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On the Markov chain models for monsoonal rainfall occurrence in different zones of West Bengal

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The probability distribution of pattern of rainfall during the monsoon season (June-September) over different regions of West Bengal (India) has been analysed with the help of Markov chain models of various orders. The analysis is based on relevant data of 25 years (1971-1995) for ten meteorological stations spread over the state. The determination of the proper order that best describes the precipitation over the region is carried out using Akaike's Information Criteria. The analysis clearly indicates that first order Markov chain model is the best one for rainfall forecasting. It is found that there is a period of occurrence of rainfall phenomenon (2-4 days) over the various stations. Moreover, the steady state probabilities and mean occurrence time of precipitation days and dry days have also been calculated for first and second order Markov chain models. The computation reveals that the observed and theoretical values of steady state probabilities are realistically matched.

Keywords: Markov chain model, Akaike's information criteria, Rainfall probability, Stationary probability, Mean recurrence time, Rainfall forecasting

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1 Introduction

The natural systems are complex and no exact laws have yet been developed that can explain precisely the natural hydrological phenomena. Rainfall itself is the one that affects weather variables affecting the growth and development of crops and spread of diseases and pests. Hence, rainfall forms the principal input to all agronomic models. The future probability of occurrence of rainfall can be used for crop planning and management and water management decisions and consequently, the risk due to weather uncertainty can be reduced. It is known that meteorological observations, such as rainfall, are separated by short interval of time either similar or highly correlated. The occurrence or non-occurrence of rainfall on a day is a simple meteorological example and the sequence of days at a particular location constitutes a time series. Tyagi *et al.*[1], Chatterjee *et al.*[2], Iyenger & Basak[3] and others attempted to predict the occurrence of rainfall in a variety of methods, such as synoptic, numerical and statistical techniques. One such statistical method, viz. time series analysis was utilized by Sengupta & Basak[4] and Iyenger[5]. The Markov models are appropriate one and frequently proposed to quickly obtain forecasts of the weather states (such as dry or wet day) at some future time using information given by the current state.

Many researchers in the past utilized Markov chain for modeling atmospheric phenomenon. Mimikou[6] reported that monthly sums of wet days are modeled better by a second order auto-regressive model than by aggregating daily precipitation generated from a Markov chain. SØrup *et al.*[7] reported theoretically that first order Markov chain is very important, whereas the 2nd order Markov chain is found to be significant. It is supported by the models developed for Sri Lanka by Perera *et al.*[8] Jimoh & Webster[9] noticed that Akaike information criteria (AIC) estimates are consistently greater than or equal to Bayesian information criteria (BIC) estimates for an order of Markov chain. Also, there is no discernible difference between the model parameters of 1st and 2nd order. For Bangladesh, Hossain & Anam[10] stressed that wet day of previous two time period influence positively the wet day of current time period in the rainy season as compared to the dry day of previous two time period. Aneja & Srivastava[11] utilized 3-state Markov chain with 5 independent parameters for analysis for Haryana, India. Dash[12] reported that first order Markov chain can adequately represent the precipitation occurrence for all months in Odisha, India. Other efforts on modeling on Markov chain performed in the past are, namely Dasgupta & De[13], Pant & Shivhare[14], Thiagarajan *et al.*[15] and Senthivelan *et al.*[16] The Markov chain models have few advantages: firstly, forecasts are available immediately after the observations because of the use of predictors only on the local information on the weather. Secondly, the chain needs minimal computation after the climatologically data have been processed. It may be also be revealed that if the record length is short, lower order chain represents the appropriate fit. However, the study of weather state in region of West Bengal, India is not adequately studied. The reliability of prediction of the amount first depends on the accuracy of the prediction of wet and dry days. The objective of this study is to simulate daily rainfall sequences (weather state) for different regional stations of West Bengal to use as inputs for crop, hydrologic and water resources models.

The yield of crop particularly in rain fed condition depends on rainfall patterns. Simple criteria related to sequential phenomenon, like wet and dry spells could be used for analyzing rainfall to obtain specific information needed for crop planning and carrying out agricultural operation[13]. Markov chains specify the state of each day as wet or dry and develop a relation between the state of the current day and the states of the preceding days. The order of the Markov chain is the number of preceding days taken into account. Most Markov chain models referred in the literature are first order, perhaps for the reason that the number of parameters would be kept minimum, yielding a better estimate. Many researchers[17,18] have used Markov chains to model the daily occurrence of rainfall. The common observations of these studies suggest that the occurrence of weather state is best described by first order Markov chain. Consequently, in the present study, attempts have been made to study the weather state over the region of West Bengal during the monsoonal season.

2 Data and Method of analysis

The daily rainfall at ten meteorological stations in different zones of West Bengal, namely, Alipore (22.51°N, 88.33°E), Balurghat (25.22°N, 88.76°E), Bankura (23.25°N, 87.07°E), Dumdum (22.62°N, 88.42°E), Jalpaiguri (26.70°N, 89.00°E), Kalingpong (27.06°N, 88.47°E), Kuchbihar (26.52°N, 89.45°E), Malda (25.00°N, 88.15°E), Midnapore (22.43°N, 87.33°E) and Purulia (23.33°N, 86.37°E) are considered.

In south-west monsoon season (1 June - 30 September), daily rainfall data for a period of 25 years (1971-1995) have been utilized in the present paper. This data is provided by India Meteorological Department (IMD), Pune. The original daily data has been transformed into binary events (0, 1) for the monsoonal period as per dry day (DD) and wet day (WD), respectively. The missing data, if any, are distributed randomly in terms of DD and WD. The binary data representing DD and WD days are well represented in terms of random variable as:

$$X_k = \begin{cases} 0, \text{ if rainfall does not occurs on the k}th\text{ day} \\ 1, \text{ if rainfall occurs on the k}th\text{ day} \end{cases}$$

where, k=1, 2, ………,etc.

The data for each season of a particular year are considered as a separate sample of the above time series. Each of the two states would pertain to one of the possible data values for a two-state Markov chain. As the process may be in any of the two states, the process may remain in the same state or move to the other state. In the second case, a transition occurs from one state to another state. The probability concerning such transition is considered as transition probability.

Let, $\{X_t, t \in T\}$ to be a Markov chain with index set T and state space S [0, 1] (DD, WD), then $\{X_t, t \in T\}$ is a two-state Markov chain. The most common form of transition probability of a two-state first-order Markov chain (following Wilks[19]) is:

$$P_{ij} = \{X_{t+1} = j \mid X_t = i\}$$

The transition probabilities for two-state second order and third order Markov chains in the above notation are:

$$P_{ijk} = \{X_{t+1} = k \mid X_t = j, X_{t-1} = i\}$$
$$P_{ijkl} = \{X_{t+1} = l \mid X_t = k, X_{t-1} = j, X_{t-2} = i\}$$

The two-state Markov chain of any order is completely determined by its initial state and a set of transition probabilities P_{ij}, P_{ijk}, P_{ijkl}. The transition probabilities are estimated using conditional relative frequencies.

The parameter estimates of the above three orders of Markov chain, in the paper, have been estimated by averaging the estimates for each of the sample and the overall estimates are then extracted by averaging the estimates from all the samples for the data previously specified. Thereafter, the proper order of the Markov chain for modeling the time series of DD and WD is assessed by AIC. Following the criteria, for a given s-state, Markov chain of order 'm' is the most appropriate model, if it minimizes the function:

$$AIC_m = -2L_m + 2s^m(s-1)$$

where,

$$L_0 = \Sigma_{j=0}^{s-1} n_j \ln(p_j)$$

$$L_1 = \Sigma_{i=0}^{s-1} \Sigma_{j=0}^{s-1} n_{ij} \ln(P_{ij})$$

$$L_2 = \Sigma_{i=0}^{s-1} \Sigma_{j=0}^{s-1} \Sigma_{k=0}^{s-1} n_{ijk} \ln(P_{ijk})$$

$$L_3 = \Sigma_{i=0}^{s-1} \Sigma_{j=0}^{s-1} \Sigma_{k=0}^{s-1} \Sigma_{l=0}^{s-1} n_{ijk} \ln(P_{ijki})$$

At the selected ten stations, the observed and expected number of dry and wet spells of different orders are compared for the chosen data period 1996-2000, using the Chi-square test[20]. The data for the years 1996-2000 were not used in the development of the Markov model but were kept reserved for the cross validation of the result obtained.

The n-step probabilities have been obtained by using first order Markov chain. In connection with some earlier works using Markov chains of first order, the transition probabilities of the chain are the elements of the matrix P^n, where P is the one-step transition matrix of the chain. After completion of four to five steps, it is generally observed that these probabilities become constant and thereby, independent of the initial state. These steady-state probabilities are noted as:

$\pi 0$ = steady state probabilities of DD

$\pi 1$ = steady state probabilities of WD

Using computational formula on conditional probability on the Markov chain of first order:

$\pi 1 = P01/(1 + P01 + P11)$

$\pi 0 = 1 - \pi 1$

The Markov chain of second order may be computed as:

$$\pi 0 = (P10P100 + P11P110)/(1 - P00P000 + P10P100 - P01P010 + P11P110)$$

$\pi 1 = 1 - \pi 0$

The expressions for the chain of higher orders are computed but not presented in the paper due to its complicated nature.

3 Results and Discussion

The statistical analysis has been done for each year. However, results discussed here pertain to average of 25 years. The estimated transition probabilities of occurrence of dry day (DD) and wet day (WD) for the Markov chain of first and second order for the occurrences of DD and WD are presented in Tables 1(a) and 1(b). It is revealed from the table that for the first order Markov chain considering all the stations, the probability of WD followed by WD (*P*11) is observed to be highest (varies from 0.6375 to

Table 1(a) — Estimation of transition probabilities of two-state Markov chains for first order

Station	*P00*	*P10*	*P01*	*P11*
Alipore	0.5122	0.2337	0.4878	0.7663
Balurghat	0.6300	0.3624	0.3695	0.6375
Bankura	0.5877	0.4122	0.5877	0.6897
Dumdum	0.5006	0.2037	0.4994	0.7962
Jalpaiguri	0.5487	0.1946	0.4512	0.8053
Kalingpong	0.5623	0.2790	0.4377	0.7209
Kuchbihar	0.4929	0.1975	0.4929	0.8024
Malda	0.5818	0.2870	0.4181	0.7130
Midnapur	0.5732	0.2956	0.5732	0.7043
Purulia	0.5417	0.2830	0.4583	0.7169

Table 1(b) — Estimation of transition probabilities of two-state Markov chains for second order

Station	*P000*	*P001*	*P010*	*P100*	*P011*	*P101*	*P110*	*P111*
Alipore	0.2901	0.5031	0.1388	0.1045	0.3518	0.4291	0.1647	0.6015
Balurghat	0.4202	0.6293	0.1428	0.2291	0.2214	0.4178	0.2036	0.1572
Bankura	0.3503	0.5810	0.1320	0.2816	0.2118	0.4794	0.1749	0.1338
Dumdum	0.2818	0.4948	0.1328	0.3677	0.1476	0.6488	0.0889	0.1147
Jalpaiguri	0.3320	0.5389	0.1155	0.3378	0.1457	0.6609	0.0905	0.1028
Kalingpong	0.3450	0.5538	0.1484	0.2934	0.1861	0.5352	0.1347	0.1439
Kuchbihar	0.2670	0.4865	0.1283	0.3801	0.1482	0.6547	0.0872	0.1099
Malda	0.3546	0.5108	0.1474	0.2727	0.1827	0.5295	0.1540	0.1338
Midnapur	0.3452	0.5697	0.1386	0.2890	0.1982	0.5051	0.1574	0.1393
Purulia	0.3150	0.5318	0.1575	0.3037	0.1848	0.5316	0.1383	0.1453

0.8053) than the probability of DD followed by DD (P00) (varies from 0.4929 to 0.6300); on the other hand, the probabilities of WD followed by DD (*P*10) and DD followed by WD (P01) lies between the corresponding *P*00 and *P*11.

Other features observed for the second order Markov chain are such as the transition probabilities *P*001 (varies from 0.4865 to 0.6293 over the stations) and *P*101 (varies from 0.4178 to 0.6609) are strikingly high whereas the probabilities corresponding to *P*000 (varies from 0.2670 to 0.4202) and *P*100 (varies from 0.1045 to 0.3801) are strikingly low. These probabilities indicate that during the south-west monsoon season, WDs are more frequent as compared to DDs. It is interesting to note that in both the orders of Markov chain (first and second), the transition probability *P*11, *P*111 and *P*001 have the greatest magnitude compared to the remaining transitions.

Having obtained the transition probabilities of the chains of different orders and taking into account various transition counts, the AIC values for the first and second order Markov chains with their corresponding log likelihood estimates are computed and presented in Table 2. It is observed that for all the stations, two-state first order chain minimizes the AIC criteria.

Table 2 — AIC scores for the model of different orders at ten stations

Station	Order I		Order II	
	L_I	AIC_I	L_{II}	AIC_{II}
Alipore	-1233.1373	2468.2747	-1791.4951	3586.9902
Balurgat	-1421.6881	2845.3762	-1986.6333	3977.2666
Bankura	-1394.5678	2791.1355	-1949.7644	3903.5288
Dumdum	-1153.3908	2308.7815	-1695.2932	3394.5864
Jalpaiguri	-1177.4917	2356.9834	-1669.3511	3342.7021
Kalimpong	-1344.4453	2690.8906	-1900.2510	3804.5020
Kuchbihar	-1119.6182	2241.2363	-1669.5520	3343.1040
Malda	-1370.1864	2742.3728	-1912.3239	3828.6477
Midnapur	-1375.6204	2753.2407	-1930.4905	3864.9810
Purulia	-1340.2660	2682.5320	-1911.4440	3826.8880

Utilizing the transition probabilities of the chains of first orders, the expected number of DD and WD spells of different lengths has been calculated for different years 1996-2000. The corresponding observed and expected values for these years have been computed; thereafter, nearness of the values has been tested using Chi-square[20] test (Table 3). The test is accepted in 90 out of 94 cases.

The steady state probabilities as obtained from the n-step probabilities are presented in Table 4. These n-step probabilities are the elements matrix of the type P^n, which stabilizes usually after 7-8 iterations where P is the one-step transition matrix. The aforesaid P^n model is undoubtedly realistic and simulates the chances of forecasting DD and WD perhaps better than any other[14,21]. Also, the corresponding matrix is computed from the computational formula from 1st order chain and is presented in Table 4. It is seen that those are nearly same in the specific cases.

The mean recurrence time for DD and WD are computed as reciprocal of steady state probabilities (from computational formula). Those mean recurrence times for DD and WD are presented in Tables 5(a) and 5(b). For example, in Alipore, mean recurrence time for DD and WD are 3.0872 and 1.4791. Those are compared with the observed mean recurrence time for the data period 1996-2000 for both DD and WD period in the respective cases. The observed mean recurrence time is found to nearly match with the computed mean recurrence time.

Table 3 — Goodness of fit test for observed and expected count (for first order Markov chain) of dry and wet spells for test data

Station	Years									
	1996		1997		1998		1999		2000	
	Dry	Wet	Dry	Wet	Dry	Wet	Dry	Wet	Dry	Wet
Alipore	2.4104	17.5902	3.4371	1.3816	5.1222	9.0401	3.4411	4.8692	6.2512	9.3211
Balurgat	5.3967	6.1301	1.7097	5.5512	3.6967	5.8705	9.5894	3.4704	5.2611	3.9118
Bankura	4.8350	0.6749	7.4075	5.5536	-	-	5.0263	6.8331	6.1243	0.3544
Dumdum	5.3923	7.0130	1.1087	5.5596	3.6966	5.8705	9.9478	3.8044	8.2014	0.0675
Jalpaiguri	2.0410	18.5095	3.3147	1.8314	5.2222	9.0409	3.4500	4.2819	9.7894	5.8705
Kalingpong	6.3967	6.1301	1.7097	5.5512	3.6967	5.8705	9.5894	3.4704	5.2611	3.9118
Kuchbihar	4.8350	0.6749	7.4075	5.5536	6.1877	2.6028	5.0263	6.8331	6.1243	10.3544
Malda	2.6310	18.6095	6.3147	1.8344	5.2222	9.0309	3.4567	4.2819	9.794	5.8705
Midnapur	4.8350	0.6749	7.4075	5.5536	-	-	5.0263	6.8331	2.6028	6.1877
Purulia	2.2860	5.1877	3.0987	1.3456	6.1236	7.2198	8.6522	15.2178	-	-

Upper 5% value of chi-square distribution with 5 degrees of freedom is 11.07.

Table 4 — Comparison of stationary probabilities obtained from N-step transition matrix and computational formula on a first order chain

Station	Stationary probability of dry day		Stationary probability of wet day	
	From matrix	From first order	From matrix	From first order
Alipore	0.3239	0.3239	0.6761	0.6761
Balurghat	0.4952	0.4952	0.5048	0.5048
Bankura	0.4952	0.4294	0.5706	0.5706
Dumdum	0.2897	0.2897	0.7102	0.7102
Jalpaiguri	0.3013	0.3013	0.6987	0.6987
Kalingpong	0.3893	0.3893	0.6107	0.6107
Kuchbihar	0.2803	0.2803	0.7197	0.7197
Maldah	0.4070	0.4070	0.5930	0.5930
Midnapur	0.4092	0.4092	0.5908	0.5908
Purulia	0.3818	0.3818	0.6182	0.6182

Table 5(a) — Mean recurrence time for dry days from stationary probability at different stations

Station	From Markov model		Observed mean recurrence				
	Stationary probability	Mean recurrence time	1996	1997	1998	1999	2000
Alipore	0.3239	3.0872	2.6000	1.3704	2.2619	1.8387	2.2820
Balurghat	0.4952	2.0195	1.6770	2.7143	2.7143	1.0213	1.2407
Bankura	0.4952	2.0195	1.4061	1.6889	2.2727	1.4102	-
Dumdum	0.2897	3.4513	2.0486	1.7500	1.6176	2.0512	1.3214
Jalpaiguri	0.3013	3.3188	2.3801	1.9687	2.7894	1.9130	2.2222
Kalingpong	0.3893	2.5688	2.2690	1.2364	2.0454	2.7945	3.1299
Kuchbihar	0.2803	3.5672	1.9648	3.3415	2.2352	2.2353	1.4444
Malda	0.4070	2.4570	2.3674	2.6000	2.1136	2.8125	2.3548
Midnapur	0.4092	2.4436	2.0952	2.8052	1.9000	2.7733	1.7460
Purulia	0.3818	2.6192	2.1880	2.4615	1.8276	2.2444	2.4167

Table 5(b)—Mean recurrence time for wet days from stationary probability at different stations

Station	From Markov model		Observed mean recurrence				
	Stationary probability	Mean recurrence time	1996	1997	1998	1999	2000
Alipore	0.6761	1.4791	2.5834	2.8513	2.4000	2.3189	2.6842
Balurghat	0.5048	1.9809	1.7335	2.0625	2.0625	1.9219	1.2414
Bankura	0.5706	1.7525	2.1406	1.9706	2.3151	-	-
Dumdum	0.7102	1.4079	2.8654	2.9062	2.7027	1.5972	2.6173
Jalpaiguri	0.6987	1.4312	2.0006	2.3151	2.2187	1.4156	2.0000
Kalingpong	0.6107	1.6374	2.2690	1.2364	2.0454	2.7945	3.1299
Kuchbihar	0.7197	1.3895	1.9300	1.5246	2.9827	2.3239	2.3521
Malda	0.5930	1.6863	2.2610	2.3871	2.1549	2.6032	2.6579
Midnapur	0.5908	1.6927	2.1179	2.0952	2.8052	1.9000	1.7460
Purulia	0.6182	1.6175	2.3059	2.6164	2.1094	2.2222	2.5757

4 Conclusions

The time series analysis of monsoonal dry days (DD) and wet days (WD) for 25 years (1970-1995) has made by two-state Markov Chain (first and second order) to visualize the probabilistic distribution of DD and WD pattern over ten regional stations of West Bengal in the south-west monsoon season. The study reveals the following:

(i) The data series consisting of the DD and WD days is best explained by the two-state first order Markov chain. The said models also very realistically simulate the fact that any weather spell (DD or WD) of shorter length is generally more frequent and longer spells are of rare occurrences.

(ii) For first and second order Markov chain, the transition probabilities P11, P001, P111 are found to be higher compared with the remaining transition probabilities. This implies that during the monsoonal season, the state of occurrence of WD is more frequent over the stations in West Bengal in south-west monsoon season. The reason seems to be very much justified.

(iii) The overall climatologically probability of rainfall on a given day in the season as obtained from the first order chain is seen to be in the range 0.5048 and 0.7102 over the stations.

(iv) The climatologically probabilities as obtained from the chain of second order is almost identical to the value obtained from the first order chain. This indicates the steady state probabilities are not only independent of the initial state of the chain but also of the order which is a significant finding in respect of the general behaviour of the Markov chain.

(v) The stationary and climatologically probability of the occurrence of DD following WD such as *P*10, *P*100, *P*101 over the state of West Bengal is observed to be very low during the season.

(vi) The first order Markov chain satisfactorily describe the process of analysis of DD and WD over time as is exposed by the sensibly close values of the observed and theoretical mean recurrence times for the test data.

(vii) As per the AIC selection criteria, first order Markov chain is least over all stations and is utilized for building up model.

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