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# Comparison of statistical Holt-Winter models for forecasting the ionospheric delay using GPS observations

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The ionosphere is a notable source of error that disrupts the accuracy of the global position system (GPS) signal to the ground by changing the speed and direction of the signal propagation and in the process causing a delay in the signal. Therefore, forecasting the ionospheric delay is very important to reduce the GPS positioning error. In this work, statistical Holt-Winter method was chosen due to its suitability in forecasting time series with repeated seasonal patterns. This involved the forecast of ionospheric delays during the period October 2009 - December 2010 using GPS Ionospheric Scintillation and TEC Monitor (GISTM) over Universiti Kebangsaan Malaysia (UKM) station, at geographic coordinates 2.55°N, 101.46°E. The comparison of Additive and Multiplicative Holt-Winter models was done in terms of month-to-month error measurement, the difference of the actual and forecasting delay and the monthly average of the forecast. The maximum difference between actual and forecasting delay was estimated to be about 2 m. The results showed that the accuracy of the Multiplicative model in forecasting the ionospheric delay is better by 2% than that of Additive model going by its small error values and higher accuracy.

**Keywords:** Ionospheric delay, Total electron content, Holt-Winter Additive model, Holt-Winter Multiplicative model

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## 1 Introduction

The ionosphere is defined as the upper region of the atmospheric layer that ranges 60-1000 km from the Earth's surface[1,2] and contains a large number of free electrons and ions. The ionospheric layer has significant effects on GPS navigation[3]; these effects are known as the ionospheric delay error and are also indicative of the dispersive nature of the ionosphere. The ionospheric delay error is regarded as a source of error for GPS applications because it corrupts the positioning and time transfer results.

Total electron content (TEC) is the main parameter of the ionosphere that causes delay in GPS signals due to its dispersive nature. The ionospheric delay error of the GPS signal varies according to several factors such as the user's location, elevation angle, time of the day, time of the year and solar cycle. During periods of high solar activity, this delay can induce a vertical ranging error of 5 - 15 m, and can also exceed 100 m in extreme cases[4], which greatly affects the performance of GPS navigation[3]. Therefore, prediction and assessment of trans-ionospheric propagation errors are necessary for precise measurements as they possess valuable information for satellite and space navigation, space geodesy and radio astronomy applications[5].

A number of empirical models have been developed to effectively forecast ionospheric variability at different locations of the world. Examples include the Bent model, Global Ionospheric Maps (GIMs) and International Reference Ionosphere (IRI) model[6]. However, using an IRI model for prediction needs to be in high density station as northern mid-latitudes[7], due to the general scarcity of data in equatorial regions, the IRI model does not provide accurate predictions, especially over Asia[8,9]. Recently, a number of techniques have been developed, as alternatives to traditional methods, for ionospheric prediction purposes such as Autoregressive Moving Average (ARMA) method and auto-covariance has been used to forecast the TEC time series over different European stations, which shows an acceptable accuracy in the mid latitude region[10,11]. In equatorial region, several researchers have been using neural network based models, autoregressive and self-consistent models for forecasting the TEC over equatorial regions[12-18].

Suwantragul *et al.*[19] demonstrated the application of the Holt-Winter statistical method to forecast the ionospheric delay using GPS TEC measurements in the equatorial region over Chiang Mai in Thailand. Using the Holt-Winter method, a five-day predicted data were generated and later compared with the observed data. The method was found to be capable of predicting the ionospheric delay and improving the positioning accuracy up to about 50%. Later, Abdullah *et al.*[20] reported 90% improvement of prediction accuracy in the ionospheric delay adopting the Holt-Winter method using GPS TEC measurements over Parit Raja station in Malaysia during September-October 2005. However, their work was only conducted over a two-month period, which is insufficient to investigate the month-to-month variation of ionospheric delay. Therefore, the research by Abdullah *et al.*[20] has been extended by the authors in this work, covering a fifteen-month period.

The aim of this paper is to present the comparison of the statistical Holt-Winter method for short-term forecasting of the ionospheric delay over a period of fifteen months, from October 2009 to December 2010 using GISTM over UKM station, at geographic coordinates 2.55°N and 101.46°E. This statistical method is applied to the time-series of GPS TEC to forecast the ionospheric delay. Holt-Winter has two models, i.e. Additive and Multiplicative models; the data is forecasted and tested with the aid of both models in an attempt to determine which of the two models is the most effective in terms of accuracy.

## 2 Statistical forecasting method

Holt-Winter is a statistical short-term method which has been used to forecast the ionospheric delay producing time series with seasonal patterns and repetitive forms. This method uses a technique called exponential smoothing, where fluctuations in the time series data are reduced, thus, providing a clearer view of the fundamentals of the series. It also provides an effective way of forecasting the future value of time series data. There are three weights or smoothing constants, viz. α, β and γ, representing the level, trend and season, respectively, which are used in both models to update components for each period of time, t [Eqs (1-3, 8- 10)]. The value of these constants lies between 0 and 1. This value is selected depending on the weight (i.e. high smooth constant mean more weight)[21,22]. The initial value for the constants in this work is 0.2. Holt-Winter method has two seasonal models, i.e. Additive and Multiplicative models. The Additive model is not affected by the changes in data series, thus, the method works best when the seasonal pattern does not change over the time while the Multiplicative model is dependent on the data size, for example, the ionospheric delay is affected by several factors, such as solar activity; when these factors increase the ionospheric delay, the seasonal component of Multiplicative model also increases. Additive model is applied using the following equations:

Level: $L_t = \alpha(Y_t - S_{t-s}) + (1-\alpha)(L_{t-1} + b_{t-1})$ … (1)

Trend: $b_t = \beta(L_t - L_{t-1}) + (1-\beta)b_{t-1}$ … (2)

Seasonal: $S_t = \gamma(Y_t - L_t) + (1-\gamma)S_{t-s}$ … (3)

Fitted: $F_t = L_{t-1} + b_{t-1} + S_{t-s}$ … (4)

Forecast: $F_{t+m} = L_t + b_t m + S_{t-s+m}$ … (5)

where, $L_t$, is the level; $b_t$, the trend; $S_t$, the season; $Y_t$, the vertical TEC (VTEC); and *t*, the time period of $L_t$, $b_t$, $S_t$, and $Y_t$ component. $F_t$ is the forecast value of a period ahead; $F_t+m$, the monthly forecasting time period; , and , level, trend and seasonal smoothing coefficients, respectively; *m*, the forecast period; and *s*, the seasonal duration. The initial value of the seasonal component $S_1$ needs to be determined using the Eq. (6), while the level of the seasonal duration, *s* is shown in Eq. (7):

$$S_1 = Y_1 - L_s, S_2 = Y_2 - L_s, \ldots, S_s = Y_s - L_s \qquad \ldots(6)$$

$$L_s = \frac{1}{s}\left(Y_1 + Y_2 + \cdots + Y_s\right) \qquad \ldots(7)$$

The equations used for the Multiplicative seasonal model are:

Level: $L_t = \alpha \frac{Y_t}{S_{t-s}} + (1-\alpha)(L_{t-1} + b_{t-1})$ … (8)

Trend: $b_t = \beta(L_t - L_{t-1}) + (1-\beta)b_{t-1}$ … (9)

Seasonal: $S_t = \gamma \frac{Y_t}{L_t} + (1-\gamma)S_{t-s}$ … (10)

Fitted: $F_t = (L_{t-1} + b_{t-1})S_{t-s}$ … (11)

Forecast: $F_{t+m} = (L_t + b_t m)S_{t-s+m}$ … (12)

The initial seasonal component value ($S_1$) and the level for seasonal duration, *s* is determined by Eqs (13 and (14), respectively:

$$S_1 = \frac{Y_1}{L_s}, S_2 = \frac{Y_2}{L_2}, \ldots\ldots, S_t = \frac{Y_t}{L_t} \qquad \ldots(13)$$

$$L_S = \frac{1}{s}\left(Y_1 + Y_2 + ... + Y_S\right) \qquad \ldots(14)$$

## 3 Error measures

The error measurement components are useful in measuring the appropriateness and accuracy of forecasting methods and to demonstrate the effectiveness of the forecasting model. In this study, three error measures are used, such as average or the mean absolute percentage error (MAPE), the average or the mean absolute deviation (MAD) and mean squared deviations (MSD). Meanwhile, MAPE measures the accuracy of the time series that has been adapted to the Holt-Winter method and it depicts the accuracy as a percentage. MAPE equation is given in Eq. (15), and the percentage of error shown in Eq. (16).

$$\text{MAPE} = \frac{1}{n}\sum_{t=1}^{n}\left|\text{PE}_t\right| \qquad \ldots(15)$$

$$PE_t = \left(\frac{Y_t - F_t}{Y_t}\right)\times 100 \qquad \ldots(16)$$

where, $n$, is the number of total observations; $Y_t$, the actual value; $F_t$, the forecast value of period $t$; and $PE_t$, the percentage of the error.

MAD is represented with the same unit as that of time series given in Eq. (17):

$$\text{MSD} = \frac{1}{n}\sum_{t=1}^{n}\left(Y_t - F_t\right)^2 \qquad \ldots(17)$$

In addition, MSD is usually calculated using the same number of total observations, $n$, regardless of the type of the model in Eq. (18):

$$\text{MAD} = \frac{1}{n}\sum_{t=1}^{n}\left|Y_t - F_t\right| \qquad \ldots(18)$$

## 4 Data processing and Measurement technique

The data were collected from October 2009 to December 2010 to forecast the ionospheric delay using GISTM located in UKM, Malaysia. The station uses the observer flickering ionosphere TEC GPS GSV4004B model system to collect data. The GPS receiver tracks up to 11 GPS satellite signals. Satellites transmit the signals in two frequency bands, which is L1 frequency (1575.42 MHz) and L2 frequency (1227.6 MHz). The phase and amplitude are measured at 50 Hz and the divergence of the code/carrier is at 1 Hz for each satellite. In this study, only measurements with elevation angles greater than 20° and lock time greater than 240 seconds are considered. By using dual-frequency pseudo-range and carrier phase, the slant TEC was determined and then converted to VTEC by using the obliquity factor. In the present study, the ionospheric delay was calculated in each of the Holt-Winter model by using Eq. (19):

$$I_k^p = 40.3\text{VTEC}\left(\frac{1}{f_2^2} - \frac{1}{f_1^2}\right) \qquad \ldots(19)$$

where, $I_k^p$, is the ionospheric delay in meter, and the carrier frequencies, $f_1$ is 1575.42 MHz and $f_2$ is 1227.60 MHz, where $f_1$, $f_2$ are $L_1$, L2 respectively.

The ionospheric delay has been forecasted for two time periods: morning to noon (0800-1200 hrs LT) and afternoon to night (1500-2100 hrs LT). These time periods were chosen to indicate the temporal variation of the ionospheric delay, i.e. the ascending and declining phases of ionospheric delay. The measurements of the first quiet three days of the previous month were used to forecast three days of the following month. The comparison between Additive and Multiplicative models were presented by computing the error measure components using Eqs (16-19) to test the accuracy of both models.

## 5 Results and Discussion

The month-to-month variations of the actual and forecasted mean ionospheric delay of both Additive and Multiplicative models during 0800-2100 hrs LT for the period October 2009 - December 2010 is shown in Fig. 1. From the figure, it can be seen that the forecasted ionospheric delay at UKM for both models shows the diurnal variation with peaks observed during the afternoon (1200-1800 hrs LT) and drops rapidly at night. The mean ionospheric delay throughout the 15-month period varies in the range 2 - 6 m. The forecasted ionospheric delay also showed seasonal variations and in general exhibits a similar trend as the actual ionospheric delay. It is observed that the error increased slightly with a significant difference between the curves of actual and forecasted delay during summer, where both models underestimated 1 m of delay in May 2010, while an overestimation of 3 m is seen in June 2010 throughout the observation period. A prominent difference of about 3 m of ionospheric delay is also observed for both models in October 2010 during

1500-2100 hrs LT. The average error between actual and forecasted delay throughout the observation period is in the range 0-2 m.

To compare the analysis between the Additive and Multiplicative models, the month-to-month error measurement, the difference of the actual and forecasting delay and the monthly average of the forecast error are presented. Figure 2 shows the month-to-month variation of the MAPE, MAD and MSD error measurement components for both Additive and Multiplicative models from morning to noon (0800-1200 hrs LT).

It can be observed that the three error measures exhibit similar trends in the month-to-month variation with a peak error in January for both models. In the period between August and February, both models show almost no difference in values, but the difference of error estimates became more apparent from March to July. Comparatively, the Additive model exhibited a slightly higher amount of error than Multiplicative model, with a difference of up to 2.5% for MAPE and around 0.05 m for both MSD and MAD.

The variation of the error measurement components from afternoon to nighttime (1500-2100 hrs LT) is shown in Fig. 3. As shown in the figure, the month-to-month variation of MAPE shows two peaks of error in January and July, while both MAD and MSD estimates do not show much monthly variation throughout the 15-month period. Meanwhile, no apparent difference is observed between the error estimates provided by the two analyzed models for all error measurement components as the maximum difference given by MAPE is 1.1% and 0.05 m for both MAD and MSD. The difference in values is almost the same from morning to noon.

Figure 4 depicts the percentage of the average forecast error obtained from October 2009 to December 2010 for both models during 0800-1200 hrs LT and 1500-2100 hrs LT. During 0800-1200 hrs LT, the overall average of forecast error is 11.76% (0.29 m), and 11.25% (0.25 m) for Additive and Multiplicative models, respectively. Meanwhile, the overall average of forecast error during the afternoon for the nighttime period for the Additive model is 14.96% (0.39 m), while 14.84% (0.37 m) is recorded for the Multiplicative model.

From the result obtained, it may be inferred that the ionospheric delay errors from the afternoon to nighttime period are, generally, larger as compared to those in the morning to noon period and the ionospheric delay normally peaks in the post-noon and decreases rapidly at night, which is expected because of the ultraviolet radiation. The ionosphere is a layer ionized by the sun radiations. This ionization is mostly due to extreme ultraviolet and thus increases the ionospheric delay, which is reported during first hours in the afternoon. Moreover, the maximum of

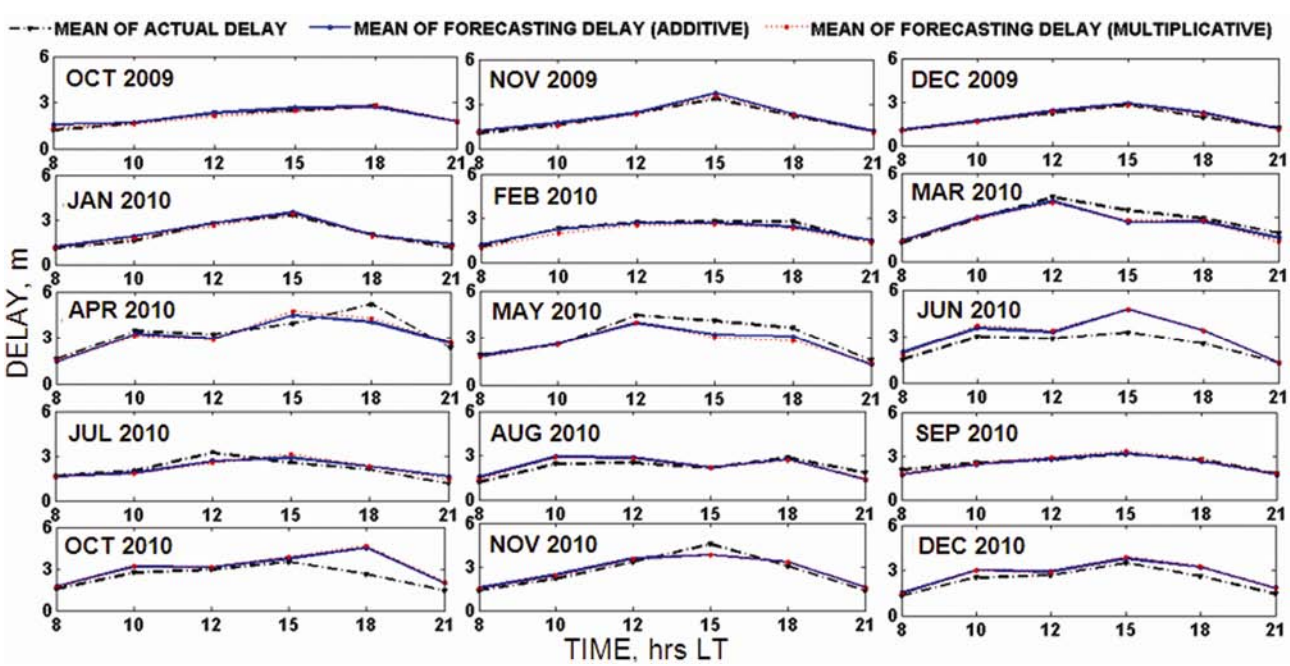


Fig. 1—Monthly variation of the actual and forecasted (Additive, Multiplicative) ionospheric delay

the ionospheric time delay forecast during summer season in June is reported in the range 20-35%, while the minimum is reported in winter. By looking at the forecasting results of both Holt-Winter models, the accuracy from morning to noon time is found to be in the range 77-91% (Additive) and 78-92% (Multiplicative), while the accuracy from afternoon to nighttime is found to be in the range 68-92% for Additive and 70-93% for Multiplicative model. From the result, the accuracy of the Multiplicative model is slightly better than that of the Additive model during both time periods by about 2% (0.05 m). This difference has made sense in term of use of one of the Holt-Winter model that can give better forecasting

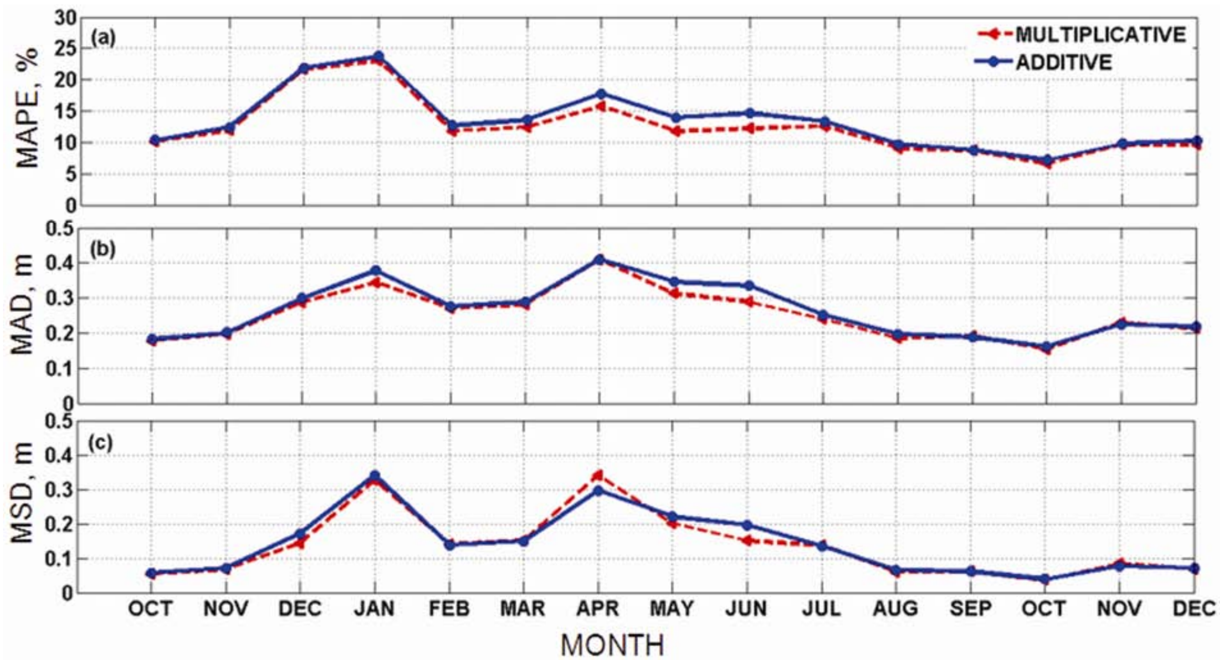


Fig. 2—Monthly variation of MAPE, MAD and MSD for Additive and Multiplicative models during 0800-1200 hrs LT

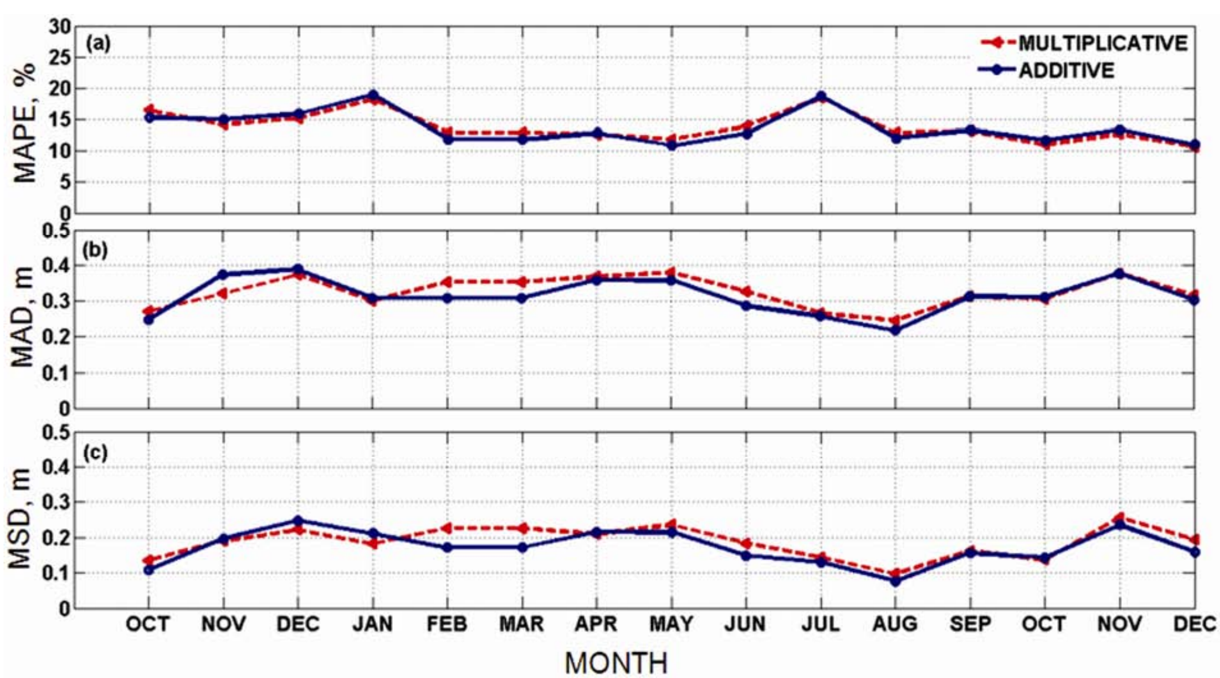


Fig. 3—Monthly variation of MAPE, MAD and MSD for Additive and Multiplicative models during 1500-2100 hrs LT

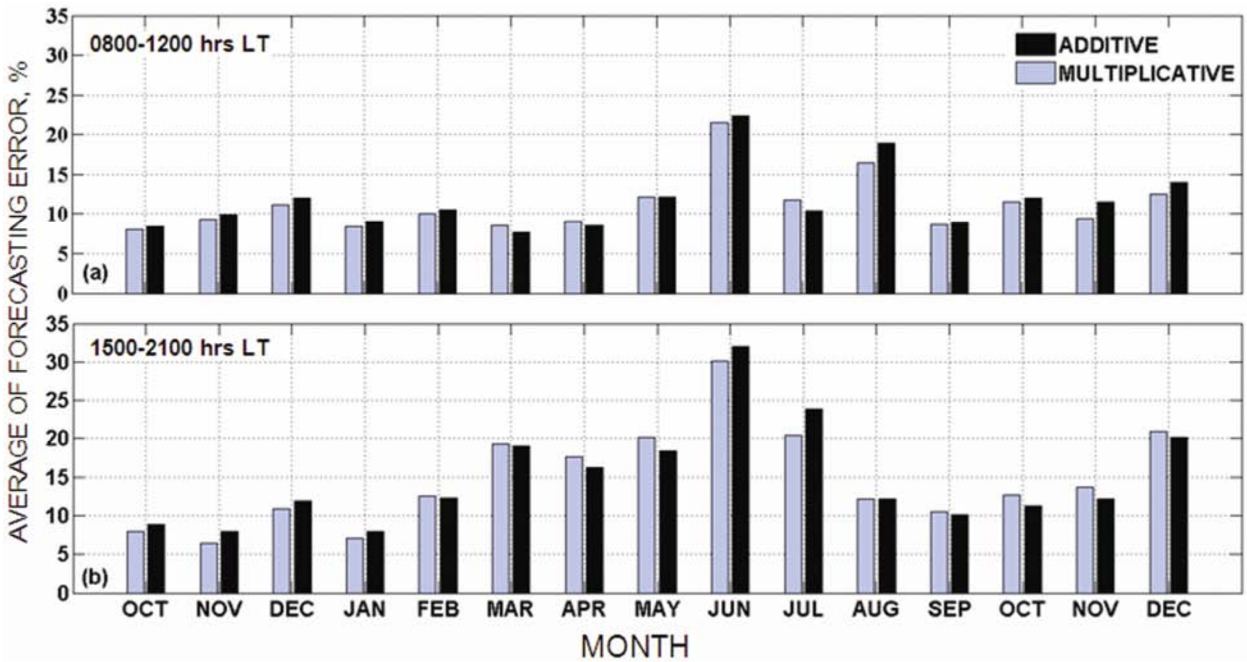


Fig. 4—Average of the forecast error obtained from October 2009 to December 2010 in the Additive and Multiplicative model during (0800-1200 hrs LT) and (1500-2100 hrs LT)

result, which is essential to correct the GPS positioning. Based on the comparative analysis of both models, it can be deduced that the Multiplicative model is more reliable for forecasting the ionospheric delay in terms of the slightly lower amount of error and better accuracy than the Additive model. A comparison of results with other forecasting techniques is made. Chan[5] has used the radial basis function neural network model to forecast the critical frequency, foF2 over 24 hours and found only 58% improvement in accuracy of foF2 prediction in the mid-latitude. In the meantime, the accuracy of the Klobuchar model[23,24] is limited to about 50-60%, while the Klobuchar model with code coefficients can only provide an accuracy of 75-85%. Therefore, the results using the Holt–Winter method over UKM in this study indicates better estimates of ionospheric delay forecast with accuracy of 70-93%, noting that the values indicated come from different data sets.

## 6 Conclusion

In this study, a comparison between Additive and Multiplicative Holt-Winter statistical method is carried out during fifteen-month period at UKM station, Malaysia and the accuracy of each model is tested. In general, the trend of the forecasted ionospheric delay for both analyzed models is similar to that of the actual delay with a very small mean error of 0-2 m. Prominent errors are found in May, June and October 2010 with values of 1-3 m. The comparative analysis between the two models indicated that the Multiplicative model (70-93%) is slightly more accurate than the Additive model (68-92%), in most of the months during the period chosen. Thus, it can be concluded that Multiplicative model is a more reliable model with a relatively higher accuracy than the Additive model in forecasting the ionospheric delay over UKM. The comparison of the Holt-Winter models is very important in knowing the best model that can forecast the delay with the least amount of error while still giving a better forecast results to improve the accuracy performance of GPS positioning by correcting ionospheric errors.

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