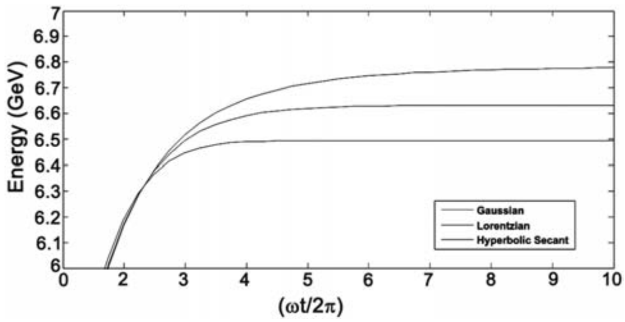

Fig. 8 — Variation of the ion energy as a function of time for a three cycle Gaussian, Lorentzian, and Hyperbolic Secant laser pulse. The parameters are: plasma density n=10 n_{cr}, n_{cr}=1.1×10^{21} cm^{-3}, normalized vector potential a_0=1836 and plasma layer thickness 1λ

Figure 7 shows the variation when the incident laser pulse has 1, 2, 3 cycles per laser pulse. Figure 8 shows a comparative study of the energy acquired by the ions when the incident laser pulses are assumed to be Gaussian, Lorentzian, and hyperbolic secant. The parameters are: plasma density n=10 n_{cr}, n_{cr}=1.1×10^{21} cm^{-3}, normalized vector potential is a_0=1836 and plasma layer thickness is 1λ.

5 Conclusions

The numerical results show that the kinetic energy gained by the ions is the maximum when the incident laser pulse temporal profile is Lorentzian. As a comparison, the energy gained by the ions by an incident Lorentzian, hyperbolic secant, Gaussian laser pulse is found to be 6.78, 6.64, 6.50 GeV, respectively. The time of acceleration for these energies is 1.8×10^{-14} s for Lorentzian and 1.5×10^{-14} s of hyperbolic secant, and 1.1×10^{-14} s for Gaussian profile. It is concluded that the Lorenztian pulse is more effective to accelerate the ions as compared to Gaussian or hyperbolic secant laser pulse.

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