



## Performance Analysis of the Swashplate Axial Piston Pump with Hydraulic Fluid Temperatures

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Variable displacement axial piston pump can be used in a hydraulic system as the primary source of fluid power, which is suitable for high pressure and high efficiency. The power can be transfer in a hydraulic system with the help of the fluid medium. The oil leakage problem in various parts of the pump, especially the internal leakages in the piston-cylinder, swash plate-slipper pad and valve plate-cylinder block, seriously affect the performance of the pump. Therefore, it is important to know the properties of the fluid and its effect on the system performance. To study the performance of an axial piston pump, a non-linear mathematical model has been developed. The developed model has been validated with the existing results. The validated pump model has been used for performance analysis of the system. Moreover, the influence of hydraulic mineral oil at different temperatures on the piston chamber pressure, output power, and leakage flow in piston-cylinder has been explored. The present investigation has been performed in MATLAB Simulink 14a environment. The simulation result shows that the pump operating temperature range can be set as 30°C to 60°C for moderate ripple and output chamber pressure.

**Keywords:** Hydraulic system, Bulk modulus, Chamber pressure, MATLAB Simulink

### Introduction

The swash-plate type axial piston pump can be used in a hydraulic system as the main fluid power source. In fluid power systems, power is transferred through hydraulic oil, therefore the oil properties like density, viscosity, and bulk modulus affect its dynamic performance. Apart from hydraulic oil properties, oil leakage also affects the performance of the hydraulic system.<sup>1</sup> Various types of hydraulic oil can be used for fluid power systems based on their properties. Hydraulic systems suffer from huge energy loss as they operate at high pressure. Researchers are attracted to energy efficiency research of hydraulic systems. The dynamic performance of hydraulic system degrades due to leakage of hydraulic fluid in the system while operation and variation of oil properties due to change of oil temperature.<sup>2-4</sup>

Many previous researchers have reported on the reduction of pressure ripple and flow ripple using groove geometry and different controller design. Only a few have studied the temperature analysis on chamber pressure and output power of an axial piston pump by using different hydraulic fluids. Kim *et al.* (2003)<sup>(6)</sup> conducted an experiment on the axial piston

pump to measure the fluid film thickness on the valve plate using hydraulic oil as a fluid medium. Javalagi *et al.* (2012)<sup>(7)</sup> studied the different hydraulic fluids available and their effect on system performance. They found that for low dynamic viscosity, the efficiency of the system decreases. The noise of the system has been significantly reduced by using mineral oil and synthetic oil. Recently, Song *et al.* (2018)<sup>(8)</sup> reported research on the influence of temperature on piston-cylinder interface leakage in an axial piston pump. The efficiency of the axial piston pump decreases with increase in temperature as contact time increases monotonically in the form of hyperbolic tangent. Kazama (2009, 2015)<sup>(9,10)</sup> and Kazama *et al.* (2010)<sup>(11)</sup> used thermocouples amplifier for measurement of temperature on bearing and seal parts of variable displacement pump. They have considered mineral oil of VG22 and a water-glycol having a temperature range of 20 to 40 degrees. Finally, they concluded that temperature raises much less using water-glycol-oil as compared to mineral oil. Cai *et al.* (2015)<sup>(12)</sup>, Feo *et al.* (2015)<sup>(13)</sup>, and Cancino *et al.* (2018)<sup>(14)</sup> studied the effect of hydraulic oil temperature on tribological behavior of steel-steel contact and performance of engine lubricated at higher temperature respectively. Along with that, Borghi *et al.* (2009)<sup>(15)</sup> and Tang *et al.* (2017, 2018)<sup>(16,17)</sup> investigated numerically the dynamic behavior of axial piston pump, motor slipper bearings,

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and lubrication characteristics of slipper pair in swash-plate type axial piston pump respectively.

Along with the analysis of different hydraulic oil on the pump's performance, few researchers have reported on the development of mathematical modelling of an axial piston pump. Noah (1999, 2000)<sup>(18,19)</sup> and Noah *et al.* (2001)<sup>(20)</sup> developed the detailed dynamic modelling and performance analysis of an axial piston pump. Gronberg (2011)<sup>(21)</sup> conducted an investigation on prediction of case temperature of swash-plate axial piston pump. Bergade *et al.* (2012)<sup>(22)</sup> established an analytical formulation of an axial piston pump by using CFD. They find that by designing a suitable geometry of the groove, both pressure ripples and flow ripples reduced. Zawistowski *et al.* (2017)<sup>(23)</sup> studied the gap flow simulation between cylinder and piston of an axial piston pump.

In the present study, the effect of temperature on the piston chamber pressure of the hydraulic system with mineral oil 46 (standard mineral oil) has been studied using MATLAB Simulink 14a environment. The hydraulic oil temperature has been varied from 20°C to 70°C and system performance has been studied for the same. It has been found from the simulated results that the increase of temperature causes a decrease of dynamic viscosity and bulk modulus of the oil. Also, with the variation of fluid property and temperature, the output chamber pressure and leakage flow for single and total pistons have been analyzed. Finally, the pump operating temperature range can be set as 30°C to 60°C for moderate ripple and output chamber pressure.

**System Description**

The schematic diagram of a swash plate axial piston pump and sectional view of the piston and

chamber has been shown in Fig. 1(a) and 1(b) respectively. The pump has nine pistons, which have been arranged along the pitch circle of the cylinder block with equally spaced. It has been considered that the pump starts from the bottom dead centre (BDC), which has been considered as zero-degree position.<sup>(24,25)</sup> The chamber is entirely full of fluid at BDC. The cylinder block along with the barrel and slipper pad (where pistons are mounted) rotates by the motion transmission from the input shaft. The first half rotation causes axial movement of the piston from BDC to TDC gives discharge from the pump. Similarly, the second half rotation causes the suction in the chamber of the pump. Geometry of the piston along with fluid properties has been shown in Fig. 1(b). During one complete revolution of the pump, the piston executes a full stroke of  $S_k$  as shown in Fig. 1(b). Dynamic response of the hydraulic system depends upon the fluid properties, which has been encountered in the present study.

**Mathematical Modeling**

A non-linear dynamic model for swash plate axial piston pump has been developed for analyzing the performance due to temperature variation of the fluid. The system model has been developed with the following consideration,

- a. The rotating speed of the shaft has been kept constant.
- b. The swash plate angle has been considered as 12°.

The kinematic flow,  $Q_i$  due to the piston movement can be expressed as,

$$Q_i = -\frac{dV_i}{dt} \quad \dots (1)$$

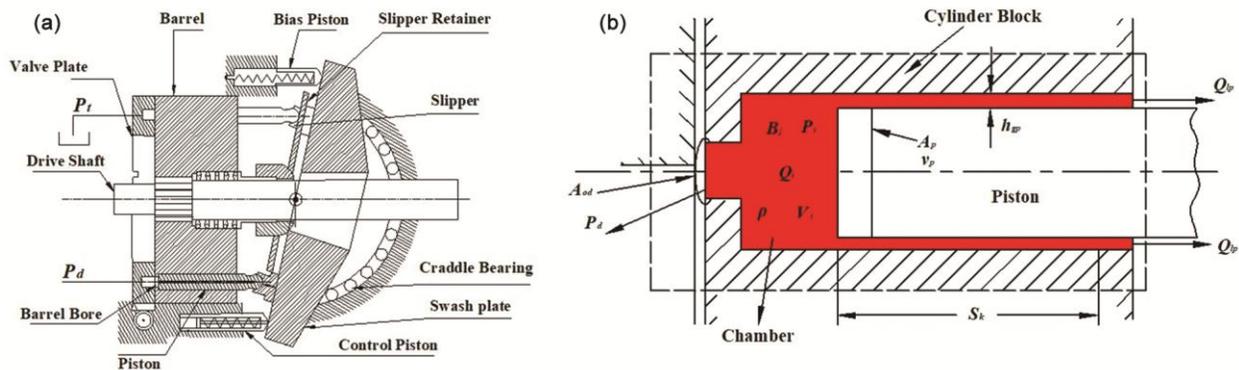


Fig. 1 — Schematic system diagram (a) Axial piston pump considered under study, (b) Control volume for the development of pressure in the piston chamber

where,  $V_i$  represents the instantaneous volume of the chamber, as shown in Fig. 1(b), and  $i$  denotes the corresponding piston ( $i=1$  to  $9$ ). The instantaneous volume  $V_i$  of the chamber can be obtained as<sup>4-6</sup>,

$$V_i = V_o + A_p R_p \tan \beta \cos(\phi_i) \quad \dots (2)$$

where,  $V_o$  represents the piston chamber nominal volume. The term  $A_p$ ,  $R_p$  and  $\phi$  are the piston effective area, cylinder block pitch circle radius and phase difference between the adjacent pistons respectively. Finally, the kinematic flow can be obtained as<sup>25,26</sup>,

$$Q_i = \omega A_p R_p \tan \beta \sin(\phi_i) \quad \dots (3)$$

$$Q_{ik} = \omega A_p R_p \tan \beta \sum_{i=1}^9 \sin(\phi_i) \quad \dots (4)$$

where,  $\omega$  represents the shaft speed.

The flow of the  $i^{\text{th}}$  piston chamber will be affected due to the fluid inertia, oil compressibility, viscous effect of the oil, etc. hence, the flow will deviate from Eq. (3) and Eq. (4). The discharge flow out of the  $i^{\text{th}}$  piston has been given by higher Reynolds number which can be obtained as<sup>5</sup> Eq. 5,

$$Q_{pn} = C_{d1} A_{dn} \sqrt{\frac{2|P_{pn} - P_s|}{\rho}} \text{sgn}(P_{pn} - P_s) \quad \dots (5)$$

where,  $C_d$  is the coefficient of discharge,  $P_{ic}$  is the instantaneous chamber pressure,  $P_d$  is the delivery pressure which is considered to be constant and  $\rho$  represents the density of the oil,  $A_{od}$  represent the instantaneous discharge flow area as shown in Fig. 1b, which can be given by Eq. (6a) through 6(f)<sup>4</sup>,

$$A_{od} = \frac{(\phi + 13)^2 \pi R_p}{360 \times 22} (R_1 - R_2), \quad -13^\circ < \phi < 9^\circ \quad \dots (6a)$$

$$A_{od} = \frac{(\phi - 9)}{360} \pi (R_1^2 - R_2^2) + \frac{22}{360} \pi R_p (R_1 - R_2), \quad 9^\circ < \phi < 17^\circ \quad \dots (6b)$$

$$A_{od} = \frac{(\phi - 9)}{360} \pi (R_1^2 - R_2^2) + \left( \frac{22}{360} \pi R_p (R_1 - R_2) - \frac{(\phi - 17)^2 \pi R_p}{360} (R_1 - R_2) \right), \quad 17 < \phi < 39^\circ \quad \dots (6c)$$

$$A_{od} = \frac{30}{360 \times 22} \pi (R_1^2 - R_2^2), \quad 39^\circ < \phi < 141^\circ \quad \dots (6d)$$

$$A_{od} = \frac{(\phi - 141)}{360} \pi (R_1^2 - R_2^2), \quad 141^\circ < \phi < 171^\circ \quad \dots (6e)$$

$$A_{od} = 0, \quad 171^\circ < \phi < 360^\circ - 13^\circ \quad \dots (6f)$$

where,  $R_1$  and  $R_2$  represents the internal and outer radius of discharge manifolds, respectively.

The schematic diagram for the derivation of chamber pressure is presented in Fig. 1(a) and 1(b). In the present model, nine pistons have been considered, which are arranged in circular patterns inside the cylinder block, and each piston is separated by an angle of  $40^\circ$ . The instantaneous chamber pressure  $P_{ic}$  in the piston can be developed by the principle of bulk modulus and conservation of mass which is given by<sup>5, 18, 25, 26</sup>,

$$\frac{dP_{ic}}{dt} = -\frac{B_j}{V_i} \left( Q_{ic} + Q_{lpm} + \frac{dV_i}{dt} \right) \quad \dots (7)$$

where,  $B_j$  represents the bulk modulus of the mineral oil ( $j = 1$  to  $4$ ),  $Q_{lpm}$  is the leakage between the cylinder and piston, which is given by Eq. (8).

$$Q_{lpm} = \frac{\pi r h_g^3}{6 \mu_j L} (P_{ic} - P_c) \quad \dots (8)$$

where,  $r$  represents piston radius,  $h_{gp}$  represents gap in piston and chamber bore,  $L$  is the length of piston,  $\mu_j$  represents dynamic viscosity of the oil ( $j = 1$  to  $4$ ), and  $P_c$  is the drain chamber pressure which is equal to atmospheric pressure and is considered to be zero throughout the simulation. Finally, the stroke  $S_{pn}$  for  $P_n$  piston is given by

$$S_{pn} = R_p \tan \beta (1 - \cos(\omega t - (n - 1)\alpha)) \quad \dots (9)$$

where,  $R_p$  represents pitch radius of piston barrel and  $t$  is time.

From the principle of conservation of mass and the definition of bulk modulus, the discharge pressure  $P_d$  from the pump can be expressed as,

$$P_d = \int \frac{B_j}{V_c} (Q_{pn} - Q_d) dt \quad \dots (10)$$

where,  $V_c$  represents the discharge chamber volume and  $Q_d$  represents the output flow from the pump. Similarly, the output flow  $Q_d$  is given by,

$$Q_d = C_{dn} A_{vn} \sqrt{2 \frac{P_d}{\rho}} \quad \dots (11)$$

where,  $C_{dn}$  represents the coefficient of discharge of the needle valve, and  $A_{vn}$  is the needle valve area.

The output power,  $P_{out}$  of an axial piston pump, can be estimated by

$$P_{out} = (P_d - P_t) Q_s \quad \dots (12)$$

where,  $P_d$  is the output pressure,  $P_t$  represents the pressure in the tank and  $Q_s$  is the output flow from the pump.

**Model validation**

In the present investigations, the piston chamber pressure of an axial piston has been validated with the Mandal *et al.* (2008)<sup>(27)</sup> results. The comparison of the present model and existing results has been presented in Fig. 2(a). To compare the present result, a similar model has been presented with the same parameters as that used by Li (2005)<sup>(5)</sup> and Mandal *et al.* (2008).<sup>(27)</sup> An axial piston pump having nine pistons and bulk modulus of 8.34 GPa has been considered for the present simulation. As seen from Fig. 2(a), the present simulation result following the same trends at the suction manifolds, but at discharge manifolds, there are minor deviations in piston chamber pressure from the Mandal *et al.* (2008)<sup>(27)</sup> results. The deviation is due to the geometry of the relief notches in the valve plate and discharge area modelling. In the present simulation, the grooves model is not considered, and output pressure from the pump is fixed to be 7.1 MPa. The present simulation result has been shown by the red curve, whereas existing results have been shown by the black curve, both the results have been compared with similar parameters except swash-plate angle. The simulation parameter for an axial piston pump model has been shown in Table 1.

**Results and Discussion**

Performance of an axial piston pump has been investigated by using mineral oil with the variation of oil temperature. The nonlinear mathematical model has been implemented in MATLAB Simulink 14a environment with ode5 (Dormand-Prince) solver. The time step size of  $2 \times 10^{-7}$  has been considered for simulation. The effect of operating parameters such as bulk modulus of hydraulic oil, swash-plate angle, and dynamic viscosity on the single piston and overall performance on total piston chamber pressure has been discussed. The hydraulic mineral oil property values have been given in Table 2. The input variable

Table 1 — Simulation parameters<sup>23,24,27</sup>

Parameters	Description	Values
$A_p$	Piston area, m <sup>2</sup>	$8.345 \times 10^{-5}$
$A_v$	Flow area of the needle valve, m <sup>2</sup>	$4.457 \times 10^{-6}$
$\alpha$	Phase delay	40°
$B$	Bulk modulus of fluid, Pa	$8.547 \times 10^8$
$\beta$	Swash-plate angle	12°
$C_{d1}$	Coefficient of flow discharge	0.675
$C_{d2}$	Coefficient of discharge	0.610
$h_g$	Piston and cylinder bore clearance, m	$60 \times 10^{-6}$
$L_o$	Leakage passage initial length, m	0.0145
$m$	Number of pistons	9
$R$	Barrel radius, m	$5.21 \times 10^{-3}$
$\rho$	Density of fluid, kg/m <sup>3</sup>	860
$R_p$	Piston pitch radius, m	0.0221
$\mu$	Absolute viscosity, N-s/m <sup>2</sup>	$5.855 \times 10^{-2}$
$V_o$	Initial piston chamber volume, m <sup>3</sup>	$1.534 \times 10^{-6}$
$\omega$	Pump rotational speed, rad/s	188.54

Table 2 — Properties of mineral oil 46 (standard mineral oil)<sup>1,16,17</sup>

Temperature	Dynamic viscosity (N-s/m <sup>2</sup> )	Bulk modulus (N/m <sup>2</sup> )
20°C	0.1275	$2.64 \times 10^9$
30°C	0.0910	$2.10 \times 10^9$
40°C	0.0671	$1.66 \times 10^9$
50°C	0.0369	$1.00 \times 10^9$
60°C	0.0109	$0.50 \times 10^9$
70°C	0.0072	$0.12 \times 10^9$

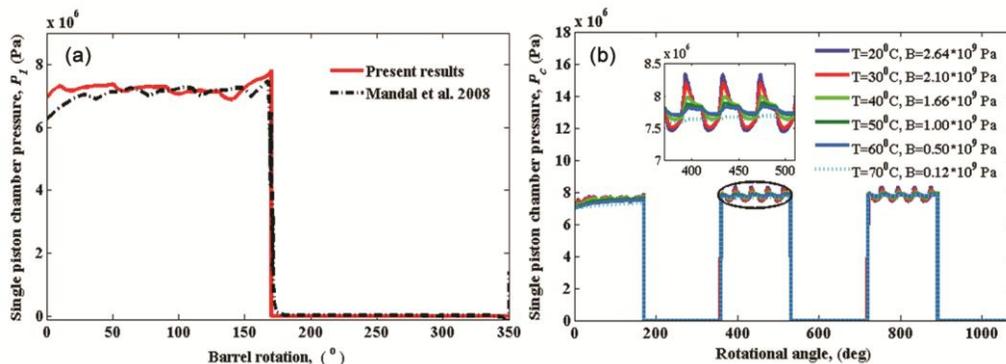


Fig. 2 — (a) Validation of single-piston chamber pressure and, (b) Effect of different temperature on single piston chamber pressure at a discharge pressure of 7.1MPa

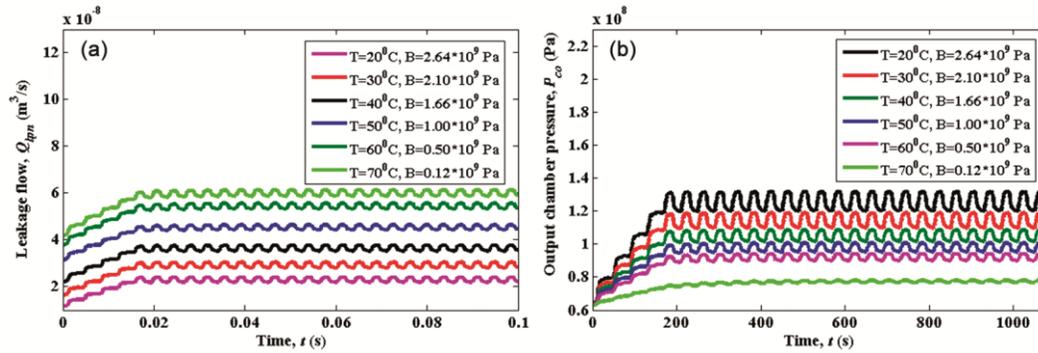


Fig. 3 — Effect of temperature on (a) Leakage flow, (b) Output piston chamber pressure

parameters given to the simulation model are dynamic viscosity and bulk modulus as applicable for particular oil and its temperature. It has been observed from Table 2, as the temperature increases the dynamic viscosity and bulk modulus decreases. This is so because when a fluid is compressed, its molecule becomes close together, and its resistance to further compression increases.

#### Effect of Hydraulic Oil Temperature on Leakage Flow and Chamber Pressure

The present investigation has been performed with six different temperatures of hydraulic mineral oil on leakage flow in piston-cylinder, chamber pressure. The total leakage flow from the cylinder-piston for mineral oil due to variation of temperature 20, 30, 40, 50, 60 and 70°C has been shown in Fig. 3(a). The bulk modulus represents the stretchability of the fluid, and dynamic viscosity describes the internal resistance of the fluid. These are the two parameters that mainly affect the performance of an axial piston pump. As bulk modulus and dynamic viscosity decreases, the piston chamber pressure and output power of an axial piston pump decreases. The leakage flow between the cylinder bore and piston has been considered, as Eq. 8. Dynamic viscosity of the fluid has been found inversely proportional to total leakage flow. As the dynamic viscosity decreases due to increase of temperature, the total leakage flow has been found to be maximum at 70°C and minimum at 20°C degrees. The effect of temperature on bulk modulus and dynamic viscosity with a fixed discharge pressure of 7.1 MPa is presented in Fig. 3(b). The black curve and light green curve shows the total chamber pressure is at 20°C and 70°C which has a mean value of 123 MPa and 78.1 MPa respectively. It has been noticed that pressure is maximum at 20°C and minimum at 70°C. The deviation from the mean value of 15 MPa at 20°C and 4 MPa at

70°C has been noticed in Fig. 3(b). Although the ripple is minimum in the case of 70°C at the same time output pressure has been found significantly less which degrades the pump performance which is not desirable. To maintain the system performance, the operating temperature of an axial piston pump in the present study has been considered in the range of 30°C to 60°C.

#### Conclusions

Mathematical model of the variable displacement axial piston pump has been implemented in the MATLAB Simulink 14a environment. The present model has been validated with the existing results. The performance analysis of an axial piston pump has been studied for mineral oil with different temperatures. In the present simulation, the variation of oil properties like dynamic viscosity, bulk modulus due to change of oil temperature has been taken into consideration. It has been observed from the simulation results that as bulk modulus and dynamic viscosity decreases, the ripples in the piston chamber and hence, output power

#### Nomenclature

$Q$	Ideal flow generated by the piston, ( $\text{m}^3/\text{s}$ )	<i>Subscript and Greek letters</i>
$r$	Piston radius, (m)	$l_{pn}$ Leakages in the nth piston
$h$	Allowance between the piston and chamber bore, (m)	$g$ Gap between piston and cylinder
$P$	Pressure from the pump, (Pa)	$i$ Number of pistons
$\rho$	Density of the oil, ( $\text{m}^3/\text{kg}$ )	$od$ Discharge port
$L$	Total piston length, (m)	$\rho$ Pump
$\varphi$	Phase difference between the adjacent piston	
$V$	Piston chamber volume, ( $\text{m}^3$ )	
$\omega$	Rotational speed of the shaft, (rad/s)	
$A$	Piston area, ( $\text{m}^2$ )	
$\beta$	Angle of the swash-plate, (Degree)	
$B$	Bulk modulus of hydraulic mineral oil, (Pa)	

reduced significantly which is beneficial for the pump. At low temperature, 20°C output chamber pressure has been high with high ripple with a deviation of 15 MPa from mean value but leakage has been found less. On the other hand, at 70°C output ripple has been found less with high leakage and less output chamber pressure with deviation of 4 MPa. The result concludes that the operating temperature can be maintained at 30°C to 60°C for moderate ripple and output chamber pressure which improve the performance of an axial piston pump.

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