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Analytical computation of unsteady MHD mixed convective heat transfer over a vertical stretching plate with partial slip conditions

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The analysis of the unsteady MHD mixed convective flow and heat transfer over an impulsively stretched permeable vertical surface in a moving fluid with partial velocity slip and thermal slip conditions in the presence of thermal radiation, internal heat absorption or generation, and injection or suction, has been studied in the present paper. The governing boundary layer equations are converted into a system of nonlinear coupled ordinary differential equations by suitable similarity transformations. The appropriate analytical solutions for the velocity and temperature fields are gotten by the DTM-BF which is an analytical method based on the differential transformation method (DTM) and basis functions. The results obtained by the DTM-BF are in good agreement with those presented by the numerical method. The effects of the various parameters which determine the velocity and temperature fields are shown by plotting graphs and discussed in detail.

Keywords: Unsteady MHD flow, Heat transfer, Analytical solution, Slip condition

1 Introduction

The momentum and thermal transport due to a stretching heated surface in a quiescent or moving fluid is of great practical interest because it occurs in a number of engineering processes such as hot rolling, metal and plastic extrusion, continuous casting, and glass fiber and paper production. The study of the flow and heat transfer adjacent to the moving surface is necessary for determining the quality of the final products of these engineering processes. Many aspects of the steady boundary layer flow and heat transfer over a horizontal moving surface have been studied by many researchers¹⁻⁵.

The mixed convective flow of different types of fluids over a stretching vertical sheet under some conditions has been investigated. Karwe and Jaluria^{6,7} have analyzed mixed convective flow over a continuous moving plate in material processes. Patil *et al*⁸ have studied the mixed convection effects over a moving vertical stretching plate in a parallel free stream. Al-Sanea⁹ discussed the effects of suction or injection on the behaviour of the steady laminar flow and heat transfer over a continuously moving vertical wall of extruded material. The MHD flow of a viscous incompressible and electrically conducting fluid over a stretching vertical surface with constant wall temperature was analyzed by Ishak et al¹⁰. Shateyi¹¹ considered steady MHD flow of a Maxwell fluid past a vertical stretching sheet in a Darcian porous medium.

All the investigations cited above considered are restricted to steady state conditions. It is important to include unsteadiness into the governing equations of any problem for the development of a more physically realistic characterization of the flow configuration. Relating to the unsteady flows and heat transfer over horizontal stretching sheets, some important investigations can be found in Refs 12-19. Recently, Mukhopadhyay²⁰ performed an analysis to investigate the effect of thermal radiation on unsteady mixed convective boundary layer flow and heat transfer over a vertical porous stretching surface in a quiescent fluid by a numerical method. Kumari and Nath²¹ investigated the effects of the magnetic field and injection/suction on the unsteady mixed convective flow and heat transfer of an incompressible electrically conducting fluid over an impulsively stretched permeable vertical surface in an unbounded quiescent fluid by solving the problem analytically using the homotopy analysis method and numerically by Keller box method. Ram $et al^{22}$. studied the effects of porosity on unsteady MHD flow past a semiinfinite vertical stretching plate with time dependent suction.

The no-slip boundary condition is one of the central tenets of the Navier-Stokes theory. However, there are some situations wherein this condition does not hold. Partial velocity and temperature slip may occur on the surface of the stretching sheet when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions. Recently, Mukhopadhyay and Andersson²³ have investigated the effects of the partial velocity and temperature slip on the flow and heat transfer over a horizontal unsteady stretching surface submerged in a quiescent fluid by numerical method. Bhattacharyya *et al*²⁴ and Mukhopadhyay²⁵ investigated slip effects on the flow and heat transfer towards an unsteady stretching sheet in a quiescent fluid.

The DTM method is an approximate analytic method for many types of nonlinear problems²⁶⁻²⁹. At the same time, it is usually difficult to obtain analytical solutions of the differential equations in an unbounded domain because the results obtained by the DTM are usually valid only in a small region. The main reason is that the series solutions obtained by the DTM are divergent as the variables of the problems go to infinity. In order to overcome this difficulty, Su *et al*³⁰. proposed an analytical method named DTM-BF which is a combination of the DTM and the approach by using base functions, and successfully solved the problem of the steady MHD mixed convective heat transfer over a permeable stretching wedge.

The DTM-BF is applied to consider the unsteady MHD mixed convective flow and heat transfer over an impulsively stretched permeable vertical surface in a moving fluid with partial velocity slip and thermal slip conditions. In the present paper, the effects of velocity slip, thermal slip, the ratio of the velocity of the stretching surface to that of the ambient fluid, thermal buoyancy, magnetic field, and thermal radiation on the momentum and heat transfer characteristics have been analyzed.

2 Mathematical Formulation

Consider an unsteady two-dimensional MHD boundary layer flow and heat transfer over a continuous stretching vertical sheet embedded in a moving viscous, incompressible, electrically conducting fluid, as shown in Fig. 1. The sheet is stretching with a velocity $U_w = ax(1-ct)^{-1}$ in the positive x direction^{20,23}. The free stream velocity far away from the sheet is $U_{\infty} = RU_{w}$, where $R \ge 0$, a > 0, c > 0 and ct < 1. The fluid is under the influence of the magnetic field B which acts in the direction normal to the stretching sheet. The induced magnetic field is negligible, which is a valid assumption on a laboratory scale under the assumption of small magnetic Reynolds number. It is also assumed that the external electric field is zero. Under these assumptions, the basic unsteady boundary layer equations governing the transfer of momentum and heat in the presence of thermal buoyancy, thermal radiation and internal absorption/generation take the following form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \dots (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho} u \qquad \dots (2)$$
$$+ g \beta_T (T - T_\infty)$$

$$\rho c_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + Q(T - T_{\infty})$$
... (3)

subject to boundary conditions:

$$u = U_w + D_u v \frac{\partial u}{\partial y}, \quad v = V_w, \quad T = T_w + D_T \frac{\partial T}{\partial y}$$
 at
 $y = 0$ (4)

$$u = U_{\infty}, T = T_{\infty}$$
 as $y = \infty$...(5)



Fig. 1 — Schematic representation of the physical model and coordinates system

where $\frac{\partial p}{\partial x} = -\rho \frac{\partial U_{\infty}}{\partial t} - \rho U_{\infty} \frac{\partial U_{\infty}}{\partial x} - \sigma B^2 U_{\infty}$ is pressure gradient, x and y axe coordinates measure along and normal to the surface, respectively. u and v are the velocity components along x and y directions, respectively, t the time, σ the electrical conductivity, T the temperature inside the boundary layer, g the gravity field, β_T the volumetric coefficient of thermal expansion, c_p the specific heat at constant pressure, α the thermal conductivity, μ the fluid viscosity, $v = \mu / \rho$ is the kinematics viscosity of the fluid, ρ the density of fluid, $T_w = T_\infty + ax^2 (2\nu)^{-1} (1-ct)^{-2}$ is the temperature of the stretching sheet, T_{∞} the temperature of the fluid outside the boundary layer, $D_u = D_{u0}(1-ct)^{1/2}$ is the velocity slip factor which changes with time, $D_T = D_{T0}(1-ct)^{1/2}$ is the thermal slip factor which also changes with time, D_{u0} and D_{T0} are the initial values of velocity and thermal slip factors, $v_w = -C(vU_w)^{1/2} x^{-1/2}$ respectively. The term represents the mass transfer on the sheet with C < 0for injection and C > 0 for suction. Q is the heat generation when Q > 0 or heat absorption when Q < 0. The variable magnetic field B is of the form $B = B_0 U_w^{1/2} (vx)^{-1/2}$.

The radiative heat flux q_r under Rosseland approximation has the form:

$$q_r = -\frac{4\sigma^1}{3k} \frac{\partial T^4}{\partial y} \qquad \dots (6)$$

where σ^1 and k are the Stefan-Boltzman constant and the mean absorption coefficient, respectively. The temperature difference within the flow is assumed to be sufficiently small so that T^4 may be expressed as a linear function of temperature, thus:

$$T^4 \approx 4T_{\infty}^3 T - 3T_{\infty}^4 \qquad \dots (7)$$

Based on Eq. (7), Eq. (6) becomes:

$$q_r = -\frac{16T_{\infty}^3 \sigma^1}{3k^1} \frac{\partial T}{\partial y} \qquad \dots (8)$$

In order to obtain similarity solutions of the problem, we introduce the following dimensionless variables:

$$\eta = U_w^{1/2} (\nu x)^{-1/2} y, \ \psi(x, y) = (\nu x U_w)^{1/2} f(\eta),$$

$$\theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}} \qquad \dots (9)$$

where $\psi(x, y)$ is the stream function that satisfies the continuity Eq. (1). Because of $u = \partial \psi / \partial y$ and $v = -\partial \psi / \partial x$, the following equations can be immediately obtained:

$$u = U_w f'(\eta)$$
 and $v = -(vU_w)^{1/2} x^{-1/2} f(\eta)$...(10)

Substituting Eqs. (8 and 9) into Eqs (2 to 5), the present problem can be expressed as:

$$f''' + ff'' - f'^{2} - A\left(f' + \frac{\eta}{2}f'' - R\right) - Mn(f' - R) \dots (11)$$
$$+ \gamma \theta + R^{2} = 0$$

$$Pr^{-1}(1+Nr)\theta'' + (\lambda - 2A)\theta - \frac{A}{2}\eta\theta' - \theta f' + f\theta' = 0$$
...(12)

with the boundary conditions:

$$f(0) = C, f'(0) = 1 + h_u f''(0),$$

$$\theta(0) = 1 + h_T \theta'(0) \qquad \dots (13)$$

$$f'(\infty) = R, \ \theta(\infty) = 0 \qquad \dots (14)$$

In Eqs (7 to 14), A = c/a is the dimensionless measure of the unsteadiness and the prime indicates differentiation with respect to η , $\gamma = g\beta_T / (2va)$ is the mixed convection parameter, $Pr = \mu c_p / \alpha$ is the Prandtl number, A is a parameter that measures the unsteadiness, $Nr = 16T_{\infty}^3 \sigma^1 / (3k\alpha)$ is the thermal radiation parameter, $Mn = \sigma B_0^2 / (\rho v)$ is the magnetic parameter, $Re_x = U_w x / v$ is the local Reynolds number, $Re_\alpha = U_w \sqrt{\alpha} / v$, $\lambda = Q\alpha Re_x / (\mu c_p Re_\alpha^2)$ is the heat source parameter. In addition, $\lambda > 0$ corresponds to heat generation, $\lambda > 0$ corresponds to

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heat absorption, $h_u = D_{u0}(av)^{1/2}$ is the dimensionless velocity slip parameter and $h_T = D_{T0}(av^{-1})^{1/2}$ is the dimensionless thermal slip parameter.

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x , which are defined as:

$$C_f = \frac{\tau_w}{\rho U_w^2 / 2}, \ Nu_x = \frac{xq_w}{\alpha (T_w - T_\infty)}$$

where the wall shear stress τ_w and the wall heat flux q_w are given by:

$$\tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}, \ q_w = -\alpha \left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Using the similarity variables as in Eq. (9), we obtain:

$$C_f R e_x^{1/2} / 2 = f''(0), N u_x / R e_x^{1/2} = -\theta'(0)$$

3 Solutions of DTM-BF

Now, we apply the DTM-BF, which was elaborated by Su *et al*³⁰., for solving the boundary value problems as in Eqs. (11-14). For this purpose, we first utilize the DTM to solve the IVP for Eqs. (11) and (12) under the following initial value conditions :

$$f(0) = C, f'(0) = 1 + 2h_u\beta_1, f''(0) = 2\beta_1 \dots (15)$$

$$\theta(0) = 1 + h_T \beta_2, \ \theta'(0) = \beta_2 \qquad \dots (16)$$

The corresponding differential transformations of the conditions given in Eqs (15 and 16) are:

$$F(0) = C, F(1) = 1 + 2h_{u}\beta_{1}, F(2) = \beta_{1} \qquad \dots (17)$$

$$\Theta(0) = 1 + h_T \beta_2, \ \Theta(1) = \beta_2 \qquad \dots (18)$$

where β_1 and β_2 are two parameters whose values are to be determined through the rest of the problemsolving process. Implementing the differential transformation for Eqs. (11 and 12) by using the fundamental operations of the DTM presented in the literature²⁶, we get the following iterative formulas of F(k) and $\Theta(k)$ which are the differential transformation functions of $f(\eta)$ and $\theta(\eta)$, respectively.

$$\Theta(k+2) = \frac{\begin{cases} (A + Mn + kA/2)(k+1)F(k+1) \\ -(AR + MnR + R^2)\delta(k) + \gamma\Theta(k) + \\ \sum_{i=0}^{k} \left[(i+1)(k-i+1)F(i+1)F(k-i+1) \\ -(k-i+1)F(i+1)F(k-i+1) \\ -(k-i+1)(k-i+2)F(i)F(k-i+2) \end{bmatrix} \right]}{(k+1)(k+2)(k+3)}$$

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In this way all the terms of F(k) and $\Theta(k)$ can be easily calculated by substituting Eqs (17) and (18) into the above iterative formulas. Then we can get the solutions of the IVP consisting of Eqs (11 and 12) and the conditions given in Eqs (15 and 16) in power series forms, i.e.,

$$f(\boldsymbol{\eta}) = \sum_{k=0}^{\infty} F(k) \boldsymbol{\eta}^k \approx \sum_{k=0}^{n} F(k) \boldsymbol{\eta}^k \qquad \dots (19)$$

$$\theta(\eta) = \sum_{i=0}^{\infty} \Theta(i) \eta^{i} \approx \sum_{i=0}^{m} \Theta(i) \eta^{i} \qquad \dots (20)$$

The next step is to express the solutions to the coupled BVP as in Eqs (11-14) as a linear form of basis functions. According to Eqs (11 and 12) and the boundary conditions [Eqs (13 and 14)], it is reasonable to assume that $f(\eta)$ and $\theta(\eta)$ are expressed by the following sets of basis functions, respectively.

$$\{f_{0,0}(\eta), f_{i,j}(\eta)_{(i=1,2,3,\cdots,j=1,2,3,\cdots,)} \} \text{ and}$$

$$\{\theta_{0,0}(\eta), \theta_{i,j}(\eta)_{(i=1,2,\cdots,j=1,2,\cdots)} \} \text{ in the form :}$$

$$f(\eta) \approx f_{N_1,N_2}(\eta) = f_{0,0}(\eta) + \sum_{j=3}^{N_1} \sum_{i=1}^{N_2} b_{i,j} f_{i,j}(\eta)$$

$$= f_{0,0}(\eta) + \sum_{j=3}^{N_1} \sum_{i=1}^{N_{j,2}} b_{i,j} \eta^j e^{ia_0\eta}$$

$$\dots (21)$$

$$\begin{aligned} \theta(\eta) &\approx \theta_{N_3, N_4}(\eta) = \theta_{0, 0}(\eta) + \sum_{j=2}^{N_3} \sum_{i=2}^{N_4} d_{i, j} \theta_{i, j}(\eta) \\ &= \theta_{0, 0}(\eta) + \sum_{j=2}^{N_3} \sum_{i=1}^{N_{j, 4}} d_{i, j} \eta^j e^{i\gamma_0 \eta} \\ &\dots (22) \end{aligned}$$

where

$$\begin{split} f_{0,\ 0}(\eta) &= C - H + R\eta + He^{a_0\eta} + b_1\eta e^{a_0\eta} + b_2\eta^2 e^{a_0\eta} \\ \text{and } \theta_{0,\ 0}(\eta) &= Le^{\gamma_0\eta} + d_1\eta e^{\gamma_0\eta} \\ (H &= (1 + 2h_u b_0 a_0 + 2b_2 h_u - b_0 - R) / (a_0 - h_u a_0^2) , \\ L &= (1 + h_T d_1) / (1 - h_T \gamma_0)) \text{ respectively satisfy the inhomogeneous boundary conditions given in Eqs (13 and 14). Besides that, the functions $f_{i,j}(\eta) = b_{i,j}\eta^j e^{ia_0\eta} \quad (i = 1, 2, 3, \cdots, j = 3, 4, \cdots) \text{ and } \\ \theta_{i,j}(\eta) &= d_{i,j}\eta^j e^{i\gamma_0\eta} \quad (i = 1, 2, 3, \cdots, j = 2, 3, \cdots) \text{ obey the homogeneous boundary conditions, respectively.} \end{split}$$$

$$f(0) = 0, f'(0) = 0, f'(\infty) = 0$$
 ...(23)

$$\theta(0) = 0, \ \theta'(\infty) = 0 \qquad \dots (24)$$

In which $a_0 < 0$ and $\gamma_0 < 0$ are two undetermined decaying parameters. In order to reduce the amount of computation, we truncate the series $f_{N_1, N_2}(\eta)$ and $\theta_{N_3, N_4}(\eta)$ to $N_i \le 4$ (i = 1, 2, 3, 4). In the present paper, we choose $N_1 = 4$, $N_2 = 3$, $N_{j, 2} = 2$ (j = 3, 4), $N_{2, 4} = 3$, $N_{3, 4} = 2$. Then, we expand the right sides of Eqs (21 and 22) as power series of η :

$$f(\eta) = C - H + R\eta + \left(\frac{Ha_0^2}{2!} + b_1a_0 + b_2\right)\eta^2 + \left(\frac{Ha_0^3}{3!} + \frac{b_1a_0^2}{2!} + b_2a_0 + \sum_{i=1}^2 b_{i,3}\right)\eta^3 + \left(\frac{Ha_0^4}{4!} + \frac{b_1a_0^3}{3!} + \frac{b_2a_0^2}{2!} + \sum_{i=1}^2 ia_0b_{i,3} + \sum_{i=1}^2 b_{i,4}\right)\eta^4 + \cdots$$
(25)

$$\theta(\eta) = L + (L\gamma_0 + d_1)\eta^2 + \left(\frac{L\gamma_0^2}{2!} + d_1\gamma_0 + \sum_{i=1}^3 d_{i,2}\right)\eta^2 + \left(\frac{L\gamma_0^3}{3!} + \frac{d_1\gamma_0^2}{2!} + \sum_{i=1}^3 i\gamma_0 d_{i,2} + \sum_{i=1}^2 d_{i,3}\right)\eta^3 + \left(\frac{L\gamma_0^4}{4!} + \frac{d_1\gamma_0^3}{3!} + \sum_{i=1}^3 \frac{(i\gamma_0)^2 d_{i,2}}{2!} + \sum_{i=1}^2 i\gamma_0 d_{i,3}\right)\eta^4 + \cdots$$
... (26)

In view of Eqs (19), (20), (25) and (26), the following equations yield:

$$\sum_{i=0}^{\infty} F(i)\eta^{i} = C - H + R\eta + \left(\frac{Ha_{0}^{2}}{2!} + b_{1}a_{0} + b_{2}\right)\eta^{2} \\ + \left(\frac{Ha_{0}^{3}}{3!} + \frac{b_{1}a_{0}^{2}}{2!} + b_{2}a_{0} + \sum_{i=1}^{2}b_{i,3}\right)\eta^{3} + \cdots$$

$$\dots (27)$$

$$\sum_{i=0}^{\infty} \Theta(i)\eta^{i} = L + (L\gamma_{0} + d_{1})\eta + \left(\frac{L\gamma_{0}^{2}}{2!} + d_{1}\gamma_{0} + \sum_{i=1}^{3}d_{i,2}\right)\eta^{2} \\ + \left(\frac{L\gamma_{0}^{3}}{3!} + \frac{d_{1}\gamma_{0}^{2}}{2!} + \sum_{i=1}^{3}i\gamma_{0}d_{i,2} + \sum_{i=1}^{2}d_{i,3}\right)\eta^{3} + \cdots$$

$$\dots (28)$$

By comparing the coefficients of like powers of η on both sides of Eqs. (27 and 28), we get a system of algebraic equations:

$$\begin{split} &\frac{Ha_0^2}{2!} + b_1a_0 + b_2 = F(2) \\ &\frac{Ha_0^3}{3!} + \frac{b_1a_0^2}{2!} + b_2a_0 + \sum_{i=1}^2 b_{i,3} = F(3) \\ &\frac{Ha_0^j}{j!} + \frac{b_1a_0^{j-1}}{(j-1)!} + \frac{b_2a_0^{j-2}}{(j-2)!} + \\ &\sum_{i=1}^2 \frac{(ia_0)^{j-3}b_{i,3}}{(j-3)!} + \sum_{i=1}^2 \frac{(ia_0)^{j-4}b_{i,4}}{(j-4)!} \\ &= F(j) \quad (j = 4, 5, 6, 7, 8, 9) \\ &L\gamma_0 + d_1 = \Theta(1) \\ &\frac{L\gamma_0^2}{2!} + d_1\gamma_0 + \sum_{i=1}^3 d_{i,2} = \Theta(2) \\ &\frac{L\gamma_0^j}{j!} + \frac{d_1\gamma_0^{j-1}}{(j-1)!} + \sum_{i=1}^3 \frac{(i\gamma_0)^{j-2}d_{i,2}}{(j-2)!} \gamma_0 d_{i,2} + \\ &\sum_{i=1}^2 \frac{(i\gamma_0)^{j-3}d_{i,3}}{(j-3)!} = \Theta(j) \quad (j = 3, 4, 5, 6, 7, 8) \end{split}$$

The values of the undetermined parameters $\beta_1 = f''(0)/2$, $\beta_2 = \theta'(0)$, a_0 , γ_0 , b_1 , b_2 , $b_{i,j}$ (i = 1, 2, j = 3, 4), d_1 , $d_{i,2}$ (i = 1, 2, 3) and $d_{i,3}$ (i = 1, 2) can be obtained by solving the above nonlinear algebraic equations. Finally, after substituting the values of the above parameters into

the expressions given in Eqs (21) and (22), then we get the DTM-BF solutions of the coupled BVP in Eqs (11)-(14). For example, the DTM-BF solutions to the coupled BVP given in Eqs (11-14) for $h_u = 0.1$, $h_T = 0.1$, C = 0.5, A = 1.2, Mn = 1, R = 2, Nr = 1, Pr = 1, $\gamma = 1$ and $\lambda = -1$ are:

$$f(\eta) = 0.2706846474 + 2\eta$$

$$+0.2293153526e^{a_0\eta}$$

- $-0.1218970384\eta e^{a_0\eta}$
- $-0.08994285063\eta^2 e^{a_0\eta}$
- $+\eta^3(0.05194808802e^{a_0\eta})$
- $+\,0.001149181495e^{2a_0\eta})$
- $+\eta^4(0.03739482413e^{a_0\eta})$
- $+0.0007061715022e^{2a_0\eta})$

 $\theta(\eta) = 0.8475608399e^{\gamma_0 \eta}$

 $-0.1259578390 \eta e^{\gamma_0 \eta}$ $+ \eta^2 (-0.1968616308 e^{\gamma_0 \eta}$ $+ 0.006710952631 e^{2\gamma_0 \eta}$ $-0.0007715643591 e^{3\gamma_0 \eta})$ $+ \eta^3 (0.05523751050 e^{\gamma_0 \eta}$ $+ 0.01023535446 e^{2\gamma_0 \eta})$

where $a_0 = -2.815590725$ and $\gamma_0 = -1.649950889$. The values of the skin friction coefficient and wall temperature gradient are f''(0) = 1.162223911 and $\theta'(0) = -1.524391601$, respectively.

4 Results and Discussion

In order to validate the accuracy of the results obtained by the DTM-BF, we also solved the BVP Eqs (11-14) numerically by using classical fourthorder Runge-Kutta scheme along with the conventional shooting method which was used successfully in the paper³¹. All the results obtained by the DTM-BF and the numerical method are presented in Tables 1 and 2 and Figs. 2 to 12, which presents a comparison between the two methods. From these Figs 2 to 12, it is observed that the results obtained by the DTM-BF and the numerical method are in very close agreement. The effects of the velocity slip parameter h_u and thermal slip parameter h_T are presented in Tables 1-2 and Figs. 2 and 3. It is shown from Table 1 and Fig. 2 that the values of the skin friction coefficient f''(0),

Table 1 — Values of
$$f''(0)$$
 and $-\theta'(0)$ for various h_u when $h_T = 0.1$, $\gamma = 1$, $\lambda = -1.0$, $Pr = 1.0$, $Mn = 1$, $C = 0.5$, $R = 2$, $Nr = 1$, $A = 1.2$

h _u	<i>f</i> "(0)		$-\boldsymbol{\theta}'(0)$	
	DTM-BF	Numerical	DTM-BF	Numerical
0	2.92171594	2.92322479	1.50281899	1.50286053
0.1	2.32444782	2.32530213	1.52439160	1.52413116
0.3	1.63522133	1.63546598	1.54774617	1.54733285
0.5	1.25659190	1.25660942	1.55996283	1.55945683
1.0	0.79329937	0.79315720	1.57438559	1.57375406
2.0	0.45558765	0.45549001	1.58452632	1.58383503
5.0	0.19921520	0.19979661	1.59118162	1.59138384

Table 2 — Values of f''(0) and $-\theta'(0)$ for various h_T when $h_u = 0.1$, $\gamma = 1$, $\lambda = -1.0$, Pr = 1.0, Mn = 1, C = 0.5,

R = 2, $Nr = 1$, $A = 1.2$	
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h_T	f''(0)		$-\theta'(0)$	
	DTM-BF	Numerical	DTM-BF	Numerical
0	2.35425155	2.35500028	1.79981199	1.79942782
0.2	2.30252294	2.30344703	1.32225506	1.32206776
0.4	2.27241089	2.27342336	1.04528717	1.04517332
0.7	2.24518688	2.24626532	0.79552646	0.79546249
1.0	2.22855042	2.22955081	0.64215868	0.64211780
2.0	2.20233839	2.20211239	0.39099060	0.39097693



Fig. 2 — Velocity profiles for various values of h_u and R when $h_T = 0.1$, $\gamma = 1$, $\lambda = -1.0$, Pr = 1.0, Mn = 1, C = 0.5, Nr = 1, A = 1.2



Fig. 3 — Temperature profiles for various values of h_r when $h_u = 0.1$, $\gamma = 1$, $\lambda = -1.0$, Pr = 1.0, Mn = 1, C = 0.5, R = 2, Nr = 1, A = 1.2



Fig. 4 — Velocity profiles for various values of *R* when $h_u = 0.1$, $h_T = 0.1$, $\gamma = 1$, $\lambda = -1.0$, Pr = 1.0, Mn = 1, C = 0.5, Nr = 1, A = 1.2



Fig. 5 — Temperature profiles for various values of *R* when $h_u = 0.1$, $h_T = 0.1$, $\gamma = 1$, $\lambda = -1.0$, Pr = 1.0, Mn = 1, C = 0.5, Nr = 1, A = 1.2



Fig. 6 — Velocity profiles for various values of γ when $h_u = 0.1$, $h_T = 0.1$, $\lambda = -1.0$, Pr = 1.0, Mn = 1, C = 0.5, Nr = 1, A = 1.2, R = 1.2



Fig. 7 — Velocity profiles for various values of A when $h_u = 0.1$, $h_T = 0.1$, $\gamma = 1$, $\lambda = -1.0$, Pr = 4.0, Mn = 1, C = 0.5, Nr = 1, R = 1.2



Fig. 8 — Temperature profiles for various values of A when $h_u = 0.1$, $h_T = 0.1$, $\gamma = 1$, $\lambda = -1.0$, Pr = 4.0, Mn = 1, C = 0.5, Nr = 1, R = 1.2



Fig. 9 — Velocity profiles for various values of Mn and R when $h_u = 0.1$, $h_T = 0.1$, $\lambda = -1.0$, Pr = 1.0, Nr = 1, $\gamma = 1$, A = 1.2, C = 0.5



Fig. 10 — Temperature profiles for various values of Pr when $h_u = 0.1$, $h_T = 0.1$, Nr = 1.0, $\lambda = -1$, Mn = 1, R = 2, $\gamma = 1$, A = 1.2, C = 0.5



Fig. 11 — Temperature profiles for various values of Nr and when $h_u = 0.1$, $h_T = 0.1$, $\lambda = -1.0$, Pr = 1.0, Mn = 1, R = 2, $\gamma = 1$, A = 1, C = 0.5



Fig. 12 — Temperature profiles for various values of λ and when $h_u = 0.1$, $h_T = 0.1$, Nr = 1.0, Pr = 1.0, Mn = 1, R = 2, $\gamma = 1$, A = 1, C = 0.5

and the thickness of the momentum boundary layer reduce with the increase of the velocity slip parameter h_u . It reveals that only part of the momentum, due to the pulling of the stretching sheet, can be transmitted to the fluid in the case of the partial velocity slip flow. In addition, it is drawn that the frictional resistance between the fluid and the surface decreases when the velocity slip starts increasing. A similar change about the surface heat transfer gradient $-\theta'(0)$ and the thermal boundary layer can be seen in Table 2 and Fig. 3 when the thermal slip parameter h_T increases. As a result, the thermal slip parameter h_T substantially decreases the heat transfer rate from the surface to the ambient fluid.

Figures 4 and 5 show the effects of velocity ratio parameter R on velocity and temperature profiles. An increase in R leads to a rise of velocity gradient in the boundary layer in the two cases of $R \in (0, 1)$ and R > 1. Moreover, it displays that the fluid velocity decreases when $R \in (0, 1)$ and has an opposite trend when R > 1 with the values of R increasing. On the other hand, the temperature is found to decrease, and the temperature gradient increases with the increasing values of R when $R \ge 0$.

The velocity profiles for different values of the mixed convection parameter γ are shown in Fig. 6. The maximum peak value of the velocity is obtained as $\gamma = 1.4$, and then decays to the free stream velocity. In addition, the minimum value of the peak values occurs in the absence of the thermal buoyancy force. That is because the thermal buoyancy force enhances fluid velocity and increases the velocity boundary layer thickness with the value of γ increasing.

Figures 7 and 8 show the velocity and temperature profiles for different values of the unsteadiness parameter A. It reveals that the gradient of the velocity increases for an increase of unsteadiness parameter A and the thickness of the boundary layer decreases with increasing values of A. For all the cases of A considered, the increase of the unsteadiness parameter A has the tendency to reduce the thermal boundary layer thickness which results in an increase in the temperature gradient in the boundary layer.

Figure 9 shows the velocity profiles for various values of the magnetic parameter Mn as R = 2 and R = 2, respectively. The velocity curves show that an increase in the magnetic parameter Mn will be to decrease the momentum boundary layer thickness and increase the velocity gradient in the boundary layer. This is entirely due to the fact that variation of Mn leads to the variation of the Lorentz force due to the magnetic field and the Lorentz force produces more resistance with the values of Mn increasing.

Finally, Figs 10-12 show the effects of Prandtl number Pr, radiation parameter Nr and heat absorption/generation parameter λ on the temperature profiles, respectively. From these Figs 10-12, it can be observed that an increase in the Prandtl number Pr leads to a reduction in the thermal boundary layer thickness, which in turns causes to decrease the temperature in the boundary layer. On the contrary, Figs. 11 and 12 show that the thickness of the temperature boundary layer decreases with the increasing of Nr or λ .

5 Conclusions

The effects of partial velocity and temperature slip on the unsteady MHD mixed convective flow and heat transfer of a moving viscous, incompressible, electrically conducting fluid past a vertical continuous stretching sheet in the presence of thermal buoyancy force, suction/injection, thermal radiation and heat generation/absorption, have been studied. By employing similarity transformation technique, the governing equations are transformed into a system of highly nonlinear coupled differential equations with boundary conditions at infinity. We derive the approximate analytical solutions of by the DTM-BF. A comparison between the approximate analytical solutions and the numerical solutions is implemented. It is found that the results obtained by the two methods are in very close agreement. The effects of various parameters on the velocity and temperature fields in the momentum and thermal boundary layers are presented.

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