



## Performance of Hydrodynamic Porous Slider Bearing with Water based Magnetic Fluid as a Lubricant: Effect of Slip and Squeeze Velocity

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This paper deals with performance of Hydrodynamic Porous inclined pad surface Slider Bearing considering slip and squeeze velocity. Water-lubricated bearings have been focused for their benefits to diminish the power loss and raise load capacity of bearing at rapid. Due to this advantage, lubricant used is water based magnetic fluid in this work. Also due to the functional property of self-lubrication, the effect of porosity is included. Under regular suppositions of hydrodynamic lubrication, the Neuringer-Rosensweig Model is considered for study. The expression of load capacity is obtained and calculated for choice of different values of squeeze velocity as well as slip velocity. It is seen that better load capacity is obtained when squeeze velocity as well as slip velocity are considered.

**Keywords:** Bearing system, Fluid pressure, Hydrodynamic lubrication, Load carrying capacity, Neuringer-Rosensweig Model

### Introduction

A bearing is a device of system factors whose function is to bear a functional load by dropping the friction among the rubbing surfaces. The hydrodynamic bearing is to develop pressure by property of comparative motion of two surfaces disconnected by a fluid film. During operating conditions, when two porous surfaces are entirely separated through lubricant film, such type of lubrication is called fluid film- lubrication. Squeeze film behavior study is useful for several fields of real life. For example, in an industry, it is useful in computer disk-drive spindles, gyroscopes, and laser scanners-printer. Until recently, many designers of these applications relied on hydrodynamic bearings (oil and gas), or ball bearings such as clutch plates, rolling elements, gears, hydraulic systems, machine tools, engines to maintain a large pressure difference between the two sides of the seal.

Squeeze velocity occurs when lubricated surfaces approach each other in fluid film region. As unusual magnetic colloids, magnetic fluids have outstanding tribological features for several applications in mechanical engineering. The anti-wear matters of very

uneven technological circumstances together with common start-up motions are intractable contest. The details can be seen in Rosensweig.<sup>1</sup> At the point when external magnetic field  $E$  is applied on film region, the fluids experience forces  $(M \cdot \nabla)E$  which depends upon  $M$  due to ferrous elements. Therefore, they are useful in several applications like detectors, sealing devices, elastic damper, bearings etc.<sup>2-6</sup>

Wu<sup>7</sup> studied squeeze film behavior for annular disks. Later on, Sparrow *et al.*<sup>8</sup> stretched the study of Wu<sup>7</sup> by adding the effect of slip velocity in the above plate. They concluded that in presence of porosity and slip, the load capacity decreases. Prakash *et al.*<sup>9</sup> analyzed the case of porous slider bearing with conventional fluid. Gupta *et al.*<sup>10</sup> found that due to transverse magnetic field on film region, the load capacity and friction could be increased. Later on, Agrawal<sup>11</sup> deliberate its effect on a porous inclined bearing with magnetic fluid as a lubricant. After that many authors namely Shah *et al.*<sup>12</sup>, Ahmad *et al.*<sup>13</sup>, Shah *et al.*<sup>14-21</sup>, Gupta *et al.*<sup>22</sup>, Patel *et al.*<sup>23</sup> have analyzed the impact of Magnetic fluid as lubricant and observed better performance of bearing overall performance over traditional fluids. The motto of this work is to test the performance of an inclined slider bearing by the assorted porous layers to both the pad surface with water based magnetic fluid lubricant in

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presence of magnetic field oblique to lower surface. Here, the presence of slip velocity (at both the porous pad surfaces) and the squeeze velocity are considered. For this work, R E Rosenweig<sup>1</sup> fluid flow model is considered.

**Mathematical Modeling**

The diagram as shown in Fig. 1 represents the slider bearing considered for study. In this bearing problem  $h$  is considered as magnetic fluid film thickness between upper and lower surfaces. Here we consider the bearing length in the  $x$  – direction as  $A$  while the bearing width in  $y$  – direction as  $B$  ( $A \ll B$ ). Also,  $l_1$  and  $l_2$  are uniform thickness of porous assorted in upper and lower surfaces respectively. Due to porosity, slip velocities are generated in the bearing. In operational mode, the squeeze velocity is generated due to vibration which is denoted by  $\dot{h}$  and is given by

$$\dot{h} = \frac{dh}{dt} \tag{1}$$

The expression of thickness of film  $h$  is represented by

$$h = h_2 - \frac{(h_2 - h_1)x}{A} \tag{2}$$

Here the applied external magnetic field vector is

$$H = H(x)(\cos\phi, 0, \sin\phi), \phi = \phi(x, z) \tag{3}$$

Also, the expression of magnitude of magnetic field vector

$$H^2 = Kx(A - x) \tag{4}$$

is used for study. Here  $K$  is proportionality constant (dimension less quantity) and the value of  $K$  is chosen between  $10^{12}$  to  $10^{16}$  to get  $H$  between the orders of  $10^5$  to  $10^7$ .

The basic flow equations for film region given by R E Rosensweig<sup>1</sup> of magnetic fluid flow model which is deliberated by Shah *et al.*<sup>16</sup> is given as:

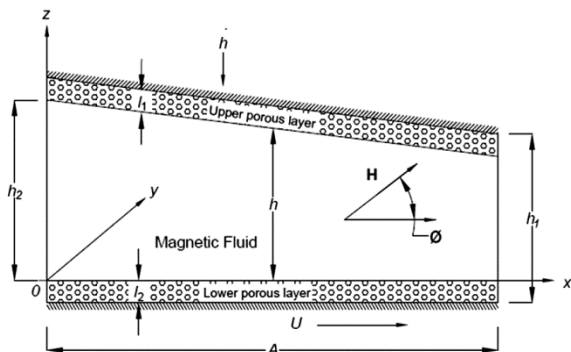


Fig. 1 — Skeletal figure of bearing system

$$\rho \left[ \frac{\partial q}{\partial t} + (q \cdot \nabla)q \right] = -\nabla p + \eta \nabla^2 q + \mu_0 (M \cdot \nabla)H \tag{5}$$

$$\nabla \cdot q = 0 \tag{6}$$

$$\nabla \times H = 0 \tag{7}$$

$$\nabla \cdot (H + M) = 0 \tag{8}$$

$$M = \bar{\mu}H \tag{9}$$

In film region, the fluid velocity is

$$q = ui + vj + wk \tag{10}$$

Taking in to consideration the Darcy’s law validity, in upper porous layer the components of velocity of fluid  $u_1, w_1$  are

$$u_1 = -\frac{\psi_x}{\eta} \frac{\partial}{\partial x} (P - 0.5\mu_0 \bar{\mu}H^2) \tag{11}$$

$$w_1 = -\frac{\psi_z}{\eta} \frac{\partial}{\partial z} (P - 0.5\mu_0 \bar{\mu}H^2) \tag{12}$$

Similarly, in lower porous layer the velocity components of velocity of fluid  $u_2, w_2$  are

$$u_2 = -\frac{\varphi_x}{\eta} \frac{\partial}{\partial x} (P - 0.5\mu_0 \bar{\mu}H^2) \tag{13}$$

$$w_2 = -\frac{\varphi_z}{\eta} \frac{\partial}{\partial z} (P - 0.5\mu_0 \bar{\mu}H^2) \tag{14}$$

The cartesian form of continuity Eq. (6) of fluid flow for the fluid flow in film region is

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{15}$$

By integrating over film region  $(0, h)$  with respect to  $z$  yields

$$\frac{\partial}{\partial x} \int_0^h u dz + w_h - w_0 = 0. \tag{16}$$

The continuity equations for porous regions are as below in Cartesian coordinates:

$$\frac{\partial u_1}{\partial x} + \frac{\partial w_1}{\partial z} = 0 \tag{17}$$

$$\frac{\partial u_2}{\partial x} + \frac{\partial w_2}{\partial z} = 0 \tag{18}$$

**Mathematical Analysis**

In present study, neglecting inertia terms and considering some usual assumptions of lubrication, the one-dimensional fluid flow equation in the film region obtained (by solving Eqs (5) – (9)) as

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta} \frac{\partial}{\partial x} (P - 0.5\mu_0 \bar{\mu}H^2). \tag{19}$$

In this study porous layer are attached on both surfaces. Due to porosity the slip velocity is

generated. In this regard, solving Eq. (19) under boundary conditions modified by Shah *et al.*<sup>16</sup> of Sparrow *et al.*<sup>8</sup>

$$u = \frac{1}{s_1} \frac{\partial u}{\partial z} + U, u = -\frac{1}{s_2} \frac{\partial u}{\partial z} \quad (20)$$

when  $z = 0, z = h$  respectively, where  $\frac{1}{s_1} = \frac{\sqrt{\alpha_x \eta_x}}{5}$ ,  $\frac{1}{s_2} = \frac{\sqrt{\beta_x m_x}}{5}$  are slip parameters, we obtain

$$u = \left\{ \frac{s_2 z^2 (s_1 + s_2 + h s_1 s_2) - h s_2 (h s_2 + 2)(s_1 z + 1)}{2 \eta s_2 (s_1 + s_2 + h s_1 s_2)} \right\} \times \frac{\partial}{\partial x} (P - 0.5 \mu_0 \bar{\mu} H^2) + \frac{s_1 (h s_1 - z s_2 + 1) U}{(s_1 + s_2 + h s_1 s_2)} \quad (21)$$

Using the above value of  $u$  in the Eq. (16), we obtain

$$\frac{\partial}{\partial x} \left[ \left\{ \frac{2 s_2 h^3 (s_1 + s_2 + h s_1 s_2) - 3 h s_2 (h s_2 + 2)(s_1 h^2 + 2)}{12 \eta s_2 (s_1 + s_2 + h s_1 s_2)} \right\} \times \frac{\partial}{\partial x} (P - 0.5 \mu_0 \bar{\mu} H^2) \right] + \frac{s_1 (2 h s_2 - h^2 s_2 + 2) U}{2 (s_1 + s_2 + h s_1 s_2)} + w_h - w_0 = 0 \quad (22)$$

Using Eqs (11) & (12), in Eq. (17), and by integration with respect to  $z$  across the upper porous film, one obtains

$$\frac{\beta_z}{\eta} \frac{\partial}{\partial z} (P - 0.5 \mu_0 \bar{\mu} H^2) \Big|_{z=h} = \frac{\beta_x}{\eta} \frac{\partial^2}{\partial x^2} (P - 0.5 \mu_0 \bar{\mu} H^2) l_1, \quad (23)$$

by Morgan-Cameron approximation given in Prakash *et al.*<sup>9</sup> and that the upper pad surface is solid. Using Eqs (13) & (14), in Eq. (18), and by integration with respect to  $z$  across the lower porous film, one obtains

$$\frac{\alpha_z}{\eta} \frac{\partial}{\partial z} (P - 0.5 \mu_0 \bar{\mu} H^2) \Big|_{z=0} = -\frac{\alpha_x}{\eta} \frac{\partial^2}{\partial x^2} (P - 0.5 \mu_0 \bar{\mu} H^2) l_2, \quad (24)$$

by Morgan-Cameron approximation given in Prakash *et al.*<sup>9</sup> and that the lower surface is solid. Considering

$$w_k = \dot{h} - w_1 \text{ and } w_0 = w_2, \quad (25)$$

the normal velocity components across the film porous boundary and combining Eqs (22) – (25), one obtains

$$\frac{\partial}{\partial x} \left\{ f_1 \frac{\partial}{\partial x} (P - 0.5 \mu_0 \bar{\mu} H^2) \right\} = \frac{\partial f_2}{\partial x}, \quad (26)$$

where

$$f_1 = \frac{1}{12 \eta s} \left[ \frac{h^2 (12 + 4 h s_1 + 4 h s_2 + h^2 s_1 s_2) + 12 s (\beta_x l_1 + \alpha_x l_2)}{12 s (\beta_x l_1 + \alpha_x l_2)} \right]$$

and

$$f_2 = \frac{s_1 U h}{2 s} (2 + h s_2) + x \dot{h}.$$

Eq. (26) is Reynolds's type equation. By using the dimensionless quantities

$$x = AX, h = h_1 \bar{h}, \beta_x = h_1^2 \bar{\beta}_x, \beta_z = h_1^2 \bar{\beta}_z, \alpha_x = h_1^2 \bar{\alpha}_x, \alpha_z = h_1^2 \bar{\alpha}_z, l_1 = h_1 \bar{l}_1, \bar{s}_1 = s_1 h_1, \bar{s}_2 = s_2 h_1, \bar{p} = \frac{h_1^2 p}{\eta A U}, \mu^* = \frac{\mu_0 \bar{\mu} K A h_1^2}{\eta U}, a = \frac{h_2}{h_1}, S = \frac{-2 \dot{h} A}{U h_1},$$

the non-dimensional form of Reynolds's Eq. (26) is

$$\frac{d}{dX} \left\{ F_1 \frac{d}{dX} (\bar{p} - 0.5 \mu^* X(1 - X)) \right\} = \frac{dF_2}{dX}, \quad (27)$$

where

$$F_1 = \bar{h}^2 (12 + 4 \bar{h} \bar{s}_1 + 4 \bar{h} \bar{s}_2 + \bar{h}^2 \bar{s}_1 \bar{s}_2) + 12 \bar{s} (\beta_x \bar{l}_1 + \alpha_x \bar{l}_2)$$

and

$$F_2 = -6 S \bar{s} X + 6 \bar{h} \bar{s}_1 (2 + \bar{h} \bar{s}_2).$$

### Solution of the Problem

It is clear that the pressure vanishes on the borders of the slider bearing in comparison to interior pressure. Using boundary conditions  $\bar{p} = 0$  when  $X = 0, 1$ , and solving Eq. (27), the expression of non-dimensional film pressure is

$$\bar{p} = \frac{1}{2} \mu^* X(1 - X) + \int_0^X \frac{F_2 - Q}{F_1} dX \quad (28)$$

where

$$Q = \left\{ \int_0^1 \frac{F_2}{F_1} dX \right\} \times \left\{ \int_0^1 \frac{1}{F_1} dX \right\}^{-1}$$

The non-dimensional load carrying capacity of the considered bearing is

$$\bar{W} = \frac{h_1^2 W}{\eta A^2 B U} = \frac{1}{12} \mu^* + \int_0^1 \left( \frac{Q - F_2}{F_1} \right) X dX \quad (29)$$

### Results and Discussions

Here the results were calculated for non-dimensional load by using the values of below mentioned parameters and also using Simpson's  $1^{1/3}$  rule with interval difference 0.1.

$$\begin{aligned} \eta_x &= 0.64, m_x = 0.81, A = 0.1 \text{ (m)}, \mu_0 \\ &= 4\pi \times 10^{-7} \left( \frac{\text{kgm}}{\text{s}^2 \text{A}^2} \right), \bar{\mu} = 0.05, K \\ &= 10^{14} (\text{A}^2 \text{m}^{-4}), h_1 \\ &= 0.0005 \text{ (m)}, h_2 = 0.001 \text{ (m)}, \eta \\ &= 0.012 \left( \frac{\text{kg}}{\text{ms}} \right), U = 1.0 \left( \frac{\text{m}}{\text{s}} \right) \end{aligned}$$

From Fig. 2, for the fixed values of  $\dot{h} = -0.5$  and  $s_1 = 10000$ , it is observed that the non-dimensional

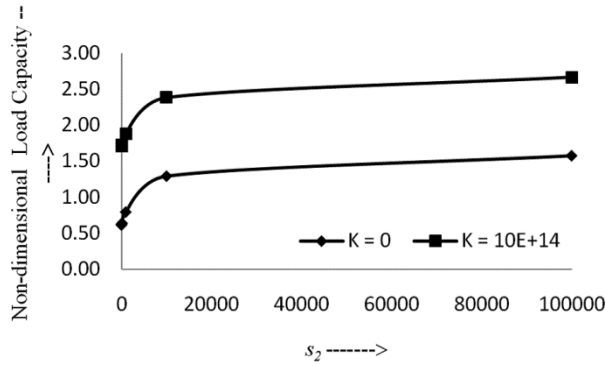


Fig. 2 —  $\bar{W}$  for different values of  $s_2$  for  $\dot{h} = -0.5$  (m/s) and  $s_1 = 10000$

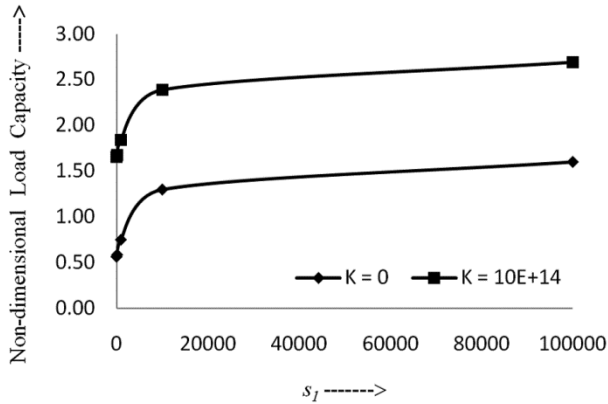


Fig. 3 —  $\bar{W}$  for different values of  $s_1$  for  $\dot{h} = -0.5$  (m / s) and  $s_2 = 10000$

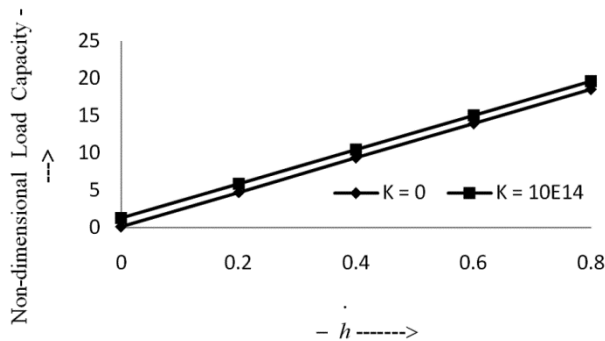


Fig. 4 —  $\bar{W}$  for different values of  $\dot{h}$  for fixed values of  $s_1 = 10000$  and  $s_2 = 10000$

load is increased as  $s_2$  increases for the upper porous layer. Also, the better non-dimensional load is attained in presence of magnetic field effect as compare to without magnetic field effect.

From Fig. 3, similar graphs can be seen for the fixed values of  $\dot{h} = -0.5$  and  $s_2 = 10000$  when one increase  $s_1$  for the upper porous layer.

It is seen from Fig. 4, the non-dimensional load increases as  $\dot{h}$  increases; here negative sign represents the upper surface approach to lower one. Also, the non-dimensional load is better obtained when using magnetic fluid as lubricant. (that means, for the value of  $K = 10^{14}$ ).

**List of symbol**

- A Length of bearing in x-direction (m)
  - B Width of bearing in y-direction (m)
  - $h_1$  Outlet film thickness (m)
  - $h_2$  Inlet film thickness (m)
  - $h$  Film thickness defined in Eq. (1)
  - $\dot{h}$  Squeeze velocity (m/s)
  - H** External magnetic field vector
  - H Magnetic field strength
  - K Quantity defined in Eq. (4)
  - $l_1$  Uniform thickness of upper porous layer (m)
  - $l_2$  Uniform thickness of lower porous layer (m)
  - M** Magnetization vector
  - P Fluid pressure in both the porous region (N / m<sup>2</sup>)
  - p Fluid pressure in film region (N / m<sup>2</sup>)
  - q Fluid velocity vector
  - $s_1, s_2$  Slip parameters for lower and upper porous regions respectively (1 / m)
  - t Time (s)
  - U Slider velocity (m/s)
  - $u, v, w$  Fluid velocity components of q in the directions of x, y, z directions respectively
  - $u_1, w_1$  Components of fluid velocity in x, z- directions in upper porous region
  - $u_2, w_2$  Components of fluid velocity in x, z -directions in lower porous region
  - W Load-carrying capacity (N)
  - $x, y, z$  Cartesian coordinates
- Greek symbols**
- $\eta$  Viscosity of fluid (N s / m<sup>2</sup>)
  - $\mu_0$  Free space permeability
  - $\bar{\mu}$  Magnetic susceptibility
  - $\mu^*$  Dimensionless magnetization parameter
  - $\rho$  Fluid density (N s<sup>2</sup> / m<sup>4</sup>)
  - $\phi$  Angle (rad)
  - $\alpha_x, \alpha_z$  Permeabilities in lower porous matrix in x and z -directions, respectively(m<sup>2</sup>)
  - $\beta_x, \beta_z$  Permeabilities in upper porous matrix in x and z -directions, respectively(m<sup>2</sup>)
  - $\eta_x$  Porosity of the lower porous region in x- direction
  - $m_x$  Porosity of the upper porous region in x- direction

Table 1 — Values of  $\bar{W}$  for various values of K

| K →               | 0       | 10 <sup>12</sup> | 10 <sup>13</sup> | 10 <sup>14</sup> | 10 <sup>15</sup> | 10 <sup>16</sup> |                                |
|-------------------|---------|------------------|------------------|------------------|------------------|------------------|--------------------------------|
| $\dot{h} = 0.0$   | 0.00000 | 0.01091          | 0.10912          | 1.09127          | 10.9127          | 109.1270         | $s_1 = 0.0$<br>$s_2 = 0.0$     |
| $\dot{h} = -0.05$ | 1.29540 | 1.30631          | 1.4045           | 2.38670          | 12.20810         | 110.4224         | $s_1 = 10000$<br>$s_2 = 10000$ |

From Table 1, it is seen that better non-dimensional load is obtained in presence of squeeze velocity as well as in the presence of slip parameters. It means that for better non-dimensional load carrying capacity, the assorted porous layers are active at both ends of inside the rubbing surfaces. Also, from the table it is clear that the non-dimensional load carrying capacity has suddenly increased for  $K = 10^{13}$  to  $10^{16}$ .

### Conclusions

This study concludes that when water based magnetic fluid is used as lubricant under oblique magnetic field to the lower surface for the porous slider pad surface bearing, the better load carrying capacity is obtained in presence of squeeze velocity and slip velocity. Hence, it is suggested to design water based magnetic fluid lubricated inclined pad surface porous bearing with slip velocity as well as squeeze velocity for better performance considering the suitable value of K.

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