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ARTICLE

Hands-on exploration of Tessellation and Fractal Mapping on a Tiling Wall to enhance interest in doing Maths in non-formal settings

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ABSTRACT

Aiming to achieve the objectives of the National Innovation Council in a learner-centric manner, the Birla Industrial & Technological Museum has been striving to incorporate interactive learning tools to reinforce its science communication wing. In its latest attempt, the impetus generated by using a Tiling Wall – both to teach the mathematics of recursive algorithm, fractal generation and tessellation along with a means of learning about the learners are explored. It is a case study highlighting the use of a Tiling Wall to illustrate mathematical algorithms like Fractal Generation and Tessellation, along with being a means of studying the participant's confidence parameter. The lessons are described in detail and so are the analyzed feedbacks received from the learners as well as their parents. The article interprets the purport of employing a Tiling Wall to foster the teaching-learning cycle.

KEYWORDS: Mathematics, Fractal, Learner Observation, Feedback Analysis, Science Communication

Introduction

Mathematicians know what mathematics is but have difficulty saying it. Yet, most agree it to be 'a precise conceptual apparatus', or better still, 'the thing that scientific ideas, as they grow toward perfection, become'[1]. As Science, Technology and Innovation continue to emerge as the major drivers of national development globally; the National Innovation Council — the *Rashtriya Avishkar Abhiyan* (RAA) — a convergent framework of the Ministry of Human Resource Development, Government of India — aims to leverage the potential for science, mathematics and technology learning in non-classroom settings, while emphasizing the primacy of the schools and classroom transactions.

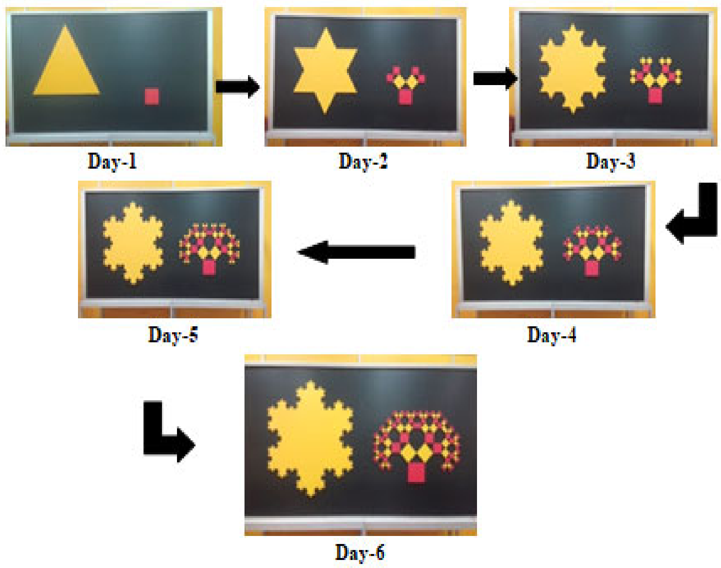


Figure 1: Progression of Tiling on subsequent Camp days

Functioning as the educational outreach wing of the National Council of Science Museums — a National level Nodal Institution associated with the *Rashtriya Avishkar Abhiyan* — the Birla Industrial & Technological Museum (BITM) introduced an out-of-school Mathematics programme for secondary-level school students. With an ambitious gamut of learning outcomes – ranging from the historical aspects of mathematics, their developments along the course of time, culminating with their modern applications and future prospects – it was imperative to assess how the programme was being taken by the participants themselves, who would explore strategies, reasoning, and aesthetic criteria for tessellations for the first time.

While the conventional Feedback Form was a lucrative option, it was deemed fit to be a second-stage validator of efficacy. An 8 feet by 4 feet landscape oriented, vertically mounted board on which students are given to generate fractal structures using recursive shape elements, emerged as a strong primary indicator and facilitator of effectiveness of such teaching-learning practices. What it was used for and how it acted as a 'crystal goblet' – meant to reveal more than it hides – is the subject matter of this communication.

Session Goals

While designing this courseware for middle-school students, we stressed on the following:

- **Games for Mathematicians:** Math games used to maintain previously mastered mathematical concepts and skills and promote computational fluency – assuring 'engagement' with every child.
- **Using What We Know:** Problem solving or challenge activities to draw upon mathematical understanding and skills – assisting 'modelling' and 'self-learning'.
- **Independent Math Work:** Materials used to teach previously mastered content incorporated into workstation tasks – fostering the culture of 'beyond text books' and 'inquiry-based learning'.
- **Developing Fluency:** Tasks that help students develop number sense and mental math skills – making use of 'peer-to-peer collaborative learning' for 'problem-solving'.
- **Expressing Mathematical Ideas:** Mathematical vocabulary and communication are the focus – supplemented with 'experiment & demonstration' and 'hands-on activity-based' learning.

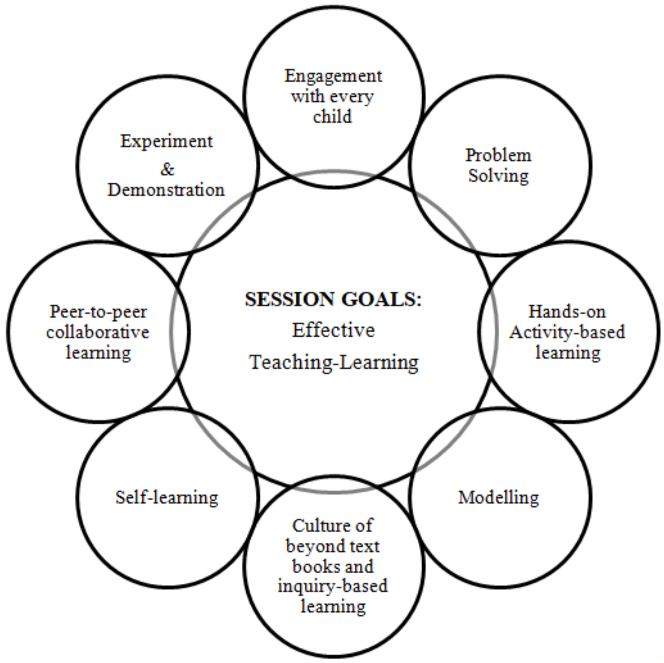


Figure 2: Mathematics Workshop Session Goals at BITM

Children's aesthetics, including sense of symmetry, influence their choices in creating tessellations and are known to have played a cognitive role in children‘s mathematical exploration of tessellation structures. As observed by Robert Scott Eberle in *Children's Mathematical Understandings of Tessellations: A Cognitive and Aesthetic Synthesis*, "The study of tessellations is a rich topic that connects many geometric concepts. Tessellations are appropriate for study in some form for all ages, from kindergarten through college. Between third grade and eighth grade, tessellations connect with much or even most of the geometry curriculum, as well as with other areas of mathematics."

In line with our session goals and relying on the sweet spot of inquiry – problems that are not easy enough to be trivial and not so hard that they're impossible to study – fractal and tessellation problems were chosen as focus points for the current Workshop.

Our Methodology

Most sessions we offer are designed to fill a standard two-hour lesson but can be made slightly shorter or longer if necessary. Sessions were delivered as group presentation and hands-on workshop for the ideal class groups. What we aspired to achieve in these lessons were the five components relating to the learner's outcomes and dispositions: facts, skills, fluency, curiosity and creativity – juxtaposed with the five methodologies of mathematics: algorithm, conjecture, generalization, isomorphism and proof.

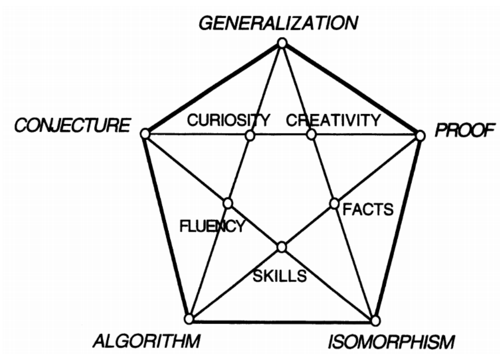


Figure 3: Components of learner's outcomes and dispositions

Supportive of the process of asking and answering productive questions, lessons and learning activities need to be designed to encourage the growth of a creative attitude and a learning environment. Providing a blank 8 feet by 4 feet landscape oriented, vertically mounted board hardly seemed to address this cardinal issue in Mathematics education. Teaching the learners the generating algorithm of recursive fractal Koch Snowflakes, Pythagoras Trees – either symmetric and asymmetric ones or aperiodic tessellation patterns of Penrose Tiling, along with the chance to modify them or stick to the plan and cover-up the board much like a graffiti wall – may also not ring a bell somewhere. But by observing the groups interact amongst themselves – teach and learn from one another, miss instructions, or read too much into them, filter in the shy ones, spot out the overly enthusiastic ones – the Tiling Wall was a treasure trove of tales for anyone willing to listen.

Yet, even with the best of our resources and experiences, the 5-fold cycle of outcomes and methodologies might cage in some young learner. We may, inadvertently assume too much and close the lines of enquiry. By simply providing them a tiling surface, some mathematical algorithms to replicate, pre-cut foam blocks and a whole lot of opportunity to utilize or exploit – we truly set them free. For, as Ron Casey, in an article 'A Key Concepts Model for Teaching and Learning Mathematics' had wondered, "…caged birds do sing, but what would be their song if they were sometimes allowed to fly"?

Figure 4: Learners seen generating a fractal from a square on a Tiling Wall

ACTIVITY 1: KOCH SNOWFLAKE

Figure 5: The finished Koch Snowflake

What we say…

In the Koch Snowflake, an infinite perimeter encloses a finite area. The perimeter of the Koch Snowflake gets bigger and bigger with each step. But what about the area? Imagine drawing a circle around the original figure. No matter how large the perimeter gets, the area of the figure remains inside the circle.

Remember the process:

1. Divide a side of the triangle into three equal parts and remove the middle section.
2. Replace the missing section with two pieces the same length as the section you removed.
3. Do this to all three sides of the triangle.

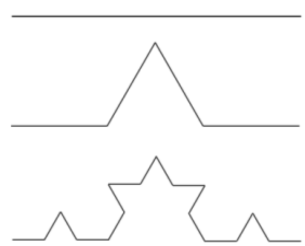


Figure 6: Koch Snowflake algorithm

What we do…

We keep pre-cut triangular foam-board pieces, ready to be put up by the learners, on the Tiling wall. The largest triangular piece is put up for them. They measure its length and work out the dimension of each succeeding triangular piece to be added to complete the Koch Snowflake. On each successive day of the Workshop, the learners need to subject this triangle to the given algorithm and convert it into the Koch Snowflake, noticing that each iteration reduces the dimension of extra triangle being added, increases their quantity and increases the perimeter of the figure, without exceeding the boundaries of the overall shape. When the learners are comfortable with placements of elements in each stage, ideas of iteration and convergence to finite area bounded by infinite element curve are easily introduced. Zeno's paradox is discussed in this context.

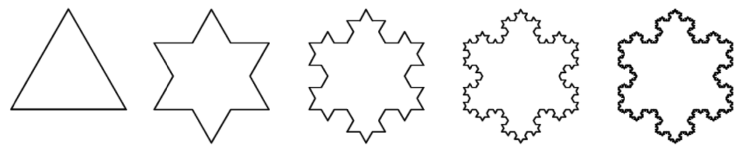


Figure 7: The stages of Koch Snowflake generation

What we observe…

We had fabricated equilateral triangular foam blocks with following dimensions:

Table 1: Details of pieces installed for Koch Snowflake activity

Side length	No. of pieces	Methodology
81 cm	1	Put up on Day-1
27 cm	3	Put up on Day-2 by students divided into 3 groups
9 cm	12	Put up on Day-3 by students divided into 6 groups
3 cm	48	Put up on Day-4 by each student individually
1 cm	192	Put up on Day-5, each student gets 4 pieces each

With each passing day, as the groups get to know each other better, the foam board pieces at their disposal get smaller but increase in quantity, the learners get more comfortable portraying the iterations. Initially, they are unsure about the size or positioning of the triangles, but once they figure out that by placing three similar pieces on one edge of the triangle and taking out the two extremes, they leave the middle triangle placed exactly in the position we want to, they repeat the algorithm with more joy than apprehension now. The shy ones, no longer able to hide behind groups as the activity shifts from being group-oriented to individual-oriented, grow more confident replicating the fractal formation with elan. That they can repeat the iterations intuitively Day-3 onwards is an added advantage.

Activity 2: Pythagoras Tree

Figure 8: Two finished Pythagoras Trees, skewed on two different edges

What we say…

The Pythagorean Tree – be it symmetric or asymmetric as shown below – is an application of the Pythagoras Theorem. The construction shows the geometric proof of the Pythagorean Theorem, that the sum of the areas of the squares along the two sides of a right triangle is equal to the area of the square along the hypotenuse.

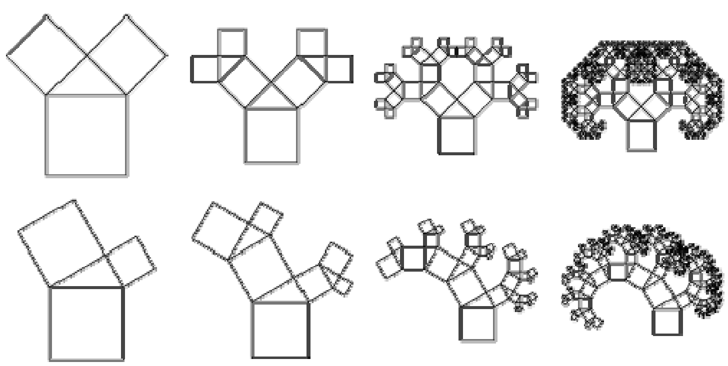


Figure 9: The stages of symmetric and asymmetric Pythagoras Tree generation

Remember the process:

1. Start with a square.
2. Choose and place two squares on top such that their corners match up pair-wise, thus forming a right-angled triangle amongst themselves.
3. Repeat step-2 for each "branch" of the tree.

All branches and leafs emerge from, and are dependent, on a starting root – the base square. The root plus the same movement repeated over and over again creates the tree.

What we do…

As before, we keep pre-cut square foam-board pieces, ready to be put up by the learners, on the Tiling wall. The base square piece is put up for them. They measure its length and work out the dimension of each succeeding square piece to be added to complete the Pythagoras Tree. The learners need to subject this triangle to the given algorithm and convert it into the Pythagoras Tree. On some instances they create the symmetric tree, while they experiment with the asymmetric one in others.

What we observe…

For the symmetric Pythagoras Tree, we had fabricated square foam blocks with following dimensions:

Table 2: Details of pieces installed for symmetric Pythagoras Tree activity

Side length	No. of pieces	Methodology
16.0 cm	1	Put up on Day-1
11.3 cm	2	Put up on Day-2 by students divided into 6 groups
8.0 cm	4	
5.6 cm	8	Put up on Day-3 by all students individually
4.0 cm	16	
2.8 cm	32	Put up on Day-4 by all students individually
2.0 cm	64	Put up on Day-5, each student gets 2 pieces each

For the asymmetric Pythagoras Tree, biased on two different sides, we had fabricated two sets of square foam blocks with following dimensions:

Table 3: Details of pieces installed for asymmetric Pythagoras Tree activity

Side length	No. of pieces	Methodology
3.2 cm	1	Put up on Day-1
4.3 cm	4	Put up by all participants on Project Day individually. The square pieces are distributed randomly. Learners measure out their pieces. They have the successive dimensions worked out and accordingly go to the Tiling wall, as per their block size, to fix it in the correct position and orientation.
5.4 cm	1	
5.8 cm	6	
7.2 cm	3	
7.7 cm	4	
9.0 cm	1	
9.6 cm	3	
10.2 cm	1	
12.0 cm	2	
12.8 cm	1	
15.0 cm	1	
16.0 cm	1	
20.0 cm	1	
25.0 cm	1	

For fabricating the symmetric Pythagoras Tree, the participants soon realize that it would take two matching pieces to join into a right angle and sit on top of the base square to form successive iterations. This leads to the perks of group participation - it starts up conversations, breaks ice during the initial stages and fosters camaraderie in subsequent ones. The happiness of self-correcting and reaching to a conclusion beyond instruction kept them forging ahead.

In case of the asymmetric Pythagoras Tree, the extended complexity of figuring out which square goes where – the larger square resting in the direction of skew of the Tree – generates more interesting conversations. Very soon the learners figure out the pattern and enthusiastically begin predicting the positions for their friend's blocks too. The more immaculate ones measure both the length and breadth of their squares, take their average, and only then decide the exact dimensions of their allotted square block. This scares the happy-go-lucky ones. They come up with something more fool-proof. They go about matching their blocks with others – in case of a perfect match, they politely ask for the dimension without taking the trouble to measure it themselves. Either way, the Tree does reach its full bloom owing to the sharp observations, skill and commitment of all the enthusiastic learners.

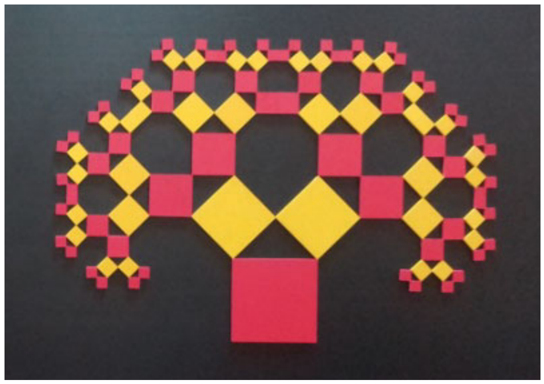


Figure 10: The finished symmetric Pythagoras Tree

Activity 3: Penrose Tiling

Figure 11: The unfinished Penrose Tiling

What we say…

Moving along the set pattern, fill in the remaining spaces with black kites and red darts – mind the white markings on them – make sure they line up.

Step back and see if you can find matching shapes.

With one placing rule, tiles dance around their five axes, creating starbursts and decagons, winding curves, butterflies and flowers – but repetition never comes. Shapes recur, yet new variations keep creeping in.

No matter how much information you have, how much you've seen of the tiling − you'll never be able to predict what happens next. We call them aperiodic tiling of the Penrose tiles.

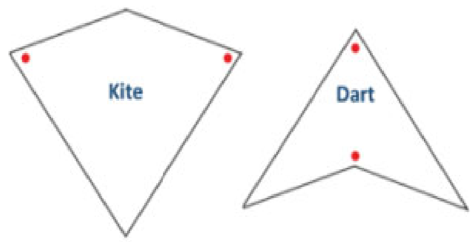


Figure 12: 'Kite' and 'Dart' shapes

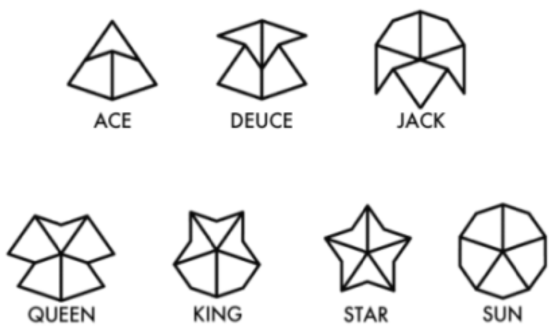


Figure 13: Allowed combinations

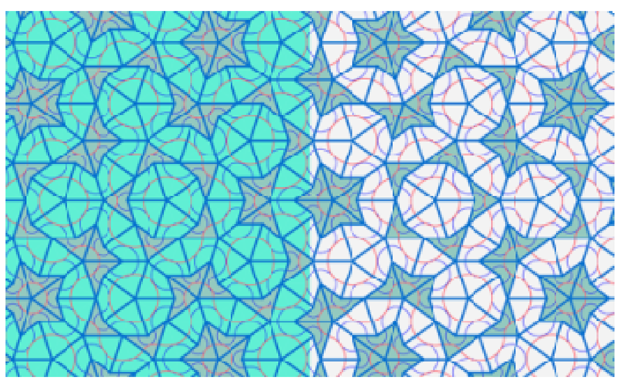


Figure 14: A classic Penrose Tiling example, picture courtesy Dominique Fung

What we do…

We prefabricate about 150 ‘kites’ and 90 ‘darts’, using the following mechanism:

1. Start with a rhombus of angles 72 and 108 degrees
2. Divide the long diagonal in the golden ratio of $(1+\sqrt{5})/2=1.61803398\ldots$
3. Join the point to the obtuse corners
4. We have a non-flying ‘kite’ and an arrow-head ‘dart’

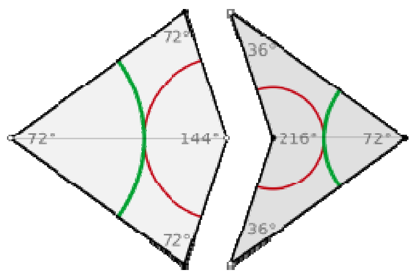


Figure 15: Construction of ‘Kite’ and ‘Dart’ shapes

We ask the learners to arrange the kites and darts – obeying the rule that vertices of same colour sit next to each other. The 7 ‘allowed’ combinations are shown as examples, alongside the Tiling pattern of squares, triangles, pentagons and hexagons. Learners are encouraged to extend on a given Penrose Tiling or try their hand at an independent tessellation. Either way, they are asked to compare and contrast their observations for the periodic tiling pattern of polygons, as compared to the aperiodic Tiling of the Penrose pieces.

What we observe…

While waiting for their turn to try out the Tiling pattern on the wall, learners play about with the Penrose pieces and observe that a ‘kite’ comprises two smaller kites and two halves of a ‘dart’. A ‘dart’ comprises a smaller ‘kite’ and two halves of a ‘dart’. Upon completion, a Penrose tiling patch must therefore be a repetitive fractal of itself. Some squeaked to realize that regular pentagons do not cover the entire available area, but the Penrose tiles, with the same 5-fold geometry, cover up any given area. Of course there were the day-dreamers who kept on playing to their fancies, without caring for the rules of the Penrose tiling. Yet, the fact that no matter how much information you have, how much you've seen of the tiling − you'll never be able to predict what happens next − the true essence of aperiodic tiling was the clear message received by all.

Truths from the Tales

This time at BITM, we made it a point to listen to the silent messages that the students using the Tiling Wall sent our way – more than 100 of them, on six opportune occasions. We even gave them a voice. Over the course of a year, we collected responses from 138 learners, aged between 10 to 16 years – both pre-camp and post-camp – covering their expectations vis-s-vis their takeaways from the Mathematics Camp. In the attempt to know the learners closely and correlate the feedbacks received, we even asked 115 parents about the habits and fancies of their wards. Feedbacks were in the form of an illustrated questionnaire – to be answered on Day 1, before the beginning of Camp, another one on the concluding day of Camp and a questionnaire for their parents.

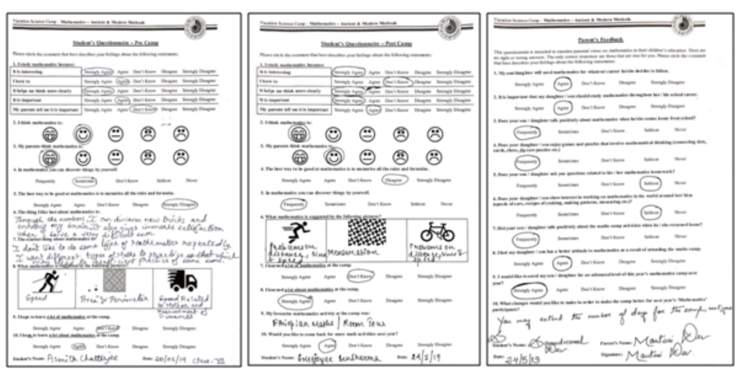


Figure 16: Submitted Pre-Camp, Post-Camp and Parent Feedback Forms from camp participants

<u>Student's Questionnaire – Pre Camp</u>

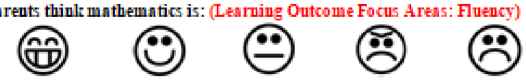
Please circle the comment that best describes your feelings about the following statements:

1. I study mathematics because: (Learning Outcome Focus Areas: Facts, Skills, Curiosity)

It is interesting	Strongly Agree	Agree	Don't Know	Disagree	Strongly Disagree
I have to	Strongly Agree	Agree	Don't Know	Disagree	Strongly Disagree
It helps me think more clearly	Strongly Agree	Agree	Don't Know	Disagree	Strongly Disagree
It is important	Strongly Agree	Agree	Don't Know	Disagree	Strongly Disagree
My parents tell me it is important	Strongly Agree	Agree	Don't Know	Disagree	Strongly Disagree

2. I think mathematics is: (Learning Outcome Focus Areas: Skills)

3. My parents think mathematics is: (Learning Outcome Focus Areas: Fluency)



4. In mathematics you can discover things by yourself. (Learning Outcome Focus Areas: Creativity)

Frequently Sometimes Don't Know Seldom Never

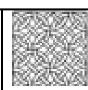
5. The best way to be good at mathematics is to memorise all the rules and formulas. (Learning Outcome Focus Areas: Facts)

Strongly Agree Agree Don't Know Disagree Strongly Disagree

6. The thing I like best about mathematics is: (Learning Outcome Focus Areas: Skills)

7. The scariest thing about mathematics is: (Learning Outcome Focus Areas: Fluency)

8. What mathematics is suggested by the following pictures? (Learning Outcome Focus Areas: Creativity)

9. I hope to learn <u>a lot of mathematics</u> at the camp. (Learning Outcome Focus Areas: Facts)

Strongly Agree Agree Don't Know Disagree Strongly Disagree

10. I hope to learn <u>a lot about mathematics</u> at the camp. (Learning Outcome Focus Areas: Curiosity)

Strongly Agree Agree Don't Know Disagree Strongly Disagree

Student's Name: **Date:**

Student's Questionnaire – Post Camp

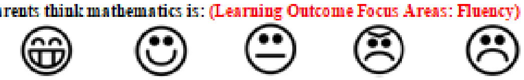
Please circle the comment that best describes your feelings about the following statements:

1. **I study mathematics because:** (Learning Outcome Focus Areas: Facts, Skills, Curiosity)

It is interesting	Strongly Agree	Agree	Don't Know	Disagree	Strongly Disagree
I have to	Strongly Agree	Agree	Don't Know	Disagree	Strongly Disagree
It helps me think more clearly	Strongly Agree	Agree	Don't Know	Disagree	Strongly Disagree
It is important	Strongly Agree	Agree	Don't Know	Disagree	Strongly Disagree
My parents tell me it is important	Strongly Agree	Agree	Don't Know	Disagree	Strongly Disagree

2. **I think mathematics is:** (Learning Outcome Focus Areas: Skills)

3. **My parents think mathematics is:** (Learning Outcome Focus Areas: Fluency)



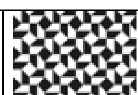
4. **The best way to be good at mathematics is to memorise all the rules and formulas.** Learning Outcome Focus Areas: Facts)

Strongly Agree Agree Don't Know Disagree Strongly Disagree

5. **In mathematics you can discover things by yourself.** (Learning Outcome Focus Areas: Creativity)

Frequently Sometimes Don't Know Seldom Never

6. **What mathematics is suggested by the following pictures?** (Learning Outcome Focus Areas: Creativity)



7. **I learned a lot of mathematics at the camp.** (Learning Outcome Focus Areas: Facts)

Strongly Agree Agree Don't Know Disagree Strongly Disagree

8. **I learned a lot about mathematics at the camp.** (Learning Outcome Focus Areas: Curiosity)

Strongly Agree Agree Don't Know Disagree Strongly Disagree

9. **My favourite mathematics activity at the camp was:** (Learning Outcome Focus Areas: Fluency)

10. **Would you like to come back for more math activities next year?** (Learning Outcome Focus Areas: Curiosity)

Strongly Agree Agree Don't Know Disagree Strongly Disagree

Student's Name: **Date:**

Parent's Feedback

This questionnaire is intended to examine parental views on mathematics in their children's education. There are no right or wrong answers. The only correct responses are those that are true for you. Please circle the comment that best describes your feelings about the following statements:

1. My son/daughter will need mathematics for whatever career he/she decides to follow.

Strongly Agree Agree Don't Know Disagree Strongly Disagree

2. It is important that my daughter / son should study mathematics throughout her / his school career.

Strongly Agree Agree Don't Know Disagree Strongly Disagree

3. Does your son / daughter talk positively about mathematics when he/she comes home from school?
(Learning Outcome Focus Areas: Fluency)

Frequently Sometimes Don't Know Seldom Never

4. Does your daughter / son enjoy games and puzzles that involve mathematical thinking (connecting dots, cards, chess, jig-saw puzzles etc.) (Learning Outcome Focus Areas: Skills, Creativity)

Frequently Sometimes Don't Know Seldom Never

5. Does your son / daughter ask you questions related to his / her mathematics homework? (Learning Outcome Focus Areas: Facts)

Frequently Sometimes Don't Know Seldom Never

6. Does your daughter / son show interest in working on mathematics in the world around her/ him (speeds of cars, recipes of cooking, making patterns, measuring etc.)? (Learning Outcome Focus Areas: Curiosity)

Frequently Sometimes Don't Know Seldom Never

7. Did your son / daughter talk positively about the maths camp activities when he / she returned home?

Frequently Sometimes Don't Know Seldom Never

8. I feel my daughter / son has a better attitude to mathematics as a result of attending the maths camp.

Strongly Agree Agree Don't Know Disagree Strongly Disagree

9. I would like to enrol my son / daughter for an advanced level of this year's mathematics camp next year?

Strongly Agree Agree Don't Know Disagree Strongly Disagree

10. What changes would you like to make in order to make the camp better for next year's 'Mathematics' participants?

Student's Name: **Parent's Name:**
Date: **Signature:**

Analysis

Chart 1: Parent's Feedback to "It is important that my ward should study mathematics throughout their school career"

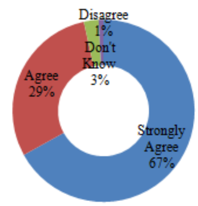
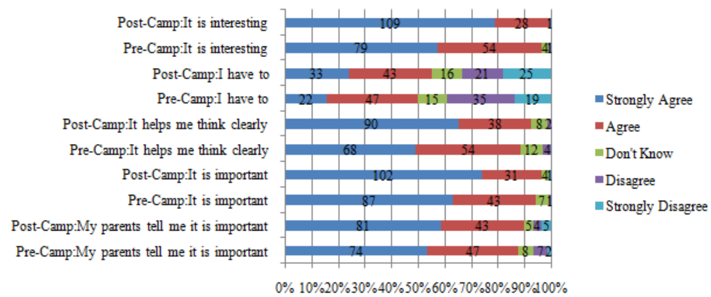


Chart 2: Student's Response to "I study mathematics because..."



- Pre-camp most students opted for traditional answers '*It is important*' (87 of 138) and '*It helps me think clearly*' (68 of 138). In contrast, post-camp, the fore-runner was the less conventional '*It is interesting*' (109 of 138) option. With 97% parents agreeing to the importance of their wards studying mathematics throughout their school career, getting them interested in it through camp activities was an achievement.

Chart 3: Parent's Feedback to "My ward will need mathematics for whatever career they decide to follow."

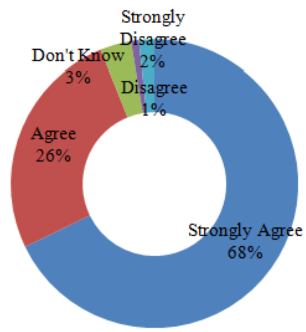
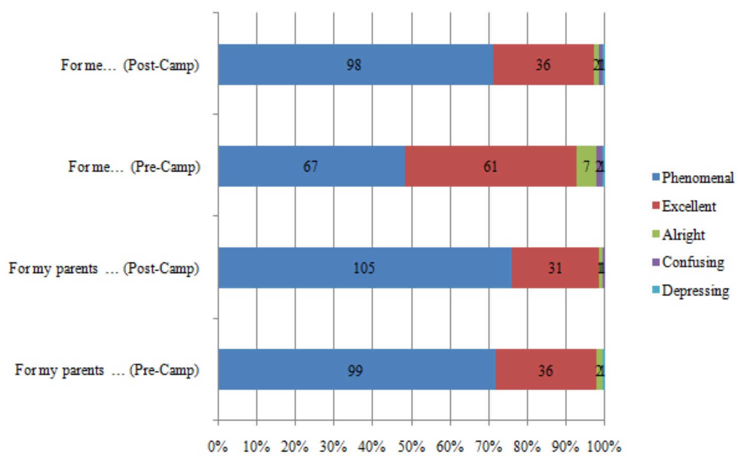
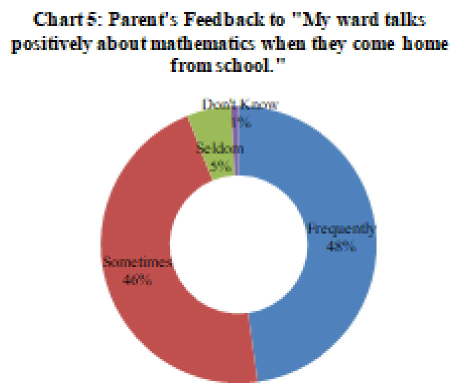
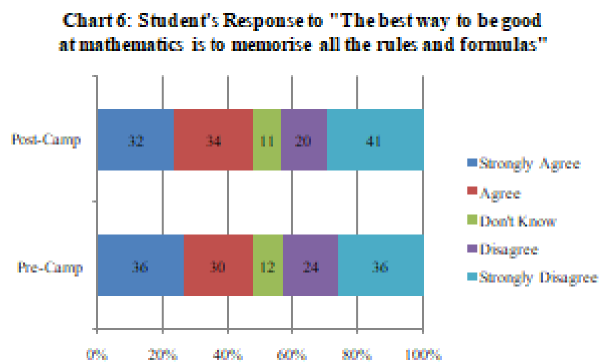


Chart 4: Student's Response to "Mathematics is..."



- With 94% of parents agreeing that mathematics will be needed by their wards for whatever career they choose to follow, along with the majority of children feeling that mathematics is important to their parents, the Camp achieved to get a significant number of more participants (31) to believe that mathematics is '*Phenomenal*'.





- While majority (48%) of the parents reported that their wards '*Frequently*' talked positively about mathematics when they came home from school, it was an achievement to make some more of them '*Strongly Disagree*' that the best way to be good at mathematics is to memorise all rules and formulas.

Chart 7: Parent's Feedback to "My ward enjoys games and puzzles that involve mathematical thinking."

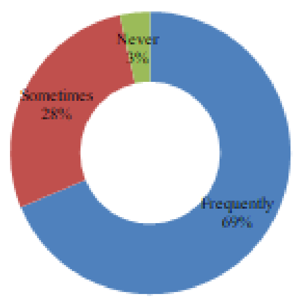
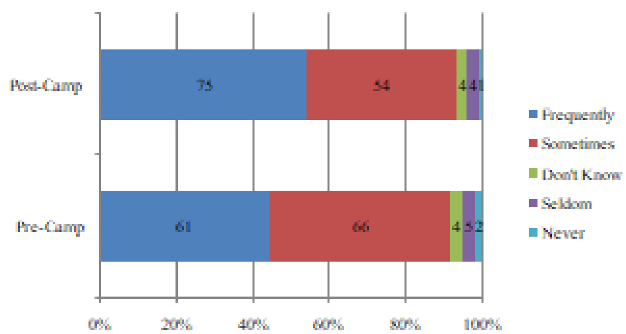


Chart 8: Student's Response to "In mathematics you can discover things by yourself"



- With 69% parents reporting that their wards already '*Frequently*' enjoyed games and puzzles that involved mathematical thinking, the Camp got some more of them to agree that in mathematics they can '*Frequently*' discover things by themselves.

Chart 9: Parent's Feedback to "My ward shows interest in working on real-world mathematics."

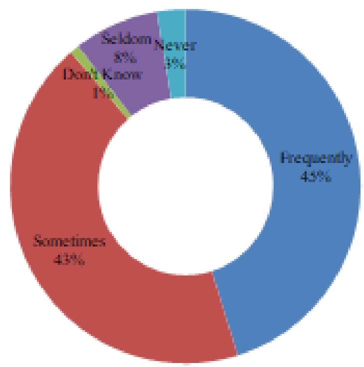
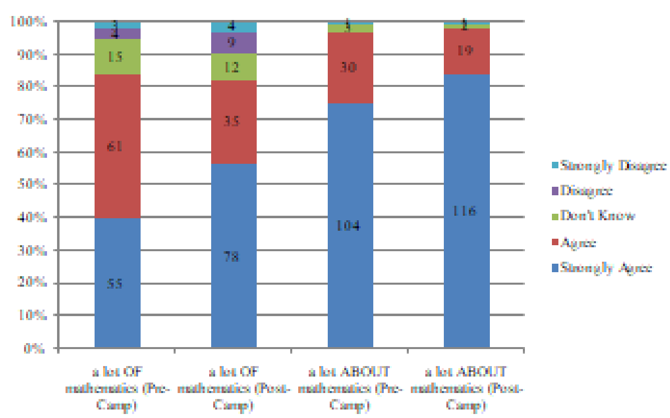


Chart 10: Student's Response to "I hope to learn ____ at the camp"



- With 88% parents admitting that the learners were intersted in working on real-world mathematics, it was clear that a significant number of participants (104 of 138) hoped to learn '*a lot ABOUT*' mathematics, as compared to the learners (55 of 138) who wanted '*a lot OF mathematics*' during the camp. Post camp analysis revealed that the activities had lived up to their expectations, as 116 of 138 learners believed thay had learnt '*a lot ABOUT*' mathematics, as compared to the 78 of 138 students who had learnt '*a lot OF mathematics*' during the Camp.

Chart 11: Parent's Feedback to "My ward talked positively about the math camp activities on returning home."

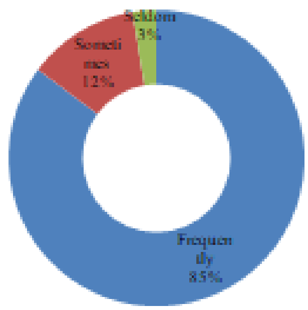


Chart 12: Parent's Feedback to "I feel my ward has a better attitude to maths as a result of attending the camp."

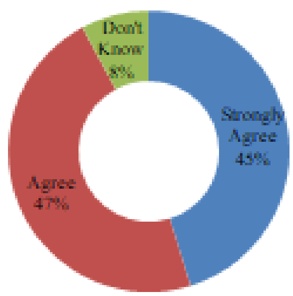
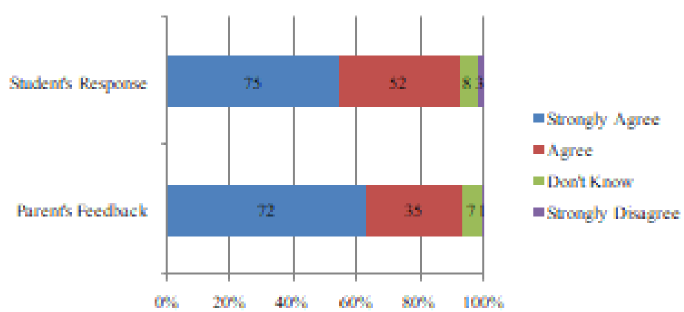


Chart 13: Parent's Feedback and Student's Response to "Would you come back for more math activities next year?"



- With 97% parents reporting that their wards talked positively about the math camp activities at home and 92% admitting that the learners had a better attitude to maths as a result of attending the Camp, it was clear that we had managed to understand the kids and deliver the lessons according to their liking. With 107 of the 115 parents agreeing to send back their wards for an advanced level of this year's mathematics camp, the current approach seems to work fine. 127 of the 138 students also reported to be happy to come back for a continuation of the activities.

Discussion – Achievements of the Tiling Wall

Supportive of small group working, flexible learning spaces with appropriate facilitator input the Tiling Wall at BITM helps achieve the following principles of learner-centered learning, as envisaged by Ian Rushton and Martyn Walker from the University of Huddersfield:

1. **The practitioner acts as a facilitator of learning rather than a teacher of knowledge:** After the initial instructions, replicating the algorithm of tiling always keeps the ball in the learner's court.
2. **Learner's prior knowledge and experiences are taken into account:** The student's power of observation sees him through the intricacies of the tessellation pattern.
3. **Learner's needs and learning preferences are identified:** Sorting out the day-dreamers from the hair-splitters helps identify specific attention seekers.
4. **Activities and resources are used to motivate and support:** With the freedom to experiment, as in the Penrose Tiling activity, the learners are encouraged to teach themselves.
5. **Learners are actively participating and reflecting in the learning process:** By thinking ahead, as in the Pythagoras Tree activity, the clarity and comprehension level of specific problems are identified.
6. **Learners are encouraged to become autonomous:** Answering the classic 'What's in it for me' question for most participants, the Tiling Wall helps them with blended learning.

7. **Learners are inspired to develop their own ideas and problem-solving skills:** With the gradual conversion of the same problem domain from group to individual activity, the interaction of learners accelerate, as does their analytical skills.
8. **Ensure formative, peer and self-assessments support learning:** By engaging in hands-on activities like the Koch Snowflake Tiling for example, complex concepts of perimeter and area, along with their interactions and applications are highlighted.
9. **Learners participate in planning, implementation and evaluation**: This they achieve in teaching and learning from each other, and suggesting ways and means to extend the problems – often in more creative ways than we could imagine.
10. **Learners develop key inclusivity skills:** From over-cautious to the flamboyant, attentive to care-free learners – the Tiling Wall gives everyone a reason to get up and explore the mathematical possibilities of the space – all we, as facilitators can do is listen to the silent messages the Wall and its users keep transmitting.

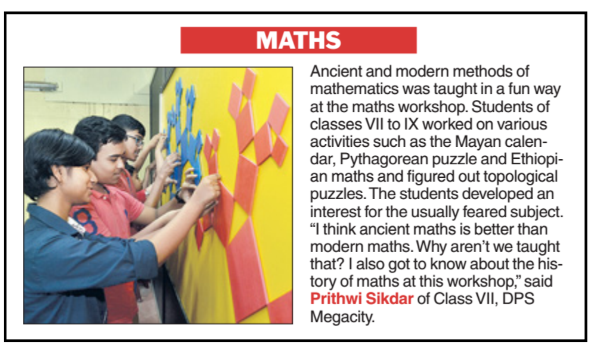
Figure 17: Participants creating complimentary Pythagoras Trees on a Tiling Wall

Conclusion

Dynamical systems are often used to describe real-world phenomena that move forward in time according to a repeated rule, like the ricocheting of a billiard ball in accordance with Newton's laws. You begin with a value, plug it into a function, and get an output that becomes your new input. Our experiment with the vertically mounted tiling surface was analogous to these dynamical problems. Akin to the insights provided by such systems, the analysis of our participant feedbacks clearly shows that we had 'listened' to the messages of the Tiling Wall, as had the learners.

Depending on the feedbacks received from the Tiling Wall activities, we had interacted more directly with the day-dreamers. The philanthropists among the lot – those who believed in helping their friends, when negotiating a particularly difficult knot perhaps – were readily roped in to help facilitate session explanations. A bigger and more complex stash of problems was maintained to keep the immaculate, the over-cautious and the mathematically inclined ones engaged. The shy ones were nudged to divulge their secret passions – music, arts, travel, literature to name a few – and the use of Mathematics in those fields were explored in subsequent activities.

The overall emphatic attitude of learners was then revalidated from the feedbacks received. In dynamic systems, you start with a value, apply the rules of the system or curve, and end up in a cycle. We too hope to merge our efforts with the rules of such systems and continue with this exercise of liberating the learner's voices -- in turn, end up learning more than we teach to our enthusiastic students.



MATHS

Ancient and modern methods of mathematics was taught in a fun way at the maths workshop. Students of classes VII to IX worked on various activities such as the Mayan calendar, Pythagorean puzzle and Ethiopian maths and figured out topological puzzles. The students developed an interest for the usually feared subject. "I think ancient maths is better than modern maths. Why aren't we taught that? I also got to know about the history of maths at this workshop," said **Prithwi Sikdar** of Class VII, DPS Megacity.

Figure 18: Press coverage of the camp activities in *Young Metro*, a supplement of *The Telegraph*, dated 26th June 2019

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