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Power Harmonic Analysis Based on Continuously Adjustable Asymmetric Window

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The study of harmonics signal analysis in electric power system is a classic and important research subject. The signal by using traditional Fourier transform methods are nearly truncated by constant coefficient symmetry window or asymmetric window. This paper propose the improved continuously adjustable asymmetric window for precise harmonic parameters measurement and the uniform formulas for calculating the parameters of harmonics and interharmonic is obtained by the asymmetric window-based phase difference correction method. The major advantages of this method is easy to implement and independent of window spectrum. The simulation analysis results proves that there is obvious high precision, effectiveness and universal applicability of this method.

Keywords: Asymmetric window, Fast Fourier transform, Harmonic signal, Interpolated algorithm, Phase difference

Introduction

Harmonic and interharmonic pollution in electric systems caused by non-linear loads, which may lead to serious problems, for instance, reduce transformer and motor service life, affect the quality of the power grid, power quality, leads to energy loss and so on.[1–3] Therefore, harmonic analysis is significant for electric system.

Fast Fourier Transform (FFT) have become an indispensable method to estimate parameters of signal with high precision. However, it is widely recognized that the defect of FFT is under the asynchronous sampling[4], so FFT cannot be used to harmonic analysis directly due to large estimation error.

Now, a lot of methods have been proposed to improve estimation accuracy, the Windowed Interpolation DFT (WIpDFT) method based on rectangular window is proposed by Jain *et al*.[4] which could effectively reduce the errors caused by leakage effects and picket fence effects. However, the performance of WIpDFT is highly dependent on the type of windows. On further researches, many classical window functions have been proposed, such as FFDN window[5], 5-RV(I) window[6], Rectangle windows[7], triangular self-convolution window[8] etc. This improved WIpDFT by using the dichotomy approach algorithm for Rectangle, Hanning, Blackman and Blackman-Harris window.[9] Meanwhile, according to the number of maximum amplitude, spectral lines requirement in interpolation, WIpDFT can also be divided into: two-point interpolation and three-point interpolation.[10,11] However, obviously, there are many drawbacks of WIpDFT, including no analytical formulas for estimating the parameters of signal as well as poor applicability. Although there is analytical formulas of the WIpDFT based on MSDW[12,13], the applicability of this method is not satisfactory.

Besides, the Phase Difference Correction (PDC) method is another powerful tool for signal processing.[14] It is first proposed by McMahon[15], hitherto, time shifting and translation of window center are two main traditional PDC methods. It is worth noting that these two methods are based on symmetric window. The asymmetric window application in PDC method[16], and this improved method provides high estimation accuracy and the good applicability that makes it the ideal choice for harmonic analysis.

In this paper, the parameters' estimation by using PDC method based on continuously adjustable asymmetric window is proposed. Firstly, we briefly discuss three traditional methods to construct asymmetric window and then introduce the novel continuous adjustable asymmetric window. Secondly,

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we present the uniform formulas for estimating the parameters of harmonic signal, and discuss how to obtain accurate harmonic frequency by numerical analysis. Finally, the improved algorithm is validated by some simulation.

Materials and Methods

Asymmetric Windows

The symmetric windows is widely used in harmonic analysis, there has been a decrease in leakage effect by windowing the signal, so the accuracy of harmonic signal parameter estimation is directly related to appropriate window in practice. However, by using symmetric windows, it brings along shortcomings like the frequency response limitations and longer time delay. Asymmetric windows can be obtained by removing the symmetry of classic windows, there is merits of this sort of windows that symmetric window does not obtain. Hitherto, three different methods, including convolution, composed and truncated method have been presented to construct the asymmetric windows in literature.[17]

Convolution Method

According to the convolution theorem, the method of obtaining asymmetric window $w_a(t)$ by convolution can be expressed as:

$$w_a(t) = w_s(t) * r(t) \qquad \ldots(1)$$

where, $w_s(t)$ is the symmetric window. By choosing $r(t)$, appropriate asymmetric window can be obtained.

Composite Function Method

Asymmetric window can also be obtained through composite function and it can be expressed as:

$$w_a(t) = w_s[g(t)] \qquad \text{...}(2)$$

where, $g(t)$ is the intermediate variable, then the different asymmetric window can be obtained by different $g(t)$. It is worth noting that, $g(t)$ should be a monotone function in t[0,1].

Truncate Method

The substance of constructing asymmetric window by truncating is to multiply the symmetric windows by a linear equation $l(t)$, which can effectively remove the symmetry constraint of symmetric windows. The most effective way of truncating is using a straight line crossing the origin $l(t) = \gamma t$ as the function. The asymmetric windows can be expressed as:

$$w_a(t) = w_s(t) \cdot l(t) \qquad \text{...}(3)$$

where, γ denotes the slope. In other words, the linear equation $l(t)$ is also considered the weighting function, the symmetric window $w_s(t)$ is weighted by $l(t)$. Before further discussion, we first discuss drawbacks of above three traditional methods for constructing asymmetric window. For convenience, functions ($g(t)$, $r(t)$, $l(t)$) are called type function (TF). From the above analysis, we can see that in the process of constructing asymmetric window, choosing the reasonable TF is crucial step. The characteristics of asymmetric window (i.e., width of main-lobe, side-lobe decaying rate, amplitude and phase characteristic) is closely related to TF, that is to say, the characteristics of asymmetric window is confirmed when the TF is designated. Obviously, the characteristics of the asymmetric window cannot be continuously adjusted in this case.

For example, the characteristics of the asymmetric window by truncating are irrelevant to the slope value γ due to the linear property of Fourier Transform. The characteristics of ideal asymmetric window should be continuously adjusted by the parameter just like Kaiser window or Dolph-Chebyshev window.

Continuous Adjustable Asymmetric Window

As discussed above, we can see that truncate method is flexible and effective way to construct asymmetric window. The way we construct continuous adjustable asymmetric window is inspired by truncate method. The asymmetric Tukey window (a = 1) by truncate method with Eq. (3) shown in Fig. 1(c), Fig. 1(a) shows symmetric window $w_s(t)$ and Fig. 1(b) shows TF $l(t) = 2n+5$. The asymmetric window (Fig. 1d) exhibit a slightly bigger main-lobe and good frequency responses.

Without loss of generality, straight line $l_c(t) = \gamma t+\varepsilon$ ($l_c(t)>0$) is used in asymmetric window construct as a type function, then

$$w_a(t) = w_s(t) \cdot (\gamma t + \varepsilon) = \gamma w_s(t) \cdot \left(t + \varepsilon/\gamma\right) \qquad \ldots(4)$$

where, γ and ε denote slope and intercept of $l_c(t)$, respectively, setting $\varepsilon/\gamma=\tau$, (4) can be rewrote as:

$$w_{a-\tau}(t) = w_s(t) \cdot (\gamma t + \varepsilon) = \gamma w_s(t) \cdot (t + \tau) \qquad \ldots(5)$$

In Eq. (5), the asymmetric window with different τ give different characteristics due to property of FT, this means that a series of asymmetric windows with

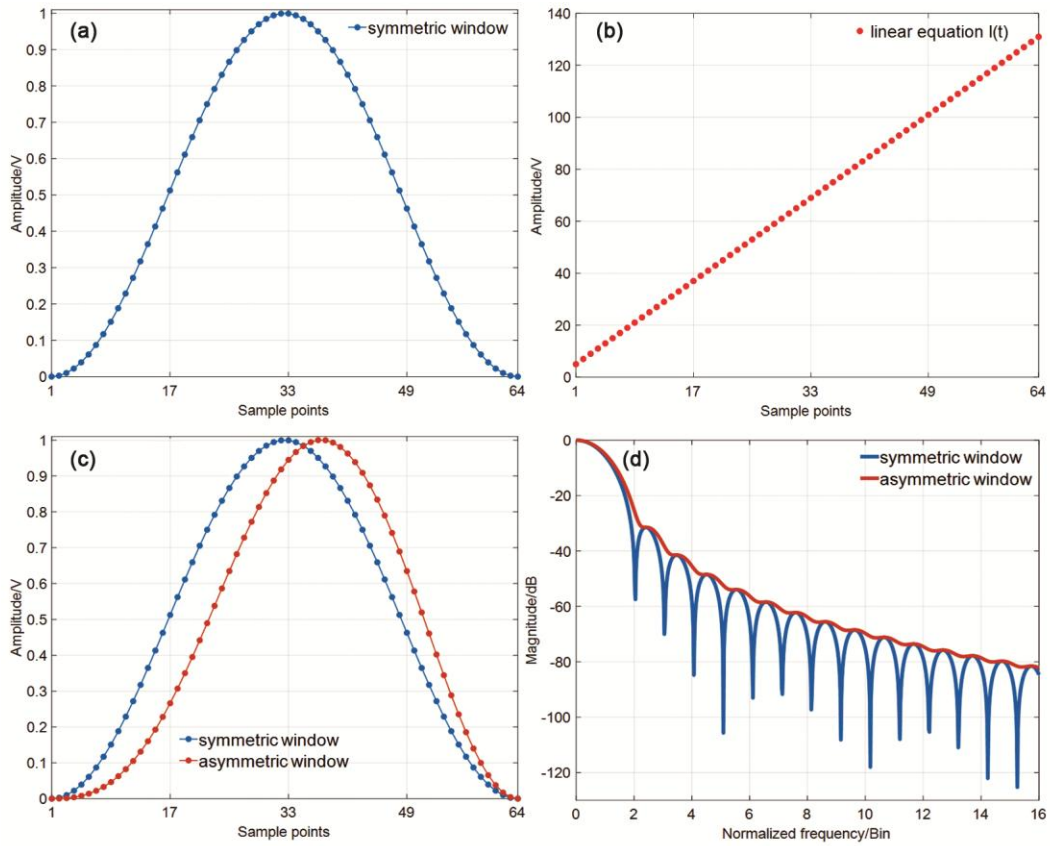

Fig. 1 — (a) Tukey window, (b) linear equation l (t) , (c)asymmetric Tukey window and (d) Magnitude frequency responses

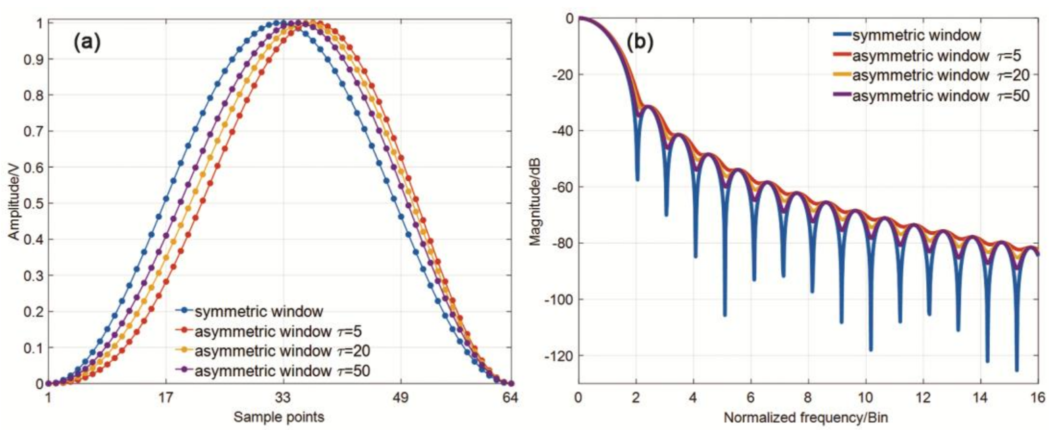

Fig. 2 — (a) Tukey window and asymmetric Tukey window(τ>0), (b) Magnitude frequency responses

different characteristics can be obtained by changing the value of τ. It is just like Kaiser window or Dolph-Chebyshev window. This continuous adjustable asymmetric window is represented by $w_{a\text{-}\tau}$. The asymmetric Blackman window with τ greater than 0 (τ = 5, τ = 20 and τ = 50) and τ less than 0 ($\gamma < 0$) ($\tau = -70$, $\tau = -90$ and $\tau = -160$) are shown in Figs 2 & 3, respectively.

According to above Figs 2 & 3, firstly, the performance of asymmetric window is strongly linked to coefficient τ, and the asymmetric window transform into symmetric window when $|\tau|\rightarrow\infty$. Secondly, the side-lobe decaying rate and the width of main-lobe can be simply adjusted by τ. In summary, the characteristics of asymmetric window can be easily and flexibly adjusted according to the changed

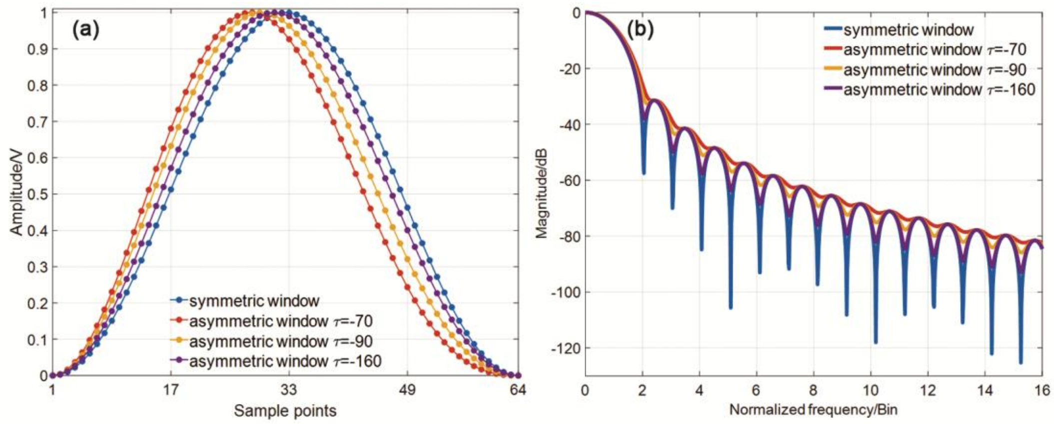

Fig. 3 — (a) Tukey window and asymmetric Tukey window (τ<0), (b) Magnitude frequency responses

τ. It is obvious that this method is very flexible and easy to implement.

Method for Harmonics and Interharmonics Analysis

Consider the Mth discrete sequence where, $y(n)$ is obtained by sampling frequency f_s from harmonic signal$y$ (t)

$$y_l(m)=\sum_{m=1}^{M} A_m \cos(2\pi n\, f_m/f_s+\theta_m) \qquad \ldots(6)$$

where, n is the sample index. Then, windowed harmonic signal $y_{m\text{-}w}(n)$ is obtained by symmetric window $w_s(n)$ of length N. Regardless the negative frequency part, the discrete Fourier transform (DFT) of $y_{m\text{-}w}(n)$ is:

$$Y_{m\text{-}w}(k)=\sum_{m=1}^{M}\frac{A_m}{2}W_s(k-k_m)e^{j[\theta_m-\xi(k-k_m)]} \qquad \ldots(7)$$

where, $W_s(k)e^{-j\xi(k)}$ is the Discrete Time Fourier Transform (DTFT) of symmetric window $w_s(n)$, k is the normalized frequency with the frequency resolution Δf ($\Delta f = f_s/N$), and then, the accurate hth harmonic frequency $f_m = k_m\Delta f$.

The multi-frequency harmonic signal is weighted by symmetric window and asymmetric window, respectively. The phase difference $\Delta\phi_m(k)$ is:

$$\Delta\phi_m(k)=\phi_{sm}(k)-\phi_{am}(k)=(\theta_m-\xi_s(k-k_m))- \\ -(\theta_h-\xi_a(k-k_m))=\xi_a(k-k_m)-\xi_s(k-k_m) \qquad \ldots(8)$$

where, $\phi_{sm}(k)$ and $\phi_{am}(k)$ are the phase of symmetric window and itself asymmetric window of spectrum line k_m. According to the conclusions as drew in[16] and the normalized phase frequency response, the solution of equation $\Delta\phi_m(k) = 0$ is the normalized theoretical frequency k_m ($\Delta\phi(k_m) = 0$), then the precise frequency of the hth harmonic is given by $f_m = k_m\Delta f$. Therefore, equation solving is the core and critical step in harmonic frequency estimation. However, it is very hard to obtain exact solution from the equation $\Delta\phi_m(k) = 0$ because there is no analytic expression.

Many methods are effective for solving nonlinear equation like $\Delta\phi_m(k) = 0$, secant method, Steffensen's method and Aitken iteration method are good choice to solve equation $\Delta\phi_m(k) = 0$. These methods don't require to calculate derivative and offer fast convergence speed. In general, simulation result indicate that it only needs 3 to 4 iterations to make the phase difference less than 10^{-5}($\Delta\varphi_m(k)<10^{-5}$) by these three methods. After k_m is solved out, the parameters f_m, A_m, and θ_m of harmonic can be worked out by following formulas:

$$f_m = k_m\cdot(f_s/N)=k_m\cdot\Delta f\ , \qquad \ldots(9)$$

$$A_m = R\cdot\sqrt{Y_{mI}^2(k_m)+Y_{mR}^2(k_m)}\,, \qquad \ldots(10)$$

$$\theta_m = \arg[Y_m(k_m)], \qquad \ldots(11)$$

where, R is resetting modulus for windowing spectrum.

Results and Discussion

Simulation Calculation and Result

In this section, some simulations are performed to illustrate the method described in above section. The simulation results were compared with two other windowed interpolation algorithms. The considered algorithms are: windowed interpolation algorithm based on FFDN window and 5-RV(I) window, respectively.

Harmonic Analysis

The following expression of the complicated harmonic signal is analyzed and the parameters of signal is the same as that in Wen *et al*.[18]

$$Y(t)=\sum_{m=1}^{21} A_m \cos\left(2\pi f_m t+\theta_m\right), \quad \ldots(12)$$

The signal is composed of 21 orders of harmonic and the frequency of fundamental harmonic f_1=50.2 Hz, the length of data is 1024 and the sampling frequency f_s = 2500 Hz. Error curves of frequency, amplitude and phase are displayed in Fig. 4. A-FFDN denotes asymmetric FFDN window, A-5-RV(I) is the same.

As it shows in Fig. 4, the precision achieved by using the proposed method based on the A-FFDN and A-5-RV(I) are superior to corresponding symmetric window by windowed interpolation algorithm in most cases. It is worthwhile to note that the precision of harmonic parameters estimation based on A-5-RV(I) are greatest because it give low peak sidelobe levels and high sidelobe attenuation rate, which can reduce the spectral leakage effectively to improve the accuracy of harmonic analysis.

Interharmonic Analysis

Interharmonic is a kind of the harmonic, unlike integer harmonic, its frequency is non-integer multiples of fundamental frequency. It obtains general characteristics of the harmonic, and also harmful to passive filter and electric motor. Therefore, it is significant to measure interharmonic exactly. For comparison to validate the proposed algorithm, the parameters of harmonics/iterharmonic are as shown[6] in Table 1. The signal contains five orders of interharmonics (1, 4, 6, 8 and 11). The sampling parameters are f_s = 1250 Hz and N = 1024. The results are shown in Fig. 5.

From the simulation results, the parameters estimation based on the proposed algorithm with asymmetric window shows better accuracy than the windowed interpolation based on FFDN and 5-RV(I), especially for interharmonic components, which shows satisfactory outcome for interharmonic analysis. Then, A-5-RV(I) exhibits higher accuracy same as simulation 4.1.

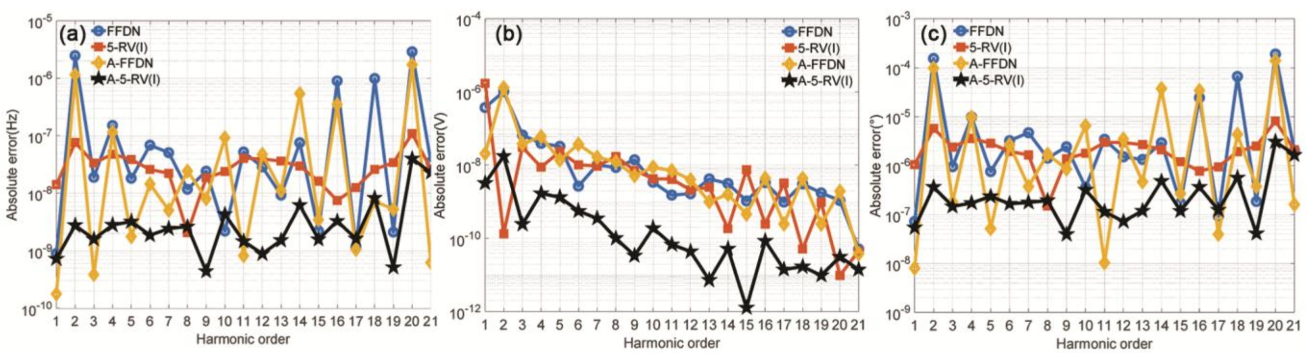

Fig. 4 — Frequency error curves of harmonic signal: (a) Frequency, (b) Amplitude, (c) Phase

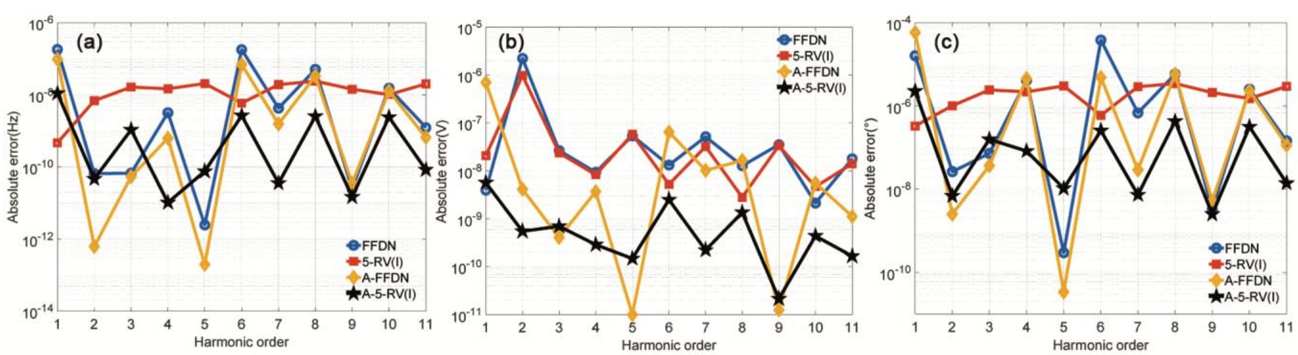

Fig. 5 — Error curves of interharmonic signal: (a) Frequency, (b) Amplitude, (c) Phase

Table 1 — Interharmonic signal

h	1	2	3	4	5	6	7	8	9	10	11
f_h(Hz)	25	50	150	175	250	330	350	380	450	500	530
A_h(V)	2.28	380	19	1.9	15.2	1.52	11.4	1.14	7.6	0.76	3.8
θ_h(°)	20	10	25	30	100	120	150	180	210	240	270

Application to Other Windows

Many classic windows are widely employed in harmonic analysis. However, because the polynomial approximation is required, this limits the windowed interpolation application in harmonic analysis. There are uniform formulas of proposed algorithm, which is fully different from that of conventional windowed interpolation algorithms.

Simulation 4.3 is present to appraise applicability of the proposed algorithm with five classic windows: Kaiser (β = 11), Chebyshev (α = 6.5), Hanning, Tukey (α = 0.75) and Blackman window. The coefficients of window (α, β) are mentioned in Harris.[19] The sampling parameters and the parameters of signal are same as simulation of interharmonic analysis. The estimation results are shown in Fig. 6.

From Fig. 6, we can clearly see: firstly, frequency, phase and amplitude of signal can be estimated satisfactorily with these windows. Secondly, the parameters precision of 1th interharmonic obtained by asymmetric Chebyshev window based on proposed algorithm is highest because of its good properties to suppress the fundamental spectrum leakage effects on adjacent weak components. Thirdly, proposed algorithm based on Bohman window has gained the highest accuracy in most cases attribute to its good side-lobe performance. In general, the proposed algorithm has more general extensive applicability than interpolated algorithms for harmonic analysis.

Simulation with Signal Containing White Noise

The anti-noise capability of the proposed algorithm is demonstrated in this simulation. The interharmonic signal in simulation 2 is superposed with Gauss white noise (μ = 0). The root mean squared error (RMSE) of 10000 times for each SNR (10 – 100 dB) of fundamental frequency are depicted in Fig. 7.

As illustrated in Fig. 7, the RMSE decreases as SNR increases, and the effect of random white noise is significant when SNR < 40. Then, FFDN[5] and A-FFDN demonstrate almost same performance under the condition of white noise for amplitude. In general, the proposed algorithm show better resistance against noise especially for frequency and phase.

Study of Fundamental Frequency Variations Influence

In this simulation, we consider the situation of fundamental frequency variations. In harmonic analysis, the precision of estimation can be affected by fundamental frequency variation. In harmonic signal model (formula 12) ,the fundamental frequency varies from 49.4 Hz to 50.4 Hz in step of 0.1 Hz, then, the results are shown in Figs 8&9.

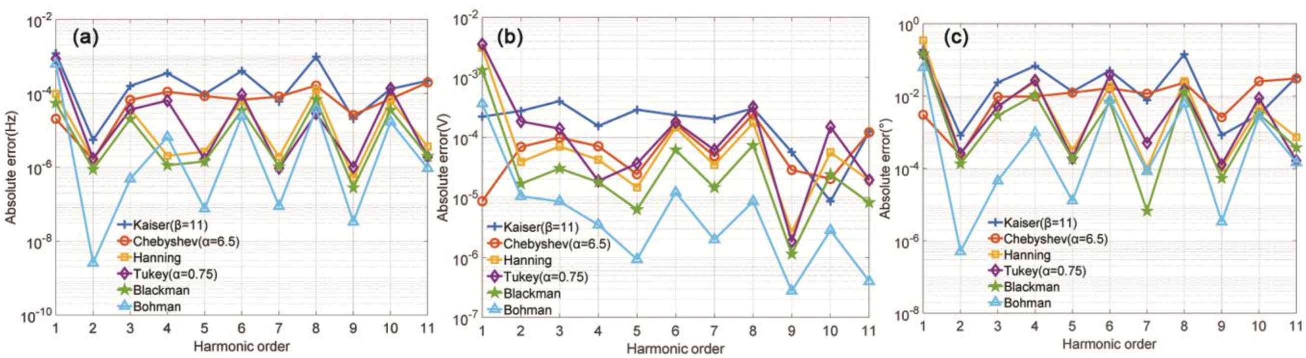

Fig. 6 — Error curves of interharmonic signal with other windows: (a) Frequency, (b) Amplitude, (c) Phase

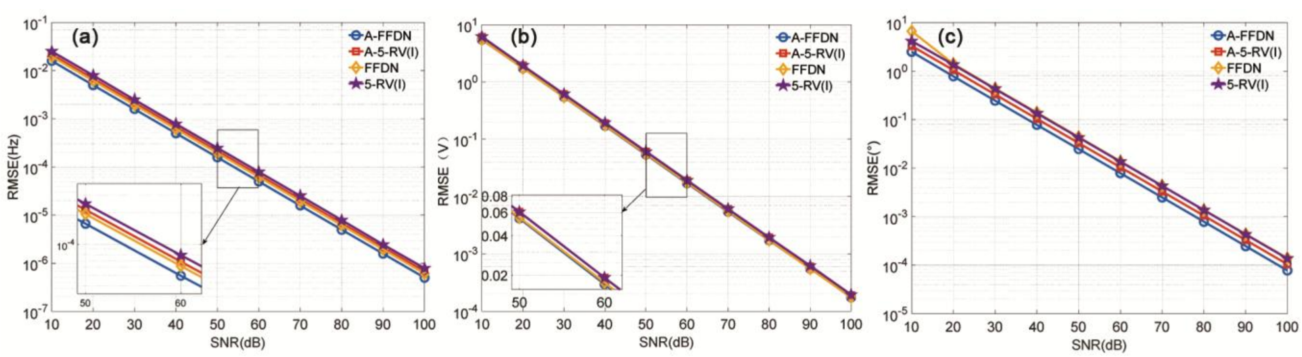

Fig. 7 — RMSE of the fundamental frequency : (a) Frequency, (b) Amplitude, (c) Phase

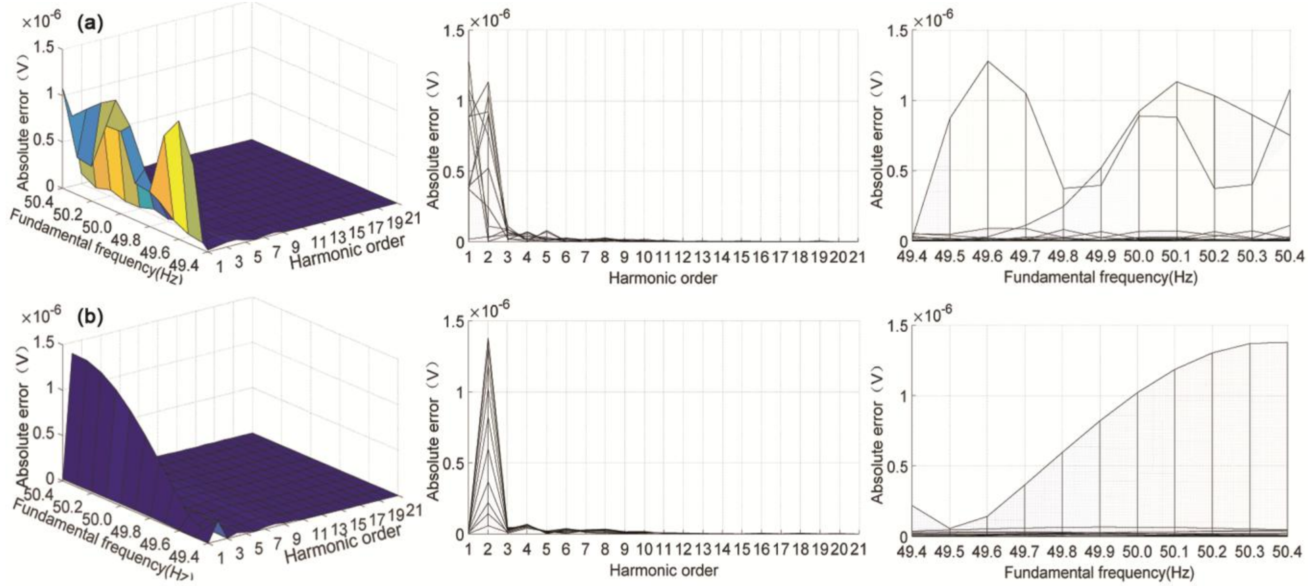

Fig. 8 — Amplitude absolute errors of the proposed method at fundamental frequency fluctuation: (a) FFDN, (b) A-FFDN

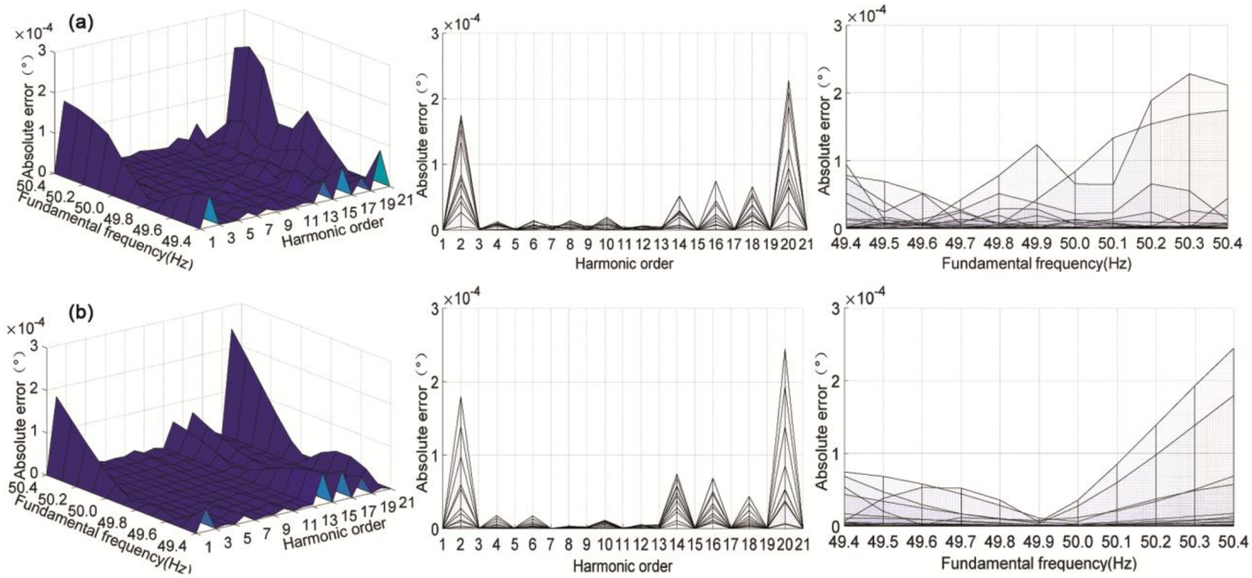

Fig. 9 — Phase absolute errors of the proposed method at fundamental frequency fluctuation: (a) FFDN, (b) A-FFDN

The simulations illustrate that, despite the FFDN[5] has a good effect on suppressing the fundamental frequency fluctuation, the proposed method could be a more preferable option as it provides better precision than FFDN. In addition, both amplitude and phase have shown large error for weak harmonic components (harmonic order = 2, 14, 16, 18, 20) this trend is particularly apparent for phase errors. This is mainly because the amplitude of this weak harmonic component is very small, and the adjacent harmonic is with relatively large amplitude. The weak signal is masked by the leakage of adjacent strong signal.

Study of τ Influence

In this section, different τ for A-5-RV(I), Gauss window (α = 7.5) and six-term fifth-order cosine window[20] was used for simulations to investigate the effect of the proposed adjustable asymmetric window. The number of iterations is 3 and $\Delta\varphi_h(k) = 10^{-5}$. From Fig. 10, it can be seen that harmonic frequency can be estimated satisfactorily by adjusting the value of τ, and when τ = 5, the proposed algorithm based on this three windows shows highest accuracy in most case. Thus, in actual engineering applications, satisfactory accuracy of frequency estimation can be obtained by adjusting the value of τ.

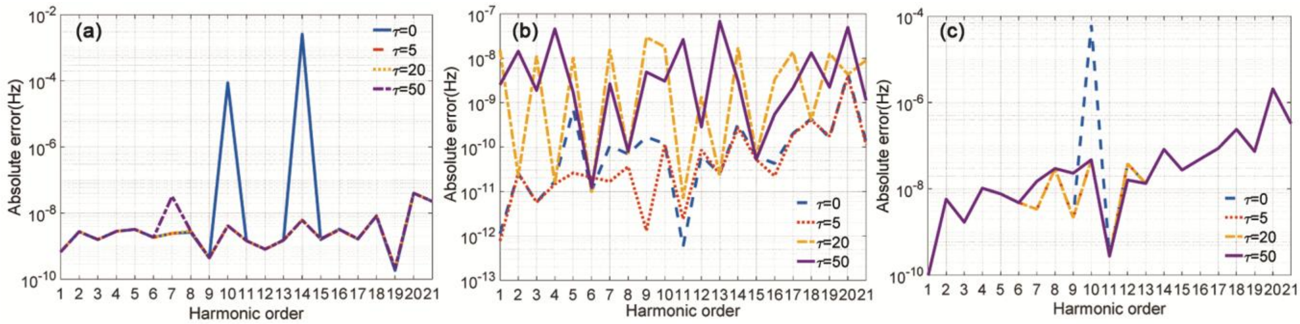

Fig. 10 — Frequency error curves for different τ: (a) A-5-RV(I), (b) Gauss (α = 7.5), (c) Six-term fifth-order cosine window

Conclusions

In this paper, the new continuous adjustable asymmetric window for parameters measurement in electric system based on PDC method is raised. Simulation results reveal that this proposed method has high precision, good anti-noise property and a wide range of applicability, which can be used to evaluate the harmonic and interharmonic in practical electric system. In the last, the proposed algorithm is slightly more complex calculations than traditional algorithm due to reduplicative iterative operation, in the future, we should seek method to optimize the computational burden.

References

1 Sujatha S & Anita R, Power quality enhancement by using SVC in real time power system, *J Sci Ind Res*, **74** (2015) 466–470.

2 Karunambigai S, Geetha K & Shabeer H A, Power quality improvement of grid connected solar system, *J Sci Ind Res*, **74** (2015) 354–357.

3 Duda K, Zieliński T P, Bień A & Barczentewicz S H, Harmonic phasor estimation with flat-top FIR filter, *IEEE Trans Instrum Meas*, **69(5)** (2020) 2039–2047.

4 Jain V K, Collins W L & Davis D C, High-accuracy analog measurements via interpolated FFT, *IEEE Trans Instrum Meas*, **28(2)** (1979) 113–122.

5 Zeng B, Teng Z S & Cai Y L, Harmonic phasor analysis based on improved FFT algorithm, *IEEE Trans Smart Grid*, **2(1)** (2011) 51–59.

6 Qian H, Zhao R X & Chen T, Interharmonics analysis based on interpolating windowed FFT algorithm, *IEEE Trans Power Deliv*, **22(2)** (2007) 1064–1069.

7 Wang K, Wen H & Li G Q, Accurate frequency estimation by using three-point interpolated discrete fourier transform based on rectangular window, *IEEE Trans Ind Informat*, **17(1)** (2021) 73–81.

8 Wen H, Zhang J, Meng Z, Guo S, Li F & Yang Y, Harmonic estimation using symmetrical interpolation FFT based on triangular self-convolution window, *IEEE Trans Ind Informat*, **11(1)** (2015)16–26.

9 Wu X G & Wang A N, Harmonic signal processing method based on the windowing interpolated DFT algorithm, *J Inf Sci Eng*, **31(3)** (2015) 787–798.

10 Fan L & Qi G Q, Frequency estimator of sinusoid based on interpolation of three DFT spectral lines, *Signal Process*, **144** (2018) 52–60.

11 Fan L, Qi G Q, Xing J, Jin J Y, Liu J Y & Wang Z S, Accurate frequency estimator of sinusoid based on Interpolation of FFT and DTFT, *IEEE Access*, **8** (2020) 44373–44380.

12 Belega D, Petri D & Dallet D, Impact of harmonics on the interpolated DFT frequency estimator, *Mech Syst Signal Process*, **66-67** (2016) 349–360.

13 Belega D & Dallet D, Multifrequency signal analysis by Interpolated DFT method with maximum sidelobe decay windows, *Measurement*, **42(3)** (2009) 420–426.

14 Wen H, Teng Z S, Wang Y & Hu X G, Spectral correction approach based on desirable sidelobe window for harmonic analysis of industrial, *IEEE Trans Ind Electron*, **60(3)** (2013) 1001–1010.

15 McMahon D R A & Barrett R F, An efficient method for the estimation of the frequency of a single tone in noise from the phases of discrete Fourier transforms, *Signal Process*, **11(2)** (1986) 169–177.

16 Luo J F & Xie M, Phase difference methods based on asymmetric windows, *Mech Syst Signal Process*, **54-55** (2015) 52–67.

17 Florencio D A, On the use of asymmetric windows for reducing the time delay in real-time spectral analysis, International Conference on Acoustics, Speech, and Signal Processing, (IEEE, Toronto, Ontario, Canada), (1991), 3261–3264.

18 Wen H, Teng Z S, Wang Y, Zeng B & Hu X G, Simple interpolated FFT algorithm based on minimize sidelobe windows for power-harmonic analysis, *IEEE Trans Power Electron*, **26(9)** (2011) 2570–2579.

19 Harris F J, On the use of windows for harmonic analysis with the discrete Fourier transform, *Proc IEEE*, **66(1)** (1978) 51–83.

20 Chen Z H, Wang L D, Wang C, Shen P & Li Z Z, An Approach for electrical harmonic analysis based on optimized composite cosine window four-spectrum-line interpolation FFT, *Power System Technology*, **44(3)** (2020) 1105–1113.