



## Reconstruction of Level Cross Sampled Signals using Sparse Signals & Backtracking Iterative Hard Thresholding

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Industry 4.0 applications involve more number of sensors or Internet of Things (IoT) devices to support automation in the industry. It involves more number of computations to analyze the sensor data collected from several critical parts of the processing units. Sparse signal processing is one which has numerous applications in area of communication and signal processing. This paper presents a novel approach to reduce the computations with the help of level cross sampling (LCS) and a backtracking based iterative hard thresholding (BIHT) algorithm for reconstruction. The process involves, an information signal is converted to a random sparse signal using non-uniform sampling at the transmitter side and then it can be reconstructed back using BIHT algorithm at receiver side. Simulation results exhibit the superior performance of the proposed BIHT reconstruction in comparison with the literature.

**Keywords:** Compressed Sensing, Frequency Domain, Reconstruction, Sampling, Sparse Signal Processing

### Introduction

Most signals that are interest in practical applications are adaptive in nature. i.e. the characteristics of the signals are randomly vary with respect to time. Speech, power, biomedical, vibration and seismic signals are the examples for non-stationary signals. The majority of the practical systems that are used to process these non-stationary signals follow the nyquist principle. The time varying nature of the signal is not taken into account with these traditional systems that may leads to system more complex in terms of computations and they required higher powers for performing the operations with these discrete samples. By considering the signal local variations in to account, the efficiency of power can be improved by adapting the intelligence in signal processing based on the local variations of the signal. In this context, data driven sampling method, called "level-crossing" is developed.<sup>1</sup> The LCS method changes the rate of sampling according to signal local characteristics.<sup>2-4</sup> Hence, this LCS method reduces the computations of the post processing section, since it only considers the relevant information of the signal.<sup>5,6</sup> In data driven applications, especially in multimedia signal processing, conventional nyquist

sampling creates more number of computations and at the same time it leads to more bandwidth requirement.<sup>7-9</sup> Conventional ADCs sends more redundant information when the slope of the signals is constant. In order to overcome this, data driven sampling techniques came in to picture.<sup>10</sup> Here the sampling is always based on the slope of the signal. If slope of the signal is constant it send very few samples where as if the slope of the signal is more it sends more number of samples in order to carry the information.<sup>11,12</sup> In order to overcome the drawbacks of conventional ADCs this paper proposes a new reconstruction algorithm for LCS signal using BIHT algorithm. The proposed approach considers the continuous time varying nature of the signal and adaptively changes the sampling frequency based on the signal characteristics.<sup>13,14</sup>

### Level Crossing Sampling Scheme

In this LCS approach, the properties of each part of the signal are analyzed initially and thereafter adaptive sampling is applied based on local-features. The LCS scheme for adaptive sampling is as shown in Fig.1. It is evident from figure that the time duration between two successive samples is non uniform in nature. Here  $q$  represents the number of levels in LCS scheme and it depends on both dynamic ranges of the signal as well as spacing between two levels.

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The mathematical representation of LCS scheme is given by

$$dt_n \leq T/2, t_n \leq T_{ref} \quad \dots (1)$$

$$t_n = t_{n-1} + dt_n \quad \dots (2)$$

$$N_i = N_i + 1 \quad \dots (3)$$

In Eq. 2  $t_n$  is the current sampling instant,  $t_{n-1}$  is the previous sampling instant and in Eq. 1,  $dt_n$  is the time elapsed between the current and the previous sampling instants.  $T$  is the fundamental time period of the signal and  $T_{ref}$  is the width of the reference window.  $N_i$  is the number of samples in  $T_{ref}$  and is given in Eq. 3.

**Backtracking Iterative Hard Thresholding(BIHT)**

BIHT algorithm involve two steps namely thresholding and backtracking. To compare with all the detailed coefficients thresholding is used a value  $\lambda$ . Thresholding is of two types, namely Hard Thresholding (HT) and the Soft Thresholding (ST). HT process assigns zero to the coefficients with absolute values are less than the threshold ( $\lambda$ ). On the other side ST is first setting to zero coefficients whose absolute values are less than  $\lambda$  and then shrinking the nonzero coefficients toward zero. Better edge preservation is provided by HT than ST but ST provides smoother results. For a given  $\lambda$  and value of wavelet coefficient  $W$ , HT is defined as in Fig. 2 (a) and ST is defined as in Fig. 2 (b). BIHT algorithm is introduced to reduce the number of iterations and compressed sensing (CS) time of iterative HT (IHT). The BIHT algorithm reconstruction probability is higher than that of IHT algorithm. With backtracking, BIHT algorithm optimizes the sub-optimal choice of supports for each iteration. The iteration procedure helps the empty space formed by the corresponding matrix columns. Reprojection from time to time, and then use the non-linear operators in the IHT algorithm an approximate solution is obtained. Let  $N$  is the length of a sinusoidal signal. The LC samples are represented by  $b$  of size  $M$ . Consider a random matrix  $A$  of size  $M*N$ . The length of a sparse signal is given by  $A^{-1}*b$ . To reconstruct the signal a matrix  $\phi$  of size  $M*N$  is consider. The original signal can be reconstructed back by multiplying the active set with inverse of  $\phi$ . The iteration to reconstruct the signal in IHT algorithm is represented by Eq. 4 as follows

$$y^{n+1} = H_s(y^n + \phi^T(x - \phi y^n)) \quad \dots (4)$$

where  $y^{n+1}$  is the reconstructed signal after n-time iteration and the non-linear operator  $H_s(\cdot)$  sets largest  $k$  elements of  $x$  to zero. If there is no such unique set, a set can be selected either randomly or based on the predefined ordering of the elements. BIHT depends on the sparsity of the signal. If only 5 or 10 percent of samples are taken from the input signal then the probability of the reconstructed signal will be reduced. The reconstruction probability is higher for higher number of samples.

**Results and Discussion**

Signals with single, two and multiple frequencies are considered for simulation to prove the superiority of the proposed BIHT reconstruction technique. In each case first the signal is sampled with LCS and then converted into sparse signal. The simulations results show the reconstruction process both in time and frequency domains. The Fig. 3 shows the reconstruction of a signal from the sparse signal time and frequency domain using BIHT. The generated sparse signal has 151 samples in this case. All the signals along with their frequency components involved in the reconstruction of double tone

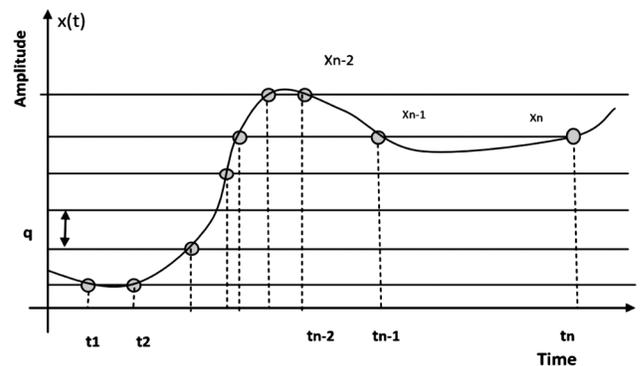


Fig. 1 — Typical LCS representation

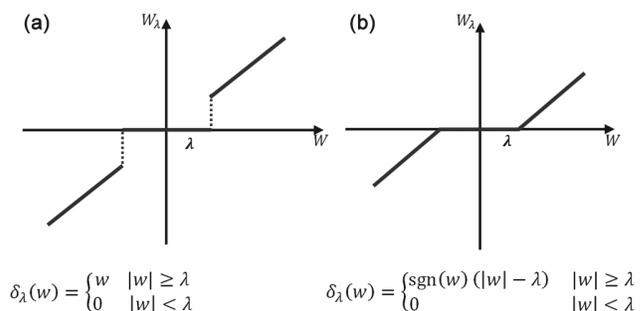


Fig. 2 — Thresholding Characteristics (a) HT (b) ST

frequency signal using BIHT are shown in Fig. 4. In this case also the sparse signal contains 151 samples. The Fig. 5 shows a multi tone signal along with their frequency components using BIHT. The generated sparse signal has 151 samples in this case. Fig. 6 shows the reconstruction of the signals from very less number of samples. It is observed that the reconstructed signal contains unknown frequency components along with the original frequencies. In this case the probability of reconstruction is very poor because of noise. The Fig. 6 shows all the signals along with their frequency components involved in the reconstruction of multi tone frequency signal using BIHT. The generated sparse signal has 75 samples in this case. From the simulations it is

observed that reconstruction of the signal with only 5 percent of the original number of samples and the signal is not reconstructed properly. So, a minimum of 10 percent of the samples should be taken from the original signal. The performance of the proposed method in terms of number of samples with uniform and implicit sampled signals is presented in Table 1. It is far better to reconstruct a signal from 151 samples than to reconstruct it from 617 samples that too with high approximation.

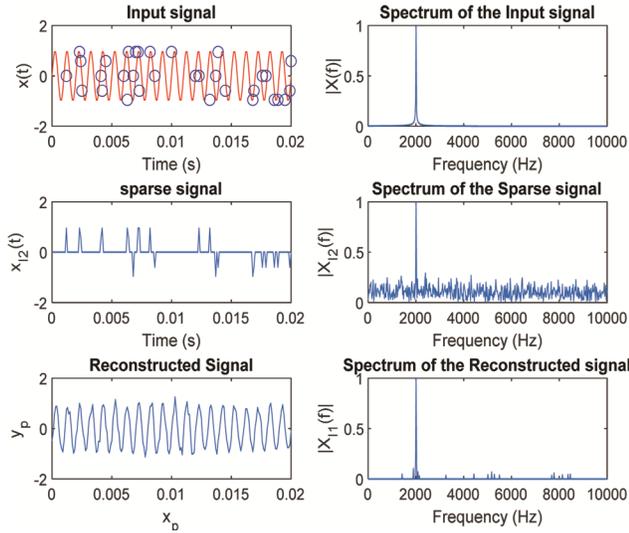


Fig. 3 — Signal Reconstruction using BIHT- Case 1 (Single Tone)

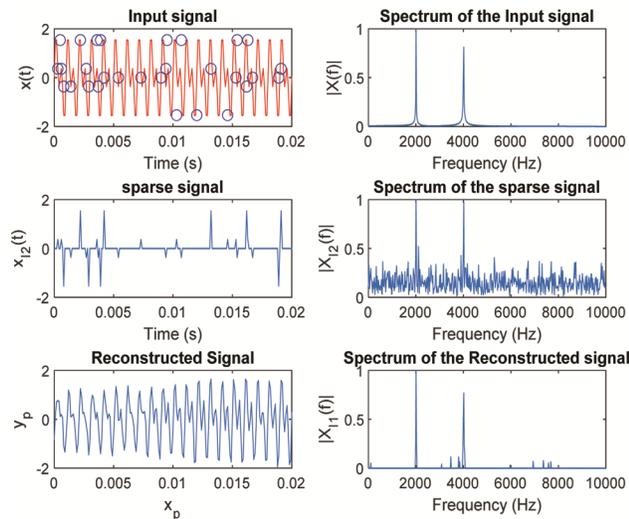


Fig. 4 — Signal Reconstruction using BIHT - Case 2 (Dual Tone)

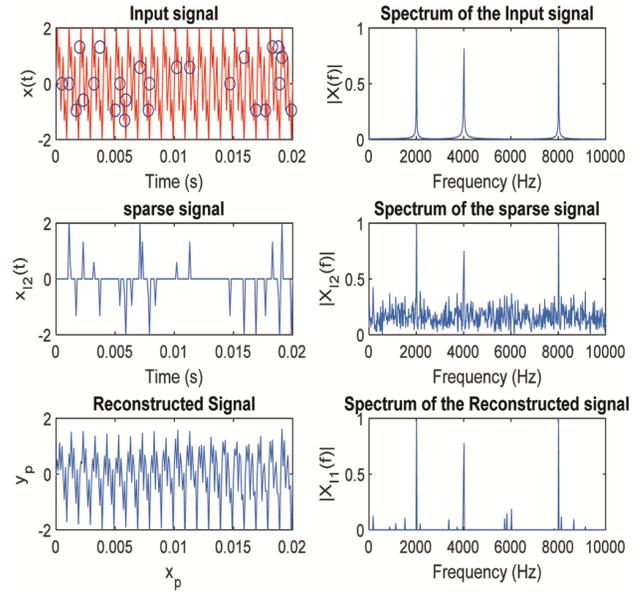


Fig. 5 — Signal Reconstruction using BIHT - Case 3 (Multi Tone)

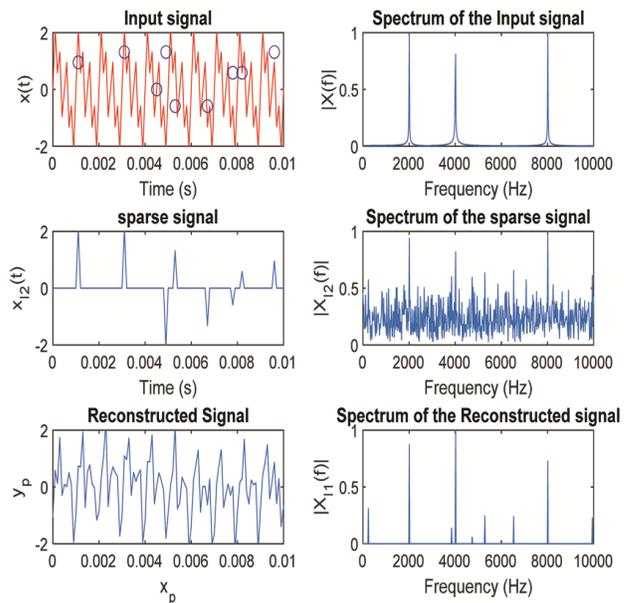


Fig. 6 — MultiTone Signal Reconstruction using BIHT with very few Samples

Table 1 — Performance of proposed method and other techniques

Type of signal	Number of Samples		
	Uniform Sampling	Implicit Sampling	Samples in Sparse Signal
Case 1 (single tone signal)	1028	617	151
Case 2 (double tone signal)	1028	205	151
Case 3 (multi tone signal)	1028	617	151

## Conclusions

Implementation of iterative algorithms using spectrally sparse signals from non-uniform sampling is presented in this paper. To reduce the complexity and bandwidth a sparse signal is generated from LC samples of the input. BIHT algorithm is used to reconstruct the original signal from sparse signal. The proposed BIHT reduces the number of iterations and processing time when compared with greedy algorithms like IHT algorithm. Probability of the reconstructed signal will also be more in BIHT when compared to other algorithms. Further, it can be extended by designing the proper filter to increase the probability of the reconstruction.

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